Risk Aversion and Stochastic Dominance: A Revealed Preference Approach

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Abstract

Theoretically, given a choice over two risky assets with equivalent expected returns, a risk averse expected utility maximizer should choose the second-order stochastically dominant asset. We develop a theoretical framework that allows for decision error, which should decrease in risk aversion. We conduct an experiment using a risk preference elicitation mechanism to identify risk averse individuals and evaluate the frequency that they choose the stochastically dominant of two lotteries. 75.76% of risk averse and 96.15% of very risk averse subjects chose at least 7 out of 10 dominant lotteries. Estimates of the effect of elicited risk aversion on the number of stochastically dominant lotteries chosen are positive and highly significant across specifications. The results suggest risk averse individuals make choices that satisfy stochastic dominance and violations are due, in large part, to decision error, which is decreasing in risk aversion.

Keywords: stochastic dominance, risk, uncertainty, experiments

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1 Introduction

This paper presents the results of an experiment intended to determine the frequency that risk averse individuals make choices that satisfy second-order stochastic dominance (SSD). Theoretically, given a choice over two risky assets with equivalent expected returns, a risk averse expected utility (EU) maximizer will choose the SSD asset. However, to our knowledge, there is no evidence that *risk averse* individuals actually satisfy SSD when making choices over risky assets. There is experimental evidence that individuals make choices that violate first-order stochastic dominance (FSD) (Birnbaum and Navarrete, 1998), as well as SSD (Levy and Levy, 2001).[1] Given the existing evidence, the validity of EU theory as a model of decision-making under uncertainty is questionable. However, before abandoning EU theory the validity of existing empirical evidence need be examined.

Since field data does not readily lend itself to a suitable investigation of the behavioral consistency of risk aversion and stochastic dominance, researchers have employed experimental methods as a means to an answer (Birnbaum and Navarrete, 1998; Levy and Levy, 2001). Despite the control that the laboratory offers, existing evidence of violations of stochastic dominance is confounded by three important factors. The first is the use of *hypothetical* questions to elicit preferences. Without satisfying the precept of induced value theory (Smith, 1982), there is no reason to believe that subjects are *truthfully* revealing their preferences. The second is the underlying risk preference of subjects has been omitted. Since satisfying SSD is only consistent with maximizing the expected utility of a *risk averse* agent, violations may simply be the result of non-risk averse preferences.[2] The third factor confounding existing evidence is decision error. A

[1]Birnbaum and Navarrete (1998) make it a point to note that their "results do not imply that people always violate stochastic dominance. Instead, they show that people systematically violate stochastic dominance in the special recipe..." The recipe entails taking a binary lottery and placing some of the probability from the lower outcome to a slightly better outcome to create a dominant lottery and placing some of the probability from the higher outcome to a slightly worse outcome to create a dominated lottery.

[2]In fact, Levy and Levy (2001) argue their results are an indication that most individuals are *not* risk

choice that violates SSD may not actually reveal the true underlying preference; it may reflect decision error (Smith and Walker, 1993). That is, an agent may not always choose the risky asset that provides the highest expected utility; they may make a mistake.[3] The use of hypothetical incentives compounds the effect of decision error (Blavatskyy, 2007; Harless and Camerer, 1994; Smith and Walker, 1993). It is important to distinguish whether an agent is revealing preferences that violate stochastic dominance as compared to providing hypothetical responses that do not truthfully reflect underlying preferences, revealing preferences that should not satisfy SSD (i.e. non-risk aversion), or the agent is simply making mistakes.

The main objective of this paper is to remove or control for the identified confounding effects and determine whether risk averse subjects make choices that satisfy SSD. First, by paying our subjects based on their decisions we increase the salience of the task (Smith and Walker, 1993). The hypothetical choices over fairly complicated gambles that are used in the previous studies (Birnbaum and Navarrete, 1998; Levy and Levy, 2001) weakens the argument that subjects incurred the cognitive costs to adequately evaluate the lotteries (Smith and Walker, 1993). Second, eliciting risk preference allows the analysis to be contingent upon the direction and degree of risk preference. Only risk averse individuals should satisfy SSD; violations of SSD by non-risk averse individuals are not inconsistent with theory. Third, adopting the contextual utility framework set forth by Wilcox (forthcoming), not only accounts for decision error, but predicts that it is decreasing in risk aversion. Several papers stress the importance of distinguishing decision error from revealed preference in order to appropriately test theoretical predictions (Ballinger and Wilcox, 1997; Blavatskyy, 2007; Hey, 1995; Hey and Orme, 1994; Harless and Camerer, 1994; Loomes et al., 2002; Smith and Walker, 1993; Wilcox, 1993).[4] Ex-

averse despite evidence that this is not the case (Andersen et al., 2006; Binswanger, 1980; Bruner et al., forthcoming; Eckel and Wilson, 2004; Goeree et al., 2003; Holt and Laury, 2002).

[3] Birnbaum and Navarrete (1998) acknowledge the possibility of decision error; They argue that random guessing will result in an error rate of 50%. Thus, the observed violations, which are in excess of 50%, cannot be attributed solely to decision error.

[4] Some may argue that decision error arises due to the small stakes in the laboratory (Levitt and List, 2007). While strong utility predicts decision error should decrease as the stakes increase, this is not the true under contextual utility (Wilcox, forthcoming). Strong utility implies the difference in the expected utilities between the two choices drives an agent's decision. Thus, scaling up the payoffs results

ploiting the heterogeneity of risk preferences in light of the contextual utility framework adds to the existing literature regarding decision cost and rewards (Smith and Walker, 1993; Wilcox, 1993, forthcoming) as well as establishing the behavioral consistency of risk aversion and stochastic dominance.

We present the results from an experiment where subjects made choices in three distinct formats of the risk preference elicitation mechanism. Two formats were designed to elicit risk preference by requiring a series of pairwise choices involving a tradeoff between risk and return; subjects revealed their risk preference through their pattern of choices. The third format was designed to determine the frequency risk averse subjects choose the SSD of two risky assets. In this format subjects faced a series of pairwise choices where the two lotteries had the equivalent expected returns but different levels of risk. The analysis required the use of a *within-subjects* design; subjects were presented with all three formats.

The results suggest that risk averse subjects make choices consistent with SSD.[5] The most frequently observed pattern of responses is 10 out of 10 times the SSD lottery is chosen. 75.76% of risk averse subjects and 96.15% of very risk averse subjects chose at least 7 out of 10 SSD lotteries. Furthermore, estimates of the effect of elicited risk aversion on the number of SSD lotteries chosen are positive and highly significant across specifications. The finding that the number of SSD lotteries chosen is increasing with risk aversion is consistent with the prediction that decision error is decreasing in risk aversion. The results from the experiment are consistent with subjects being noisy EU maximizers.

