How many 2-colorings of K5 have no 1-colored K3 ?

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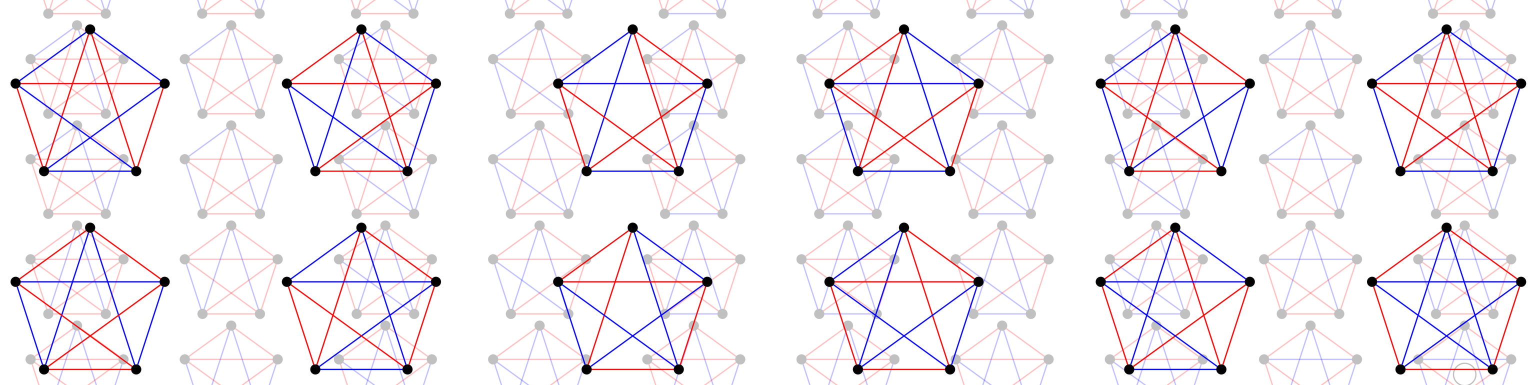
Introduction

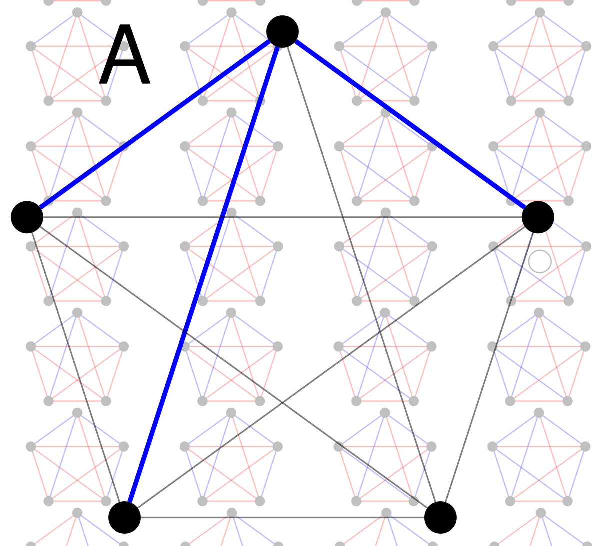
Of the 1024 possible 2-colorings of K5, only 12 have no 1-colored triangles.



Claim 1

If any 3 edges match, then there is a 1-colored triangle.

A



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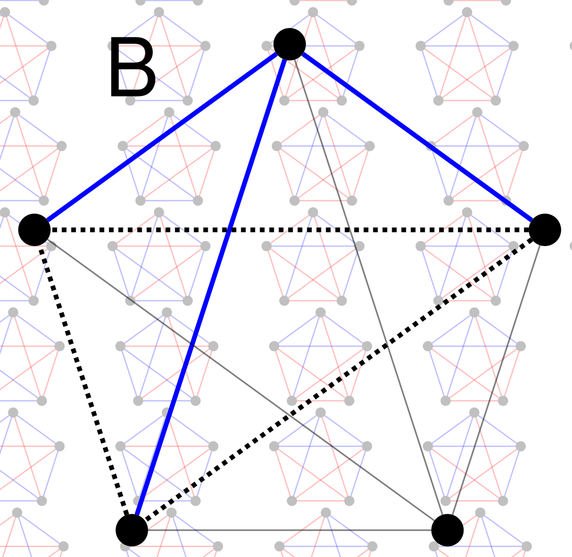

