Ramsey Interest Group
Meeting notes: 10/16/2017

RIG is changing nights and place and time! The next meeting will be on Tuesday, November 7 at 6:45 pm in 304 Walker.

Item 1: Last time, we discussed guessing colorings to try to improve the lower bounds for finite Ramsey numbers. Are the odds of this working good or awful? We can check for known results. For example, there are 2^{10} different 2-colorings of the edges of K_5, some of which have no monochromatic triangles (K_3). How many of these colorings have no monochromatic triangles? As a percentage of colorings, this would tell us how likely we are to guess at random a coloring showing that $r(3,3) > 5$.

This time we showed that there are 24 2-colorings of K_5 with no monochromatic triangles. This was achieved by proving the following three claims.

Claim 1: Suppose we have a 2-coloring of K_5 with no monochromatic triangles. Then no vertex has three edges that match.

Claim 2: Suppose we have a 2-coloring of K_5 with no monochromatic triangles. Then the coloring must consist of a red 5-cycle and a blue 5-cycle.

Claim 3: There are exactly 24 2-colorings of K_5 consisting of a red 5-cycle and a blue 5-cycle.

Conclusion: Only $24/1024 \sim 2.34\%$ of the 2-colorings of K_5 have no monochromatic triangles. The odds of picking one at random are not particularly good.

Item 2: We know that $r(3,3,3) = 17$, that is, that every 3-coloring of K_{17} has has a monochromatic triangle, but some 3-colorings of K_{16} have no monochromatic triangles [1].

Question: Can we use what we know about 2-colorings of K_5 to construct a 3-coloring of K_{16} with no monochromatic triangles? (Note: There must be at least one.)

Question: How many of the 3^{16} 3-colorings of K_{16} have no monochromatic triangles?

Question: We know that $51 \leq r(3,3,3,3) \leq 62$ [1]. Can we find a 4-coloring of K_{51} with no monochromatic triangle? (Note: There may not be one.)

[1] Stanisław P. Radziszowski, *Small Ramsey numbers*, Electron. J. Combin. **1** (1994), Dynamic Survey 1, 30. http://www.combinatorics.org/ojs/index.php/eljc/article/view/DS1/pdf.