The paper is organized as follows. Section 2 presents the theoretical framework in order to formalize testable hypotheses. Section 3 describes the experimental design that

in scaling up the difference in expected utilities, which should reduce observed decision error. However, the contextual utility model of Wilcox (forthcoming) adopted in this paper assumes an agent divides the difference in expected utilities by the range of possible utility outcomes. Thus, scaling up the payoffs has no effect. There is reasons to believe the contextual utility model is the more appropriate specification for our data. The contextual utility framework explains the observed decrease in decision error associated with increased risk aversion, which can not be explained under strict utility. Thus, we disagree with the argument that decision error is the result of small stakes. We return to this point in the next section.

[5]97.2% of our subjects satisfy FSD in decisions intended to verify rationality.

is used to test the theoretical predictions. Section 4 presents the results from the experimental sessions. Section 5 summarizes the results and discusses their implications.

2 Theoretical Framework

The analysis considers the choice between two risky assets, $k = A$ or B. Each asset is defined by its return, R, which is assumed to be a Bernoulli random variable where $R = R_k$ with probability p_k and $R = 0$ with probability $1 - p_k$. The expected return from an asset is $E[R] = p_k R_k$. The following section will examine the properties of the distribution that are required to satisfy SSD. Then we will demonstrate that satisfying SSD is equivalent to maximizing the EU of a risk averse individual. The theoretical demonstration of the equivalence of risk averse and SSD is analogous to that presented in Hadar and Russell (1969) taking into consideration the particular distribution used in the experiment. We then take into account subjects' ability to make mistakes and demonstrate that decision error is decreasing in the magnitude of constant relative risk aversion. The theoretical framework that follows provides the necessary foundation to analyze the choices made in the experimental setting.

2.1 Stochastic Dominance of Bernoulli Random Variables

Consider how the rules of stochastic dominance apply to the returns on our two risky assets, R_A and R_B. Each return has a cumulative distribution function, $F_k(R)$, that is defined as follows:

$$F_k(R) = \begin{cases} 0 & \text{if } R < 0 \\ 1 - p_k & \text{if } 0 \leq R < R_k \\ 1 & \text{if } R \geq R_k \end{cases}$$

Following Hadar and Russell (1969) and Rothschild and Stiglitz (1970), R_A is said to second-order stochastic dominate R_B iff

$$\int_0^{R_A} [F_B(R) - F_A(R)]dR \geq 0 \qquad \text{for all R, with at least one strict inequality.}$$

Thus we may state the following:

Lemma 1 *For equivalent expected returns, asset A second-order stochastically dominates asset B when R_A is less than R_B when the returns are Bernoulli random variables.*

Proof: Using the definitions for $F_A(R)$ and $F_B(R)$, asset A SSD asset B iff

$$\int_0^{R_A}(1-p_B)dR > \int_0^{R_A}(1-p_A)dR.$$

Rewriting this condition,

$$(1-p_B)R_A > (1-p_A)R_A.$$

If $E[R_A] = E[R_B]$, then $p_A R_A = p_B R_B$. Substituting $p_A = p_B \frac{R_B}{R_A}$ into the condition above, asset A second-order stochastically dominates asset B when $R_A < R_B$. Q.E.D.

Figure 1 demonstrates the result from Lemma 1. The figure shows that when the high return from asset B, R_B, exceeds the high return from asset A, R_A, such that $E[R] = p_B R_B = p_A R_A$, the area under the cumulative density function from 0 to R_A for asset B exceeds that of asset A. Thus, in order for the two assets to have equivalent expected returns, it must be that asset B has more probability mass assigned to lower returns resulting in asset A being SSD.

2.2 Stochastic Dominance and Expected Utility

Agent i's EU from asset k is $EU_i = p_k U_i(R_k)$, where $U_i(R_k)$ is a monotonically increasing function of R_k.[6] If $U_i(R_k)$ is concave, $U_i''(R_k) < 0$, then agent i is risk averse; if $U_i(R_k)$ is convex, $U_i''(R_k) > 0$, then agent i is risk seeking; if $U_i(R_k)$ is linear, $U_i''(R_k) = 0$, then agent i is risk neutral. Thus, we may state the following:

[6] We assume the normalization $U_i(0) = 0$ throughout the analysis.

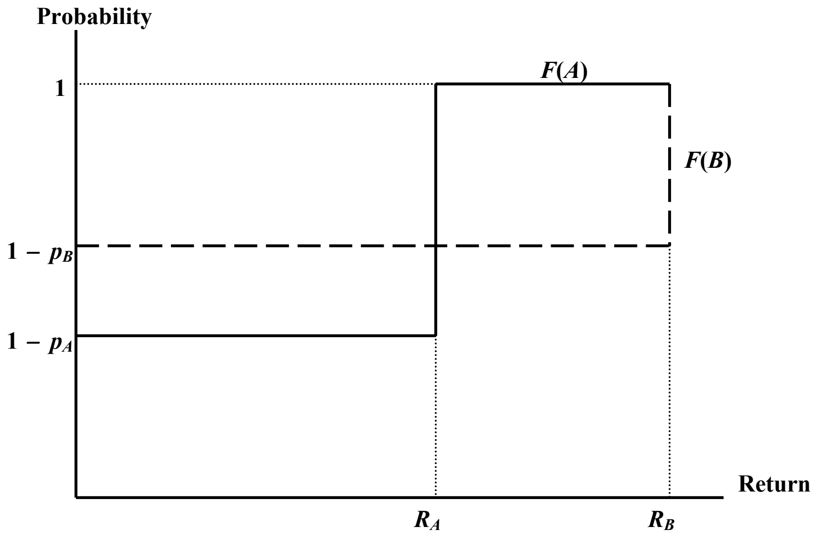

Figure 1: **Cumulative distribution functions of assets with binary returns**

Lemma 2 *For equivalent expected returns, the expected utility from asset A is greater (less) than the expected utility from asset B for a globally risk averse (seeking) agent when R_A is less than R_B when the returns are Bernoulli random variables.*

Proof: Assuming $E[R_A] = E[R_B]$, then $p_A R_A = p_B R_B$. Thus, agent i's EU from asset A can be written as $p_B \frac{R_B}{R_A} U_i(R_A)$. Subtracting the EU of asset B from the EU of asset A

$$EU_i(R_A) - EU_i(R_B) = \frac{U_i(R_A)}{R_A} - \frac{U_i(R_B)}{R_B}$$

When $U_i''(R_k)$ is negative (positive) for all R_k, the above expression is positive (negative) for R_A less (greater) than R_B. Q.E.D.