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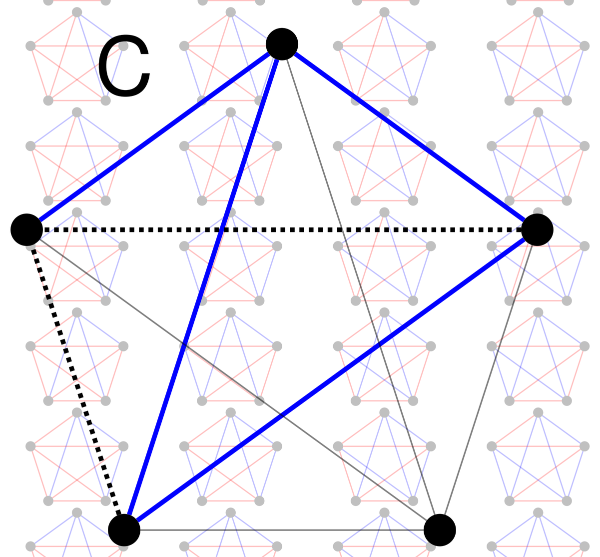

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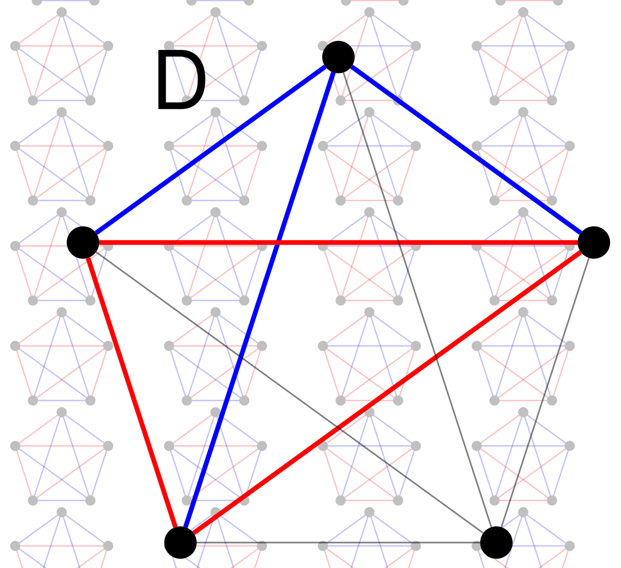
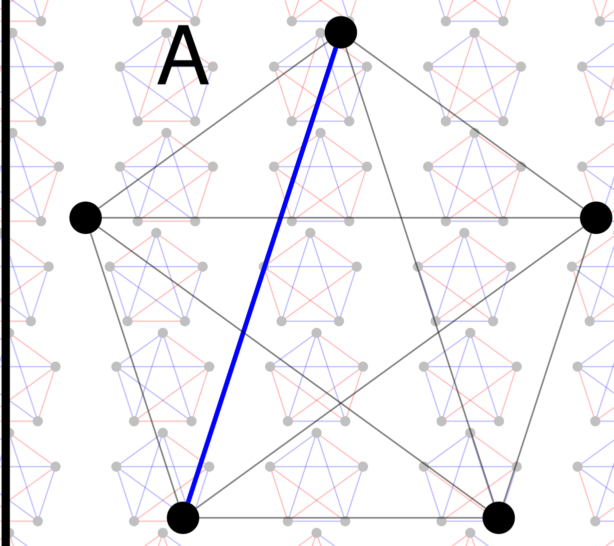


Claim 2

If G has no 1-colored triangles, then G has a 1-colored 5-cycle.

