Mathematical Appendix

Following MRW, the estimating equations presented in section 2 are derived as follows. Begin with the production function (1) in the text, which is repeated here for convenience:

$$Y_t = K_t^{\alpha} H_t^{\beta} (A_t L_t)^{1-\alpha-\beta} . \tag{A1}$$

Recall that labor grows at the exogenous rate n; that is, $L_t = L_0 e^{nt}$. The level of technology grows according to

$$A_t = A_0 e^{gt} \tag{A2}$$

where g is the exogenous rate of technical progress. Individuals invest constant fractions of income, s_k and s_h, in physical and human capital, respectively. The rate of depreciation is assumed to be constant and equal to δ for both types of capital. Thus, the equations describing the evolution of physical and human capital are

$$\frac{dK_t}{dt} = s_k Y_t - \delta K_t \text{ and } \frac{dH_t}{dt} = s_h Y_t - \delta H_t . \tag{A3}$$

Dividing through by AL in (A1) and (A3) allows quantities to be expressed in per effective worker terms, where lower-case letters with "hats" denote quantities per effective worker:

$$\hat{y}_t = \hat{k}_t^{\alpha} \hat{h}_t^{\beta} , \tag{A4}$$

$$\frac{d\hat{k}_t}{dt} = s_k \hat{y}_t - (n + g + \delta) \hat{k}_t , \text{ and } \frac{d\hat{h}_t}{dt} = s_h \hat{y}_t - (n + g + \delta) \hat{h}_t . \tag{A5}$$

In the steady state, investment in physical and human capital equals the amounts required to maintain constant levels of physical and human capital per effective worker. That is, $dk_t/dt = dh_t/dt = 0$ in (A5) which implies that the steady-state levels of physical and human capital per effective worker can be written as

$$\hat{k}^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n + g + \delta} \right)^{1/1-\alpha-\beta} \text{ and } \hat{h}^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n + g + \delta} \right)^{1/1-\alpha-\beta} . \tag{A6}$$

The natural logarithm of the (constant) steady-state level of output per effective worker is obtained by substituting (A6) into (A4) and taking logs:

$$\ln \hat{y}^{*} = -\left(\frac{\alpha + \beta}{1 - \alpha - \beta}\right) \ln (n + g + \delta) + \left(\frac{\alpha}{1 - \alpha - \beta}\right) \ln s_k + \left(\frac{\beta}{1 - \alpha - \beta}\right) \ln s_h . \qquad (A7)$$

Since the levels of physical capital, human capital, and output *per effective worker* are constant in the steady state, the levels of these variables in *per worker* terms will grow at the constant rate g. Similarly, the aggregates K, H, and Y grow at rate $(n + g)$ in the steady state. An equation analogous to (A7) for output per worker can be written by making use of

$$\ln \hat{y}_t = \ln Y_t - \ln L_t - \ln A_t = \ln y_t - \ln A_0 - gt . \qquad (A8)$$

Substituting the steady-state level of output per effective worker into the left-hand-side of (A8) and solving for the steady-state value of $\ln y_t$ provides

$$\ln y^{*} = \ln \hat{y}^{*} + \ln A_0 + gt \qquad (A9)$$

where $\ln^{*}$ is given by (A7).

Transitional dynamics in the model are described using a linear approximation of the economy's path toward the steady state. This implies

$$\frac{d \ln \hat{y}_t}{dt} = \phi (\ln \hat{y}^{*} - \ln \hat{y}_t) \qquad (A10)$$

where the parameter $\varphi = (n + g + \delta)(1 - \alpha - \beta)$ defines the speed at which output per effective worker approaches its steady-state value. Integrating (A10) from some initial time 0 to time T produces

$$\ln \hat{y}_T = (1 - e^{-\phi T}) \ln \hat{y}^{*} + e^{-\phi T} \ln \hat{y}_0 . \qquad (A11)$$