Figure 2 demonstrates the result from Lemma 2 for a globally risk averse agent. The figure plots two scenarios. In the first scenario, the high return from asset A, R_A, exceeds the high return from asset B, R_{B1}, yet each has the same expected return, $E[R_1] = pR_{B1} = p_{A1}R_A$. In the second scenario, the high return from asset B, R_{B2}, exceeds the high return from asset A, R_A, while each still has the same expected return, $E[R_2] = pR_{B2} = p_{A2}R_A$. As the graph demonstrates, the EU from asset B is greater under the first scenario while the EU from asset A is greater under the second scenario is greater for a globally risk averse agent.

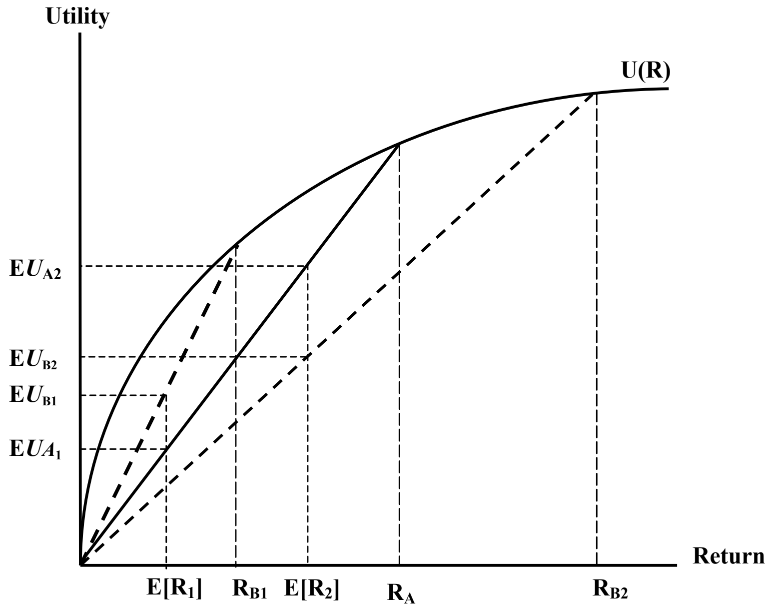

Figure 2: **EU from assets with Bernoulli returns for a globally risk averse agent**

Combining lemmas 1 and 2 we can state the following:

Proposition 1 *For equivalent expected returns, if the expected utility from asset A is greater than the expected utility from asset B for a globally risk averse agent, then asset A second-order stochastically dominates asset B when the returns are Bernoulli random variables.*

Proof: From lemma 1, the necessary condition for asset A to be SSD is $R_A < R_B$. From lemma 2, the necessary condition for the EU from asset A to be greater than the EU from asset B is also $R_A < R_B$. Thus, satisfying lemma 2 necessarily satisfies lemma 1. Q.E.D.

Thus, a globally risk averse EU maximizer should choose the SSD asset when faced with a choice between two risky assets whose returns are Bernoulli random variables. Of course, this implies that an agent never makes a mistake.

2.3 Accounting for Decision Error

A more likely case is that an agent will make mistakes. Previous literature (Ballinger and Wilcox, 1997; Blavatskyy, 2007; Hey, 1995; Hey and Orme, 1994; McFadden, 1974; Loomes et al., 2002; Wilcox, 1993) assumes each agent i maximizes his *stochastic* EU from asset k is $EU_i(R_k) = p_k U_i(R_k) + \varepsilon_{ik}$, where ε_{ik} is a stochastic noise parameter. That is, agent i formulates a noisy estimate of his EU.[7] This formulation of EU changes agent i's decision rule such that he does not always choose the SSD lottery. If asset A is SSD, the agent will choose asset A iff:

$$EU_i(R_A) = p_A U_i(R_A) + \varepsilon_{iA} > p_B U(R_B) + \varepsilon_{iB} = EU_i(R_B)$$

Then the probability that agent i chooses A can be written as

$$P[p_A U_i(R_A) - p_B U_i(R_B) > \varepsilon_i] \tag{1}$$

where $\varepsilon_i = \varepsilon_{iB} - \varepsilon_{iA}$ such that $\varepsilon_i \sim (0, \sigma_i)$; we adopt the Fechner model of stochastic choice under risk.[8] The decision rule in equation (1) says that the probability an agent chooses the SSD asset is equal to the probability that the difference in stochastic EU is positive; the difference in EU exceeds the noise. While the experiment eliminates much of the noisiness of the naturally occurring world by virtue of the induced value framework, it still remains that the agent must cognitively evaluate the difference in EU from the lotteries. One may think of the noise parameter as capturing the cognitive cost associated with refining ones estimate of the difference in EU (Smith and Walker, 1993).

[7] We still maintain that ex post the agent knows their utility from the realized outcome.

[8] Several papers investigate various specifications of the error term and/or preferences in equation (1) (Blavatskyy, 2007; Hey, 1995; Buschena and Zilberman, 2000; Loomes et al., 2002).

However, recently an alternative to this specification of probabilistic decision making has been proposed by Wilcox (forthcoming) in which the context of the decision enters into the agents decision rule. Rather than the difference in stochastic expected utility determining the agents choice, the difference is scaled by the range of possible outcomes; the choice is determined by "contextual utility" (Wilcox, forthcoming). In this framework, the probability that agent i chooses A can be written as

$$P[\frac{p_A U_i(R_A) - p_B U_i(R_B)}{U_i(R_B)} > \varepsilon] \tag{2}$$

for $R_B > R_A$. We adopt this framework due to the convenient implication it has regarding the relationship of decision error and risk aversion.

Assume agent i's preferences over potentially random distributions of income are given by the popular constant relative risk aversion (CRRA) utility function where the expected utility from asset k is $EU_i = p_k R_k^{1-r_i}$. The parameter r_i measures the relative risk preference of the respondent, where $r_i = 0$ if the respondent is risk-neutral; $0 < r_i < 1$ if the respondent is risk averse; $r_i < 0$ if the respondent is risk seeking. According to equation (2), the probability agent i chooses A is

$$P[choose\ SSD] = P[\frac{p_A R_A^{1-r_i} - p_B R_B^{1-r_i}}{R_B^{1-r_i}} > \varepsilon_i] \tag{3}$$

for $R_B > R_A$. We restate the Wilcox's proposition regarding the relationship between risk aversion and likelihood of choosing the SSD lottery, "stochastic risk aversion" (Wilcox, forthcoming).

