

Ramsey Interest Group
Meeting notes: 9/18/2017

The next meeting will be on Monday, October 10, 2017 at 7:30 pm in 302 Walker.

In his paper *On a problem of formal logic*, Ramsey proves his theorem in order to construct an algorithm for deciding whether or not certain statements of predicate calculus are logically valid.

Question 1: What is the class of predicates for which Ramsey's algorithm works?

We know the classic Ramsey number value $r(3, 3) = 6$. This means that:

- (1) there is a 2-coloring of the edges of K_5 (5 vertices with every possible edge) with no monochromatic K_3 (triangle), and
- (2) every 2-coloring of K_6 contains a monochromatic K_3 .

If we write B_{ij} for "the edge from vertex i to vertex j is blue," then statement (1) corresponds to the fact that the disjunction

$$\forall_{1 \leq i < j < k \leq 5} (B_{ij} \leftrightarrow B_{jk} \leftrightarrow B_{ik})$$

is **not** a tautology. Also, statement (2) corresponds to the fact that

$$\forall_{1 \leq i < j < k \leq 6} (B_{ij} \leftrightarrow B_{jk} \leftrightarrow B_{ik})$$

is a tautology. The only difference in the formulas is changing the index limit 5 to a 6.

Question 2: How can we confirm these facts from propositional calculus? Can we automate the truth table construction? Is there a short proof of the second disjunction in L ?

Question 3: Can we adapt these notions to verify that $r(4, 4) = 18$? Can we improve the bounds $43 \leq r(5, 5) \leq 48$?