A



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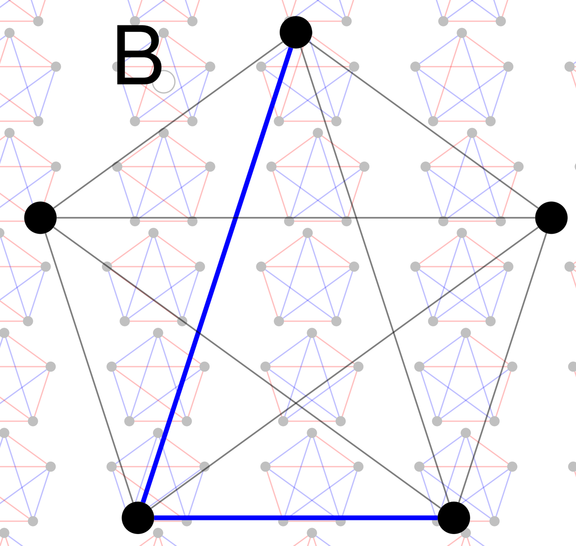

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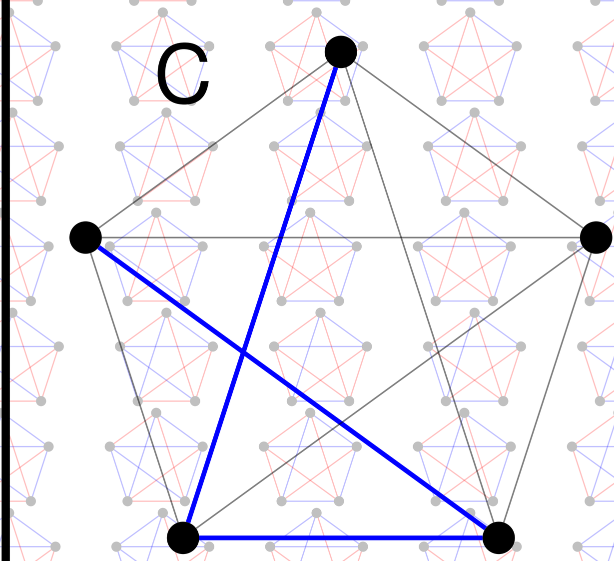

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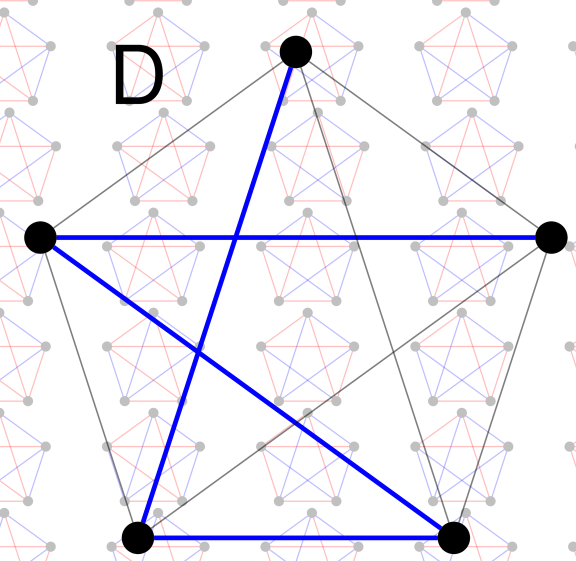
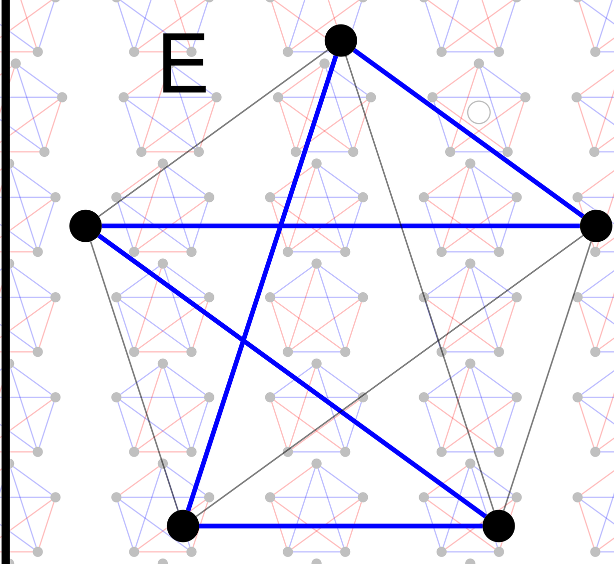