Proposition 2 *For a given $\varepsilon_i \sim (\mu, \sigma_i)$, the likelihood of a risk averse agent choosing the stochastically dominant lottery increases with the magnitude of constant relative risk aversion.*

Proof: According to equation (3), the probability of choosing the SSD is increasing in the difference in contextual expected utility, $\frac{p_A R_A^{1-r_i} - p_B R_B^{1-r_i}}{R_B^{1-r_i}}$. Taking the derivative of equation (3) with respect to r_i,

$$\frac{\partial \frac{p_A R_A^{1-r_i} - p_B R_B^{1-r_i}}{R_B^{1-r_i}}}{\partial r_i} = p_B (\frac{R_B}{R_A})^{r_i} \ln[\frac{R_B}{R_A}] > 0$$

The difference in contextual EU is increasing in r_i. Thus, the probability of choosing the SSD lottery, is increasing in the magnitude of constant relative risk aversion. Q.E.D.

Proposition 2 is consistent with Smith and Walker (1993), who argue that errors arise due to decision cost; it may be too cognitively costly for a subject to evaluate the available choice set, resulting in suboptimal decisions.[9] In this setting, ε_i captures the agent i's cognitive cost of evaluating the difference in EU from the two lotteries. However, Proposition 2 implies that the reward, the difference in contextual EU from the two lotteries, "the V-distance" (Wilcox, forthcoming), is increasing in risk aversion. Thus, the more risk averse a subject is, the more they stand to gain from choosing the SSD asset.[10]

Now suppose the agent faces a series, $j = 1 \ldots J$, of choices between two risky assets, A and B, where one asset is SSD each decision j. The decision process that drives the agent's choice for a single decision is the same process for all J decisions. Thus, we may model the probability that an agent chooses X out of J SSD assets in the series. Let z_i^* be the latent difference in contextual expected utility which is a function of the degree of constant relative risk aversion r_i. Then our structural model can be written as

[9] As already stated, some argue that errors arise due to small stakes (Levitt and List, 2007). If the probability of choosing the SSD asset is as stated in equation (1), then errors should decrease with an increase in stakes. However, if the probability of choosing the SSD asset is as stated in equation (2), then errors should be unaffected the size of the stakes. To see this, suppose that the returns to assets A and B are scaled up by a factor $\delta > 1$. Furthermore, suppose that the expected utility from a risky asset is given by $EU_i = p_k R_k^{1-r_i}$. Then equation (1) can be rewritten as $P[p_A(\delta R_A)^{1-r_i} - p_B(\delta R_B)^{1-r_i} > \varepsilon_i] = P[\delta^{1-r_i}(p_A R_A^{1-r_i} - p_B R_B^{1-r_i}) > \varepsilon_i]$. Thus, the difference in strong expected utility is increasing in the size of the stakes. However, equation (2) can be written as $P[\frac{p_A(\delta R_A)^{1-r_i} - p_B(\delta R_B)^{1-r_i}}{(\delta R_B)^{1-r_i}} > \varepsilon_i] = P[(p_A R_A^{1-r_i} - p_B R_B^{1-r_i} > \varepsilon_i]$ which is identical to equation (2). Thus, the size of the stakes is predicted to have no effect.

[10] This argument can be made for risk-seeking preferences as well. That is, the difference in expected utility will be increasing in the magnitude risk seeking as well.

$z_i^* = \beta r_i + \varepsilon_i$.[11] Let z_i be the observed number of times that agent i chooses the SSD asset. Then we observe $z_i = X$ if $\tau_{X-1} \leq z_i^* < \tau_X$ for $X = 1$ to J where τ_m denotes the cutoff value of the latent variable for $z_i = m$. Thus, $P(z_i = X|r_i) = F(\tau_X - \beta r_i) - F(\tau_{X-1} - \beta r_i)$ where F denotes the cdf of the error term ε. The model is estimated with both an ordered probit and ordered logit regression.[12] The assumption that the model does not contain a constant means the model is identified (Long, 1997).

3 Experimental Design

The experiments were conducted to investigate the extent to which *risk averse* subjects satisfy SSD when choosing between two lotteries. Since we must determine which subjects are risk averse, subjects were presented with two formats of a risk preference elicitation mechanism in which subjects faced a tradeoff between risk and return. They were also presented with a format intended to determine how often the risk averse subjects choose the SSD of two lotteries; the lotteries had equivalent expected returns but different levels of risk. The analysis required a *within-subjects* design; subjects were presented with all three series.

The lottery variation (LV) format was designed to investigate SSD. It required subjects to make 10 decisions between two lotteries. The expected values of the lotteries were increased through the series so that initially one lottery dominated the other and then vice versa. The expected value of the first series of lotteries was increased by probability variation (PV), the outcomes of the lottery were held constant, \$0 or \$10, while the probability of a payout was varied from 0.10 to 1.0 in increments of 0.10. The expected value of the second series of lotteries was increased by reward variation (RV), the probability of a payout was held constant, 0.50, while the reward varied from \$2.00 to \$20.00 in \$2.00 increments. The decisions for the LV format are shown in Table 1. Since the lotteries had

[11] In the following section we demonstrate the importance of the contextual utility framework as it pertains to this econometric specification.

[12] The ordered probit specification assumes $\varepsilon \sim N(0,1)$ and $F = \int_{-\infty}^{\varepsilon} \frac{1}{\sqrt{2\pi}} \exp(-\frac{t^2}{2})dt$. The ordered logit specification assumes $\varepsilon \sim N(0, \frac{\pi^2}{3})$ and $F = \frac{\exp \varepsilon}{1+\exp \varepsilon}$. We include subject demographics to account for subject heterogeneity.

the same expected value, it was necessary for subjects to rely on higher moments of the distribution of payouts to distinguish between the lotteries. As Proposition 1 implies, for risk averse subjects choosing the lottery with the higher EU is consistent with choosing the SSD lottery.