To facilitate estimation of the model, we transform output per effective worker to output per worker by first substituting (A11) into the left-hand-side of (A8) to obtain

$$(1 - e^{-\phi T}) \ln \hat{y}^{*} + e^{-\phi T} \ln \hat{y}_0 = \ln y_T - \ln A_0 - gT . \qquad (A12)$$

Again using the definition in (A8) to substitute for $\ln_0$ on the left-hand side of (A12), solving for $\ln y_T$, and subtracting $\ln y_0$ from both sides provides

$$\ln y_T - \ln y_0 = (1 - e^{-\phi T}) \ln \hat{y}^{*} + (1 - e^{-\phi T}) \ln A_0 + gT - (1 - e^{-\phi T}) \ln y_0 . \qquad (A13)$$

Finally, substituting for $\ln^{*}$ from (A7) results in the equation that is estimated in empirical tests of the neoclassical model, written as equation (2) in the text:

$$\ln y_T - \ln y_0 = \pi_0 + \pi_1 \ln y_0 + \pi_2 \ln (n + g + \delta) + \pi_3 \ln s_k + \pi_4 \ln s_h \qquad (A14)$$

where $\pi_0 = (1 - e^{-\varphi T}) \ln A_0 + gT$,

$$\pi_1 = -(1-e^{-\varphi T}),$$
$$\pi_2 = -(1-e^{-\varphi T})[(\alpha+\beta)/(1-\alpha-\beta)],$$
$$\pi_3 = (1-e^{-\varphi T})[\alpha/(1-\alpha-\beta)],$$
and $$\pi_4 = (1-e^{-\varphi T})[\beta/(1-\alpha-\beta)].$$

We now formalize the potential effect of institutions on growth described by what is known as the investment channel. The equation describing the evolution of physical capital characterizes investment behavior in the economy. Now let the saving share s_k be a function of F_t so that

$$\frac{dK_t}{dt} = s_k(F_t)\,Y_t - \delta K_t \qquad \text{(A15)}$$

where $s_k' > 0$. It is immediately clear that this change does not alter the steady-state solutions given in (A6) and (A7) or the estimable equation in (A14), except for the fact that s_k is now a function of F_t, as described in section 2.1.1 of the text.

To account for the effect of institutions on total factor productivity, (A2) is augmented as follows:

$$A_t = A_0 e^{gt} F_t^{\rho}. \qquad \text{(A16)}$$

This specification combines the effect of institutions with the traditional exogenous component of growth in productive efficiency. Given (A16), (A8) is rewritten as

$$\ln\hat{y}_t = \ln Y_t - \ln L_t - \ln A_t = \ln y_t - \ln A_0 - gt - \rho\ln F_t. \qquad \text{(A17)}$$

Then, after reworking the transitional dynamics, (A12) becomes

$$(1 - e^{-\phi T})\ln\hat{y}^* + e^{-\phi T}\ln\hat{y}_0 = \ln y_T - \ln A_0 - gT - \rho F_t. \qquad \text{(A18)}$$

Substituting (A17) into (A18) for ln0, solving for $\ln y_T$, and subtracting $\ln y_0$ from both sides provides a revised version of (A13):

$$\begin{aligned}\ln y_T - \ln y_0 = {} & (1 - e^{-\phi T})\ln\hat{y}^* + (1 - e^{-\phi T})\ln A_0 + gT \\ & + \rho(1 - e^{-\phi T})\ln F_t - (1 - e^{-\phi T})\ln y_0.\end{aligned} \qquad \text{(A19)}$$

Finally, substituting ln* from (A7) into (A19), the equation to be estimated is now written

$$\ln y_T - \ln y_0 = \pi_0 + \pi_1\ln y_0 + \pi_2\ln(n + g + \delta) + \pi_3\ln s_k + \pi_4\ln s_h + \pi_5\ln F \qquad \text{(A20)}$$

where the π_i, $i = 0,\ldots,4$, are as defined in (A14) and $\pi_5 = \rho(1-e^{-\varphi T})$. Clearly, the estimating equation now includes both a measure of investment and institutions, as in equation (3) of the text.