Introduction to Convolution

Problem 1 *Given the image I and kernel K, by hand find the (5,3) pixel value of the output image using zero, periodic and reflexive boundary conditions.*

$$I = \begin{array}{|c|c|c|c|c|}\hline 17 & 24 & 1 & 8 & 15 \\ \hline 23 & 5 & 7 & 14 & 16 \\ \hline 4 & 6 & 13 & 20 & 22 \\ \hline 10 & 12 & 19 & 21 & 3 \\ \hline 11 & 18 & 25 & 2 & 9 \\ \hline \end{array} \quad \textit{and} \quad K = \begin{array}{|c|c|c|}\hline 8 & 1 & 6 \\ \hline 3 & 5 & 7 \\ \hline 4 & 9 & 2 \\ \hline \end{array}$$

The reading introduced a MATLAB *function called* `conv2` *which can be used for convolving images. This function may be used as follows:*

- `C = conv2(I, K, 'valid');` *will compute only those pixel values that are completely defined. What should the size of C be?*
- `C = conv2(I, K, 'same');` *will use zero boundary conditions as described above.*

For example, compute:

```
>> I = [17 24 1 8 15;23 5 7 14 16;4 6 13 20 22;10 12 19 21 3;11 18 25 2 9];
>> K = [8 1 6;3 5 7;4 9 2];
>> C_valid = conv2(I, K, 'valid')
>> C_same = conv2(I, K, 'same')
```

Problem 2 *Each of the kernels K_1, K_2 and K_3 have different properties, and are used for different purposes. To see how each of these perform in detecting edges, we will conduct some experiments in* MATLAB.

$$K_1 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \quad K_3 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}.$$

(a) Create a MATLAB *script m-file with the following statements:*

```
I = imread('testpat2.tif');
K1 = [-1 0 1; -2 0 2; -1 0 1];
K2 = [-1 -2 -1; 0 0 0; 1 2 1];
K3 = [0 -1 0;-1 4 -1;0 -1 0];
C1 = conv2(I, K1, 'same');
C2 = conv2(I, K2, 'same');
C3 = conv2(I, K3, 'same');
subplot(2,2,1), imshow(I),  title('Original Image')
subplot(2,2,2), imshow(C1), title('Edges detected by K1')
subplot(2,2,3), imshow(C2), title('Edges detected by K2')
subplot(2,2,4), imshow(C3), title('Edges detected by K3')
```

(b) Repeat the experiment for the images `logo.tif`, `saturn.tif`, `spine.tif`, *and* `mri.tif`.

Problem 3 *Carefully examine the results of conv2 on your images. Notice that K1 was most effective at detecting those vertical edges in the original image where pixels changed from white to black (from L to R). On the other hand, those vertical edges where pixels changed from black to white were undetected.*

Build a new kernel, K1a, that will detect the vertical edges that K1 missed. (Once you understand why certain edges are detected by convolution with K1 and why others are not, this should be easy! Note: this understanding comes from knowing what happens (at the pixel level) during convolution.)

Problem 4 *Analogous to the problem with K1, K2 has trouble detecting certain edges as well. Describe the problem, and build a new kernel K2a that will detect the edges that K2 misses.*