Table 1: **Decisions for LV Format**

Decision	**PV Series**	**RV Series**
1	10% chance of \$10	50% chance of \$2
2	20% chance of \$10	50% chance of \$4
3	30% chance of \$10	50% chance of \$6
4	40% chance of \$10	50% chance of \$8
5	50% chance of \$10	50% chance of \$10
6	60% chance of \$10	50% chance of \$12
7	70% chance of \$10	50% chance of \$14
8	80% chance of \$10	50% chance of \$16
9	90% chance of \$10	50% chance of \$18
10	100% chance of \$10	50% chance of \$20

However, taking a stochastic approach to modeling EU permits decision error. Wilcox's Proposition states that this decision error is decreasing in the magnitude of constant relative risk aversion. This conclusion relies upon the assumption that individuals evaluate the contextual expected utility of rather than the strong expected utility. To see why this is the case, Figure 3 plots the difference in strong expected utility and the difference in contextual expected utility as a function of the magnitude of constant relative risk aversion. The upper graph plots the difference in strong expected CRRA utility, $|p_{PV}R_{PV}^{1-r_i} - p_{RV}R_{RV}^{1-r_i}|$, as a function of risk aversion for the lottery pairs 3, 6, and 9 in Table 1. Clearly these differences are not monotonically increasing in risk aversion. The lower graph plots the difference in contextual expected CRRA utility, $\frac{|p_{PV}R_{PV}^{1-r_i} - p_{RV}R_{RV}^{1-r_i}|}{R_{max}^{1-r_i}}$, as a function of risk aversion for the lottery pairs 3, 6, and 9 in Table 1. Clearly these differences are monotonically increasing in risk aversion. Furthermore, the differences are practically linear which is consistent with the specification of the ordered probit and ordered logit models.

The other two formats presented subjects with 10 decisions intended to determine each subject's risk preference. The PV format required subjects to choose between the

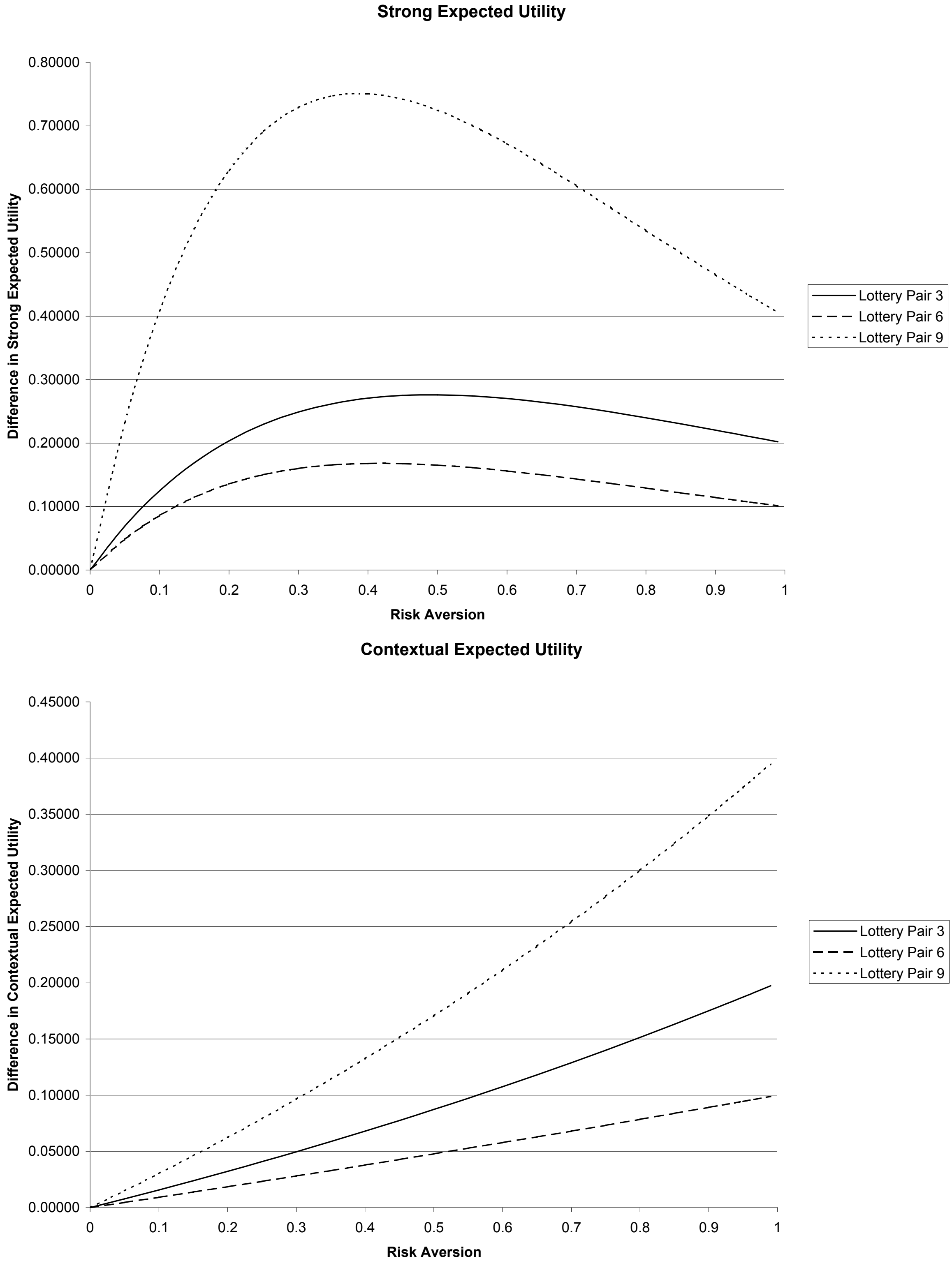

Figure 3: **Difference in strong EU and contextual EU as a function of risk aversion**

lotteries in the PV series and a guaranteed \$5. The RV format required subjects to choose between the lotteries in the RV series and a guaranteed \$5. Since subjects faced a tradeoff between risk and return estimates of the degree of risk preference are obtained. Subjects should be induced switch from choosing \$5 to choosing the lottery as the expected value of the lottery increases. The decision at which a subject switches provides an interval estimate of risk preference. A risk averse subject may switch anywhere after decision 4 in the PV and RV formats. However, only switching at decision 5 or 6 is consistent with satisfying SSD, in the LV format.[13]

Experimental sessions consisted of three stages. In each stage, one of the three formats was presented to a subject. Subjects were presented with all three elicitation formats. Thus, subjects made 30 decisions in the experiment. In order to control for order effects (Harrison et al., 2005), the order in which the three formats were presented was randomized within a session yielding six orthogonal treatments.[14] Table 2 presents the experimental design. Additionally, framing effects (Kahnemann and Tversky, 1979) were controlled for by stating each outcome as a gain. However, not every outcome was *actually* a gain. There was an opportunity cost associated with the lottery in the PV and RV formats, the guaranteed amount the subject had forgone. Thus, a \$0 payout in a lottery resulted in an ex post net loss of \$5. Table 2 presents the experimental design.