E



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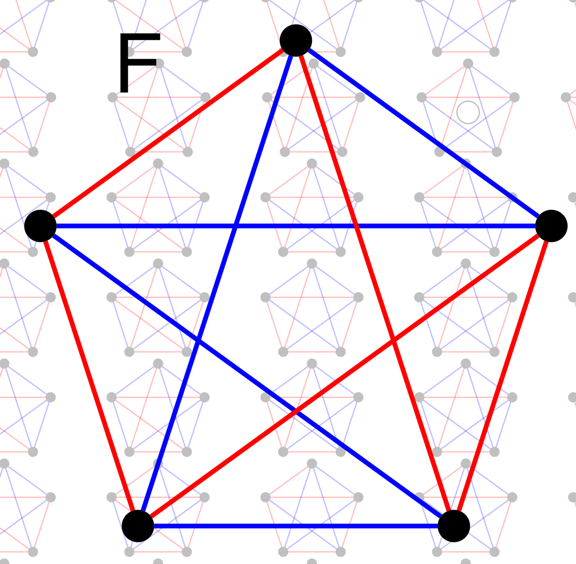


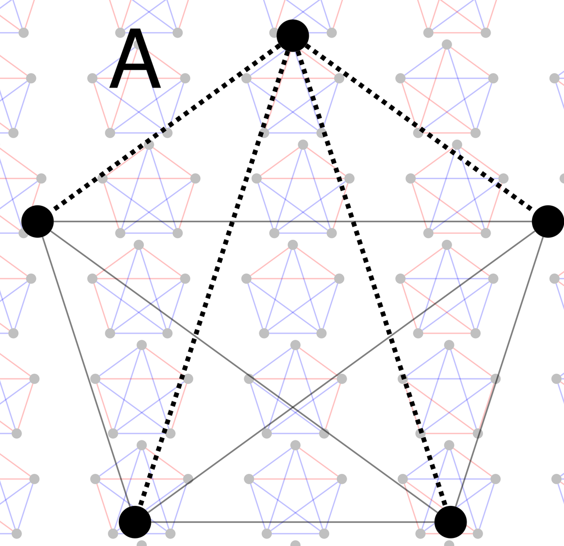
E: 1-colored 5-cycle

F: Remaining edges form a 5-cycle

Claim 3

There are 12 ways to construct a 1-colored 5-cycle.

A



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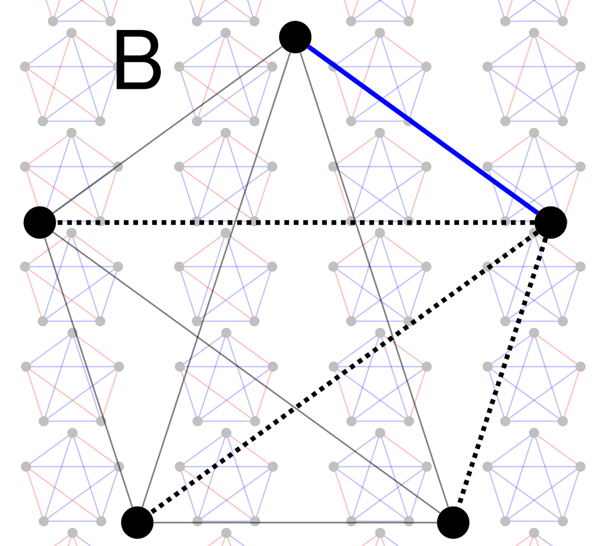