Table 2: **Experimental Design**

T1	Stage 1 = PV	Stage 2 = LV	Stage 3 = RV
T2	Stage 1 = RV	Stage 2 = LV	Stage 3 = PV
T3	Stage 1 = PV	Stage 2 = RV	Stage 3 = LV
T4	Stage 1 = RV	Stage 2 = PV	Stage 3 = LV
T5	Stage 1 = LV	Stage 2 = PV	Stage 3 = RV
T6	Stage 1 = LV	Stage 2 = RV	Stage 3 = PV

Prior to making any decisions, subjects were presented with instructions on the computer screen. Screen images are included in the Appendix and on the author's website. Subjects were informed in advance that they would be making 30 decisions, 10 in each of the three stages. Furthermore, subjects were told before they saw any instructions that only *one* of their decisions would determine their earnings in the experiment.[15] This was

[13]Option 5 is equivalent in the formats so switching at either Option 5 or 6 is consistent with risk aversion.

[14]The previous evidence of order effects Harrison et al. (2005) pertains to varying the magnitude of payoffs, which is constant in our experiment. Thus we have no prior beliefs about the existence, let alone the direction or magnitude of order effects in this experiment.

[15]Specifically, the following verbal instruction was provided to subjects before beginning the instruc-

done to raise the cost of decision error. The selection of the decision that determines their payoff was presented as a compound lottery; the computer first selects the stage of the experiment (each has a $\frac{1}{3}$ chance of being selected) and then the decision of the stage is selected (each has a $\frac{1}{10}$ chance of being selected). Thus, we assume that preferences conform to the Independence Axiom (Samuelson, 1952). The evidence in the literature suggests that 'random lottery selection' is incentive-compatible for simple choice sets (Ballinger and Wilcox, 1997; Starmer and Sugden, 1991; Wilcox, 1993). Subjects were given specific instructions pertaining to the stage and shown an example decision screen prior to making any decisions for a particular stage. Upon completion of the decisions in a stage, subjects received instructions and examples concerning the subsequent stage. After completion of the final stage, subjects were shown the stage and the decision that was selected by the computer for payment. If the selected stage involved either the PV or the RV formats and the subject chose the lottery for the selected decision, the subject was informed of the outcome of the lottery, otherwise they were informed they would receive the guaranteed amount. If the selected stage involved the LV format, subjects were informed of the outcome of the chosen lottery.

Subjects then provided demographic information by answering questions on the computer. There is evidence of demographic effects in the literature (Andersen et al., 2006; Eckel and Wilson, 2004). We control for collected demographics in our analysis. Upon completion of the debriefing questions, subjects were again shown their earnings for the experiment and asked to fill out a receipt form. Subjects were paid individually in private. At no time was any deception used in the experiment.

The experiment was conducted at XXXX. The subject pool is composed of volunteer students at the university. Subject's were recruited by email via the lab's Online Recruitment System for Experimental Economics (ORSEE) (Greiner, 2004). The experiment was programmed and conducted with the software Z-Tree (Fischbacher, Forthcoming).

tions. "Before we begin with the instructions, I would like to bring one thing to your attention. As you will read in the instructions, you are going to make several decisions in this experiment. However, only ONE of these will actually determine your earnings for this experiment! So, it is important that you take each decision seriously since a single mistake can be quite costly!"

Experimental sessions lasted approximately 35 minutes and average earnings were $12 including a $5 show-up fee. A total of 106 subjects participated.

4 Analysis and Results

Proposition 1 implies that risk *averse* subjects should choose the SSD lottery for each decision in the LV format. Thus, we must first determine risk preference. We use the choices in the PV and RV formats to estimate risk preferences.

4.1 Determination of Risk Preference

Table 3: **Risk Preference Classification Based on Lottery Choices**

Number of Safe Choices	**PV Risk Parameter Range**	**RV Risk Parameter Range**	**Classification**	**Number of Subjects PV**	**Number of Subjects RV**
2	-1.322 < r ≤ -0.737	-∞ < r ≤ -2.802	bet kids' college fund	2	3
3	-0.737 < r ≤ -0.322	-2.802 < r ≤ -0.475	risk loving	3	2
4	-0.322 < r ≤ 0.000	-0.475 < r ≤ 0.000	slightly risk loving	20	18
5	0.000 < r ≤ 0.263	0.000 < r ≤ 0.208	slightly risk averse	28	24
6	0.263 < r ≤ 0.485	0.208 < r ≤ 0.327	risk averse	19	21
7	0.485 < r ≤ 0.678	0.327 < r ≤ 0.404	very risk averse	21	14
8 or more	0.678 < r ≤ 1.000	0.404 < r ≤ 1.000	stay in bed	13	24

Since subjects make 10 decisions in each format, we are able to construct bounds on the implied risk aversion parameter based on the number of safe choices. Table 3 shows the ranges of the implied risk aversion parameter in columns 2 and 3 for the PV and the RV formats, respectively, assuming a CRRA utility function.[16] The table indicates that the majority of subjects exhibited risk aversion, 76% and 78% of subjects in the PV and the RV formats, respectively. However, some subjects only exhibited risk aversion in one format.[17] 64 out of 106 subjects exhibited risk averse preferences in both formats. We will focus our attention on these 64 subjects for the analysis of the decisions in the LV format.

[16] We assume the utility function is $U(R) = p_k R_k^{1-r}$ as stated in section 2.3.

[17] 8 subjects exhibited risk loving preferences in the PV format and risk averse preferences in the RV format. 9 did the opposite. 12 subjects exhibited risk loving preferences in both formats. The remaining subjects were inconsistent in one or both of the formats.

4.2 SSD Choices in LV Format

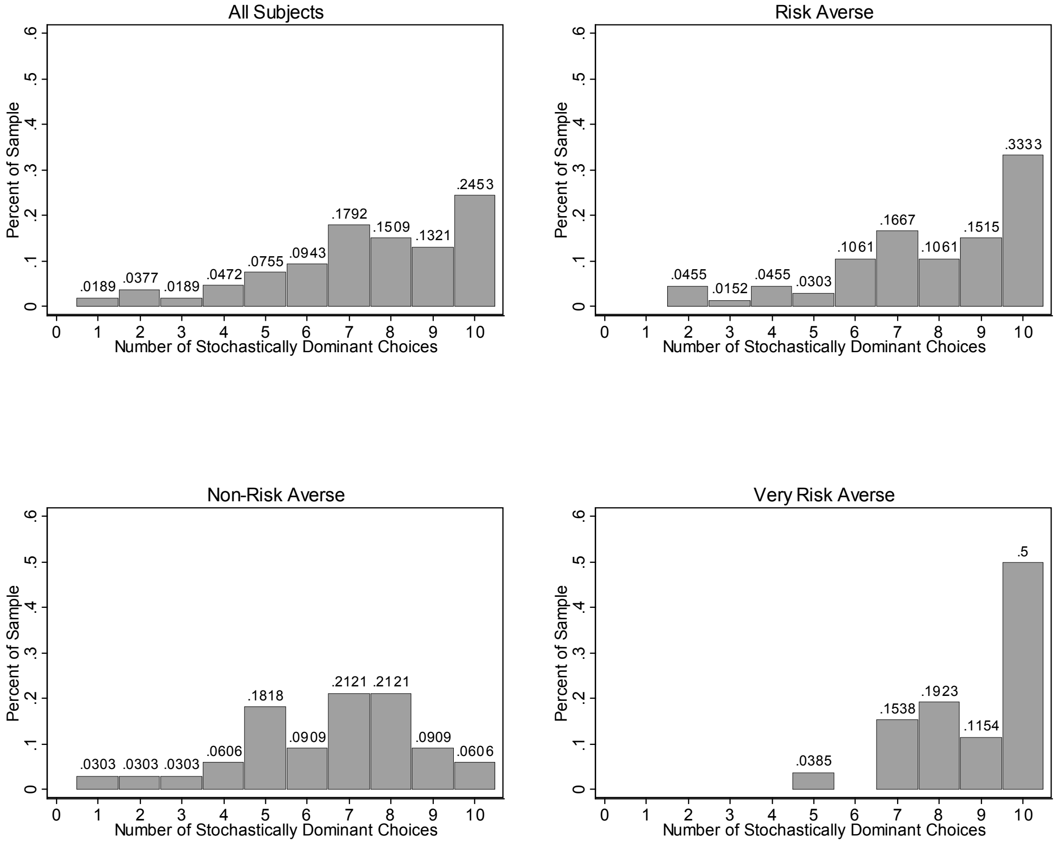

Figure 4: **Histogram of number of SSD lotteries chosen**

Figure 4 presents histograms of the number of the number of SSD lotteries chosen by subjects of various risk preference. The upper left graph plots the distribution of choices for the entire sample. Notice that the distribution is very dispersed. Suppose one were to naively consider anything other than 10 out 10 responses a violation of SSD. Then only 24.53% of subjects make choices that satisfy SSD. However, this result is confounded by the inclusion of non-risk averse subjects, who theoretically should not satisfy SSD, and error naively being categorized as violations. The remaining graphs in Figure 4 demonstrate how these confounding factors influence the results.

Proposition 1 implies that risk *averse* subjects maximizing their expected utility should choose all 10 SSD lotteries in the LV format. The upper right graph in Figure 4 plots the distribution of SSD lotteries chosen for unambiguously risk averse, those that exhibited risk aversion in both the PV and RV formats. The results are striking;

the most frequently observed pattern of responses for the risk averse subjects is 10 out of 10 times the stochastically dominant lottery is chosen. Allowing for up to 3 mistakes, 75.76% of risk averse respondents chose at least 7 out of 10 stochastically dominant lotteries. By contrast, the pattern of responses is much more dispersed for non-risk averse subjects. The lower left graph in Figure 4 is the distribution for non-risk averse subjects, those that did not exhibit risk averse preferences in either the PV or RV format or both. A nonparametric Kolmogorov-Smirnov (K-S) test rejects the null hypothesis of the equivalence of the two distributions (risk averse and non-risk averse) at the 1% level of significance. Thus, it appears risk averse subjects choose the SSD lottery more often than not and more often than non-risk averse subjects. Furthermore, it suggests that not accounting for risk preference confounds the results.

According to Proposition 2, noisiness in expected utility is the source of decision error, the likelihood of which should decrease with risk aversion.[18] The lower right graph in Figure 4 presents a histogram of the number of SSD lotteries chosen by *very* risk averse, those that chose the guaranteed $5 at least 7 times in either the PV or RV formats. 50% of these subjects choose the stochastically dominant lottery 10 out of 10 times. 96.15% of these subjects choose the stochastically dominant lottery at least 7 out of 10 times. Comparing this to the distribution for all risk averse subjects, a K-S test rejects the null hypothesis of the equivalence of the two distributions at the 1% level of significance. This is consistent with the hypothesis that increasing risk aversion decreases the likelihood of error.

While most choices satisfy SSD, there were a nontrivial number of violations. The graph on the left in Figure 5 plots the proportion of RV series lotteries chosen for each decision by the risk averse and the very risk averse. Recall that theoretically, with no error, subjects should choose the RV series lottery for decisions 1 - 4 and the PV series lottery for decisions 6 - 10.[19] There is an overwhelming pattern in the error rate across decisions; for ever single decision the SSD lottery is chosen more frequently by very risk

[18]Although Proposition 2 also implies decision error should decrease with risk seeking, we do not observe sufficient variation in the magnitude of elicited risk seeking to explore this implication.

[19]The choice for decision 5 is irrelevant since the lotteries are identical.

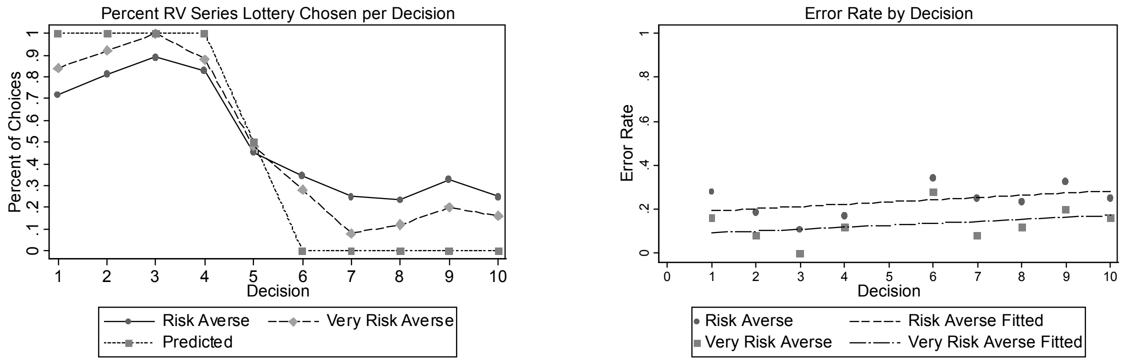

Figure 5: **Proportion of sample that choose PV series lottery and error rate for each decision**

averse subjects relative to all risk averse subjects. This pattern becomes clearer in the second graph on the right in Figure 5. The graph plots the error rate by decision for both the risk averse and the very risk averse, where the error rate is calculated as the percentage of choices of the dominated lottery. For ever decision the error rate is lower for the very risk averse relative to all risk averse subjects. Thus, not only do the very risk averse make fewer errors overall, they make fewer errors for every single decision. Again, this pattern is consistent with the behavior described in Proposition 2. In general, the error rate appears relatively constant across decisions with a slight upward trend. The lack of strong evidence of systematic violations of SSD suggests that inconsistencies are due, in large part, to error associated with noisiness in EU.