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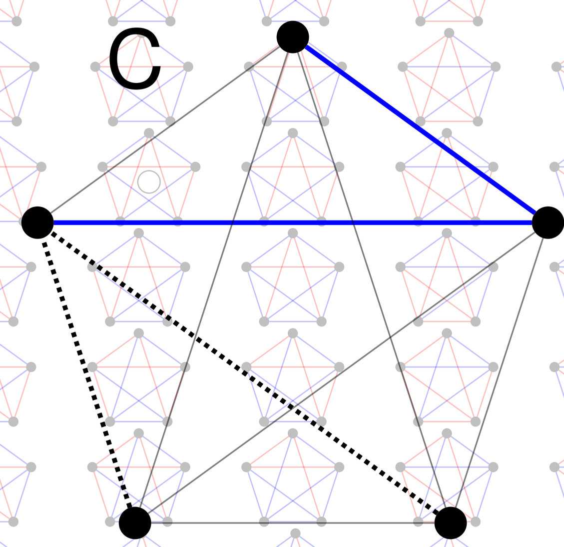

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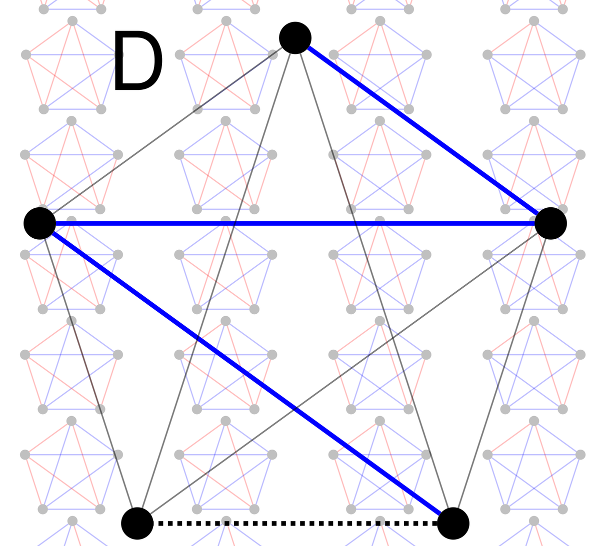

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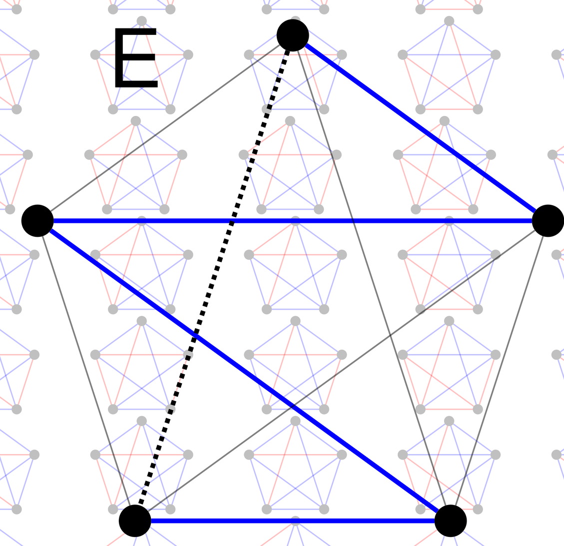

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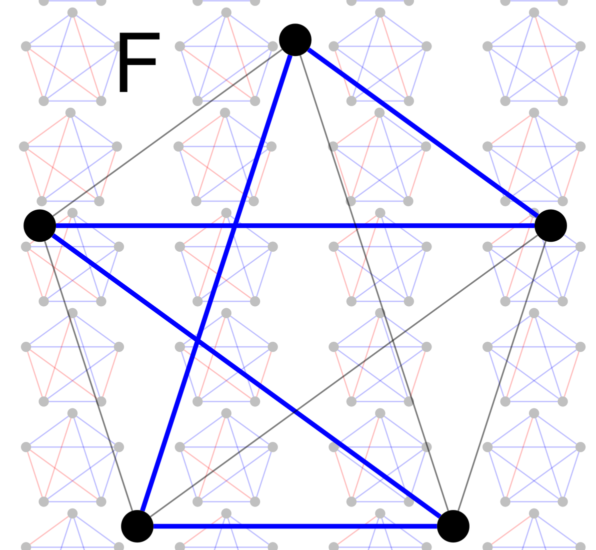

$$\frac{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1}{2} = 12$$