We conduct a more rigorous test of the hypothesis that decision error decreases with the degree of risk aversion. Table 4 provides summary statistics as well as descriptions of the variables used in the econometric models. We estimate the effects of risk aversion, *PV Risk Category*, on the likelihood of choosing the SSD lottery more frequently, *Number of SSD Choices*. The choice of which responses, PV or RV, to use as the independent variable is arbitrary as the responses in the PV and RV formats both elicit risk preference. The results are robust to the inclusion of either set of responses as a proxy for risk preference. We control for the demographic variables listed in the table to account for subject heterogeneity.

Table 4: **Descriptive Statistics For Number of SD Lotteries Chosen**

Variable	Obs	Mean	Std. Dev.	Min	Max	Description
Number SSD Choices	64	7.84	2.22	2	10	Number of times SD lottery chosen
PV Risk Category	64	6.38	1.14	5	9	Number of times $5 chosen in PV format
Age	64	20.39	3.00	18	31	Age in years
Male	64	0.44	0.50	0	1	1 = male; 0 = female
Economics Courses	64	2.56	5.30	0	33	Number of economics courses completed
Employment	64	0.47	0.50	0	1	1 = student only; 0 = work part-time
Previous Experiment	64	0.72	0.45	0	1	1 = previously participant$^{\alpha}$; 0 = first time participant

$^{\alpha}$Equals one if subjects previously participated in a different experiment.
None of the subjects participated in this experiment more than once.

Table 5: **Estimation of Number of SSD Lotteries Chosen**

Coefficients	OLS 1	OLS 2	Ordered Probit 1	Ordered Probit 2	Ordered Logit 1	Ordered Logit 2
PV Risk Category	1.214*** (0.042)	0.782*** (0.185)	0.376*** (0.126)	0.394*** (0.130)	0.598*** (0.217)	0.637*** (0.227)
Age		0.160** (0.064)		0.086* (0.051)		0.163* (0.095)
Male		-0.181 (0.541)		-0.145 (0.282)		-0.191 (0.481)
Employment		0.259 (0.526)		0.231 (0.291)		0.542 (0.493)
Previous Experiment		-0.935 (0.613)		-0.661** (0.334)		-1.073* (0.584)
Economics Courses		0.084* (0.050)		0.040 (0.028)		0.062 (0.044)
Log-Likelihood						

Standard errors in parenthesis
Significance levels : $*$: 10% $**$: 5% $***$: 1%
64 observations

Table 5 presents the results from the linear probability models as well as the ordered probit and ordered logit specifications. Proposition 2 implies the estimated coefficient on risk aversion should be positive. Not only is this the case for all specifications, but it is highly significant across specifications. It is also robust to the inclusion of demographic variables. This is strong evidence that decision error is decreasing in risk aversion, as proposed.

Using the predicted values from the models provides another means of presenting the result. Table 6 presents the mean predicted probabilities of choosing 10 out of 10 SSD lotteries based on the number of times the guaranteed $5 is chosen in the PV format. As the table demonstrates, the more safe choices that a subject makes in the PV format (i.e. the more risk averse), the more likely they are to choose 10 out of 10 SSD lotteries. Across the ordered probit and ordered logit models, the predicted probability ranges from a low of 16.1%, for 5 safe choices, to a high of 75.6%, for 9 safe choices, suggesting that risk aversion plays an important role in the decision error process. The result is robust across specifications.[20]

Table 6: **Predicted Probabilities of Choosing 10 out of 10 SD Lotteries**

PV Risk Category	**OLS 1**	**OLS 2**	**Ordered Probit 1**	**Ordered Probit 2**	**Ordered Logit 1**	**Ordered Logit 2**
5 Safe Choices	0.077	0.142	0.161	0.195	0.165	0.206
	(0.000)	(0.069)	(0.000)	(0.124)	(0.000)	(0.126)
6 Safe Choices	0.171	0.169	0.269	0.230	0.263	0.229
	(0.000)	(0.061)	(0.000)	(0.124)	(0.000)	(0.126)
7 Safe Choices	0.283	0.271	0.406	0.415	0.394	0.420
	(0.000)	(0.019)	(0.000)	(0.107)	(0.000)	(0.115)
8 Safe Choices	0.352	0.330	0.554	0.566	0.542	0.562
	(0.000)	(0.034)	(0.000)	(0.140)	(0.000)	(0.145)
9 Safe Choices	0.326	0.362	0.696	0.752	0.683	0.756
	(0.000)	(0.001)	(0.000)	(0.134)	(0.000)	(0.127)

Standard deviations in parenthesis

5 Conclusions

Experimental findings of violations of EU theory have been used to motive a vast literature of theories of decision-making under uncertainty. However, once confounding

[20]The mean predicted probability decreases slightly from 8 to 9 safe choices for the OLS 1 model. This result does not concern us given that it disappears with the inclusion of demographics.

effects were accounted for, such as decision errors, EU theory seemed to perform reasonably well (Harless and Camerer, 1994; Hey and Orme, 1994). As with other violations of EU theory, existing evidence of violations of stochastic dominance is confounded.

After removing or controlling for confounding effects, we find risk averse subjects make choices consistent with stochastic dominance. The most frequently observed pattern of responses is 10 out of 10 times the SSD lottery is chosen. Violations appear to be due, in large part, to simple decision error rather than revealed preference. By adding a stochastic noise parameter to expected utility, we demonstrate that the likelihood of a decision error should be decreasing in the magnitude of constant relative risk aversion. Our results support this proposition.

Experimental findings have been used to motivate theorists to develop and employ stochastic utility models that incorporate individuals ability to make mistakes. The argument put forward by Smith and Walker (1993) regarding decision cost and decision rewards has stimulated the literature to place considerable attention on modeling decision cost and decision error (Blavatskyy, 2007; Ballinger and Wilcox, 1997; Buschena and Zilberman, 2000; Hey, 1995; Loomes et al., 2002). Wilcox (1993) exploits heterogeneity in decision costs to demonstrate that error rates increase as the ratio of decision rewards to decision costs declines. This paper approaches the problem from the reward side. By exploiting heterogeneity in decision rewards, in a contextual utility framework increasing constant relative risk aversion raises the reward to choosing the SSD asset, we find an equivalent result. The growing and consistent evidence suggests that the argument made by Smith and Walker (1993) has merit.

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