Handout--Econ 3010
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Note. Present value tables for Topic 7 (Chapter 10) are at the back of this handout.

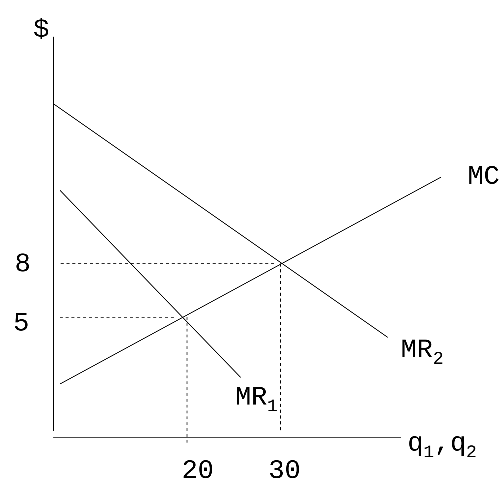
Part 1. Price discrimination

Consider price discrimination (PD) where the firm can segment the market---break consumers up into different sub-markets---but cannot perfectly PD. We will focus on one case only: where the firm can segment into 2 sub-markets and MC is constant. The latter assumption allows us to use simple numerical problems. If MC ≠ a constant, we must determine a combined MR for the 2 sub-markets. Otherwise we cannot find the profit-maximizing outputs (qs).

For example, **Figure 1** shows what happens if we set each sub-market's MR = MC. This CAN NOT be profit-maximizing because the relevant cost is for the 50th unit, since cost is independent of where the good is sold (there is no cost differential between the sub-markets). Given $q_1 = 20$ and $q_2 = 30$, if $\Delta q_1 = -1$ & $\Delta q_2 = 1$, $\Delta TC = 0$, $\Delta TR = \$3$, and $\Delta \pi = \$3$. Now if MC is constant = \$x, simply setting each sub-market's MR = MC is o.k. We do not need to derive the combined MR because we know MR = \$x in each sub-market for profit maximization. The conditions for profit-maximizing PD are:

(1) MR must be = in each sub-market, and (2) MR = MC.

Figure 1



Requirements for PD. (1) segment market, (2) sub-markets must have different elasticities of demand, and (3) prevent resale.

An example.

MC = \$2 (TC = 2Q), $Q = q_s + q_{ns}$, s = students, and ns = non-students. The demand is for concert seats. First suppose the profit-maximizing choice of q_s and q_{ns} yields Q < capacity of the hall. Later we will consider what happens with a capacity constraint.

Inverse demands: $P_s = 30 - q_s/20$ & $P_{ns} = 60 - q_{ns}/10$. Now find TR in each sub-market (so we can derive MRs).

$$TRs = 30q_s - \frac{q_s^2}{20} \text{ and } TR_{ns} = 60q_{ns} - \frac{q_{ns}^2}{10}.$$

Take the derivatives to get MRs:

$$\frac{\partial(TR_s)}{\partial q_s} = 30 - \frac{q_s}{10} \text{ and } \frac{\partial(TR_{ns})}{\partial q_{ns}} = 60 - \frac{q_{ns}}{5}.$$

Now set MR = MC in each sub market:

$$30 - \frac{q_s}{10} = 2 \Rightarrow q_s = 280,$$

$$60 - \frac{q_{ns}}{5} = 2 \Rightarrow q_{ns} = 290.$$

Use the demand schedules to get $P_s = \$16$ and $P_{ns} = \$31$. Compute E_P^D at the profit-maximizing quantities: ($E_P^D = \frac{\partial Q}{\partial P}\frac{P}{Q} = \frac{1}{\text{slope}}\frac{P}{Q}$). Thus, for students, slope = -1/20, and for non-students slope = -1/10.

$$E_{P_S}^D = -20 \times \frac{16}{280} \approx -1.14 \text{ \& } E_{P_{ns}}^D = -10 \times \frac{31}{290} \approx -1.069.$$

The highest $|E_p|$ gets the lowest P. Since MRs must be equal in each sub-market, we must have:

$$P_s = \left(1 + \frac{1}{E_{P_S}^D}\right) = P_{ns}\left(1 + \frac{1}{E_{P_{ns}}^D}\right).$$

If, for example, $P_s < P_{ns}$, $1/E^D_{p_s} > 1/E^D_{p_{ns}}$, which implies $|E^D_{p_s}| > |E^D_{p_{ns}}|$---the sub-market with the lowest P must have the largest E^D_p.

A capacity constraint. Suppose the hall only seats 400. Since $q_s+q_{ns} = 570$, we can sell 400, but who gets the tickets and what are the Ps? In this problem, MC = ∞ because of the capacity constraint (MC = \$2 for $Q \leq 400$). MC is now irrelevant. Think!! What must be true for the profit-maximizing choices of q_s and q_{ns} (using the constraint $q_s+q_{ns} = 400$), and what are the profit-maximizing Ps?

Part 2. Versioning

Suppose a seller can not practice price discrimination, but can offer different versions of its product. The trick here is to get one to implicitly reveal his or her buyer reservation price by the choice of a particular version of the good.

Consider a problem where there are 2 types of customers, A and B. For simplicity, let TVC = 0. Since TFC does not affect output or quality decisions, we can simply ignore all cost. The fraction α of potential buyers are As. Each buyer, A or B, buys at most one unit. The question is whether one buys and what quality is purchased. **Figures 2 & 3** show the demand for quality, x. Suppose the seller knows there are As and Bs, knows α, knows how demand looks, but may not know who is an A or a B. To capture all consumer surplus, so the consumer is just willing to buy, the seller could offer 2 versions of its product: an x_1 quality version at P_1 = G, and an x_2 quality version at P_2 = G+L+M.

However there is a problem here. An A who buys x_2 at P_2 = G+L+M, has CS = 0, but, if the same person buys x_1 at P_1 = G, then CS = L. Thus what is the highest P_2 the seller can charge for x_2 and induce As to just be willing to buy x_2?

Consider **Figure 4**. If the seller reduces the low quality version to x_0, each B will now pay only G-R, so R is lost from each of them. However, what can now be charged for x_2, given P_0, and when does it pay to decrease the quality of the low quality version from x_1 to x_0?

Figures 2 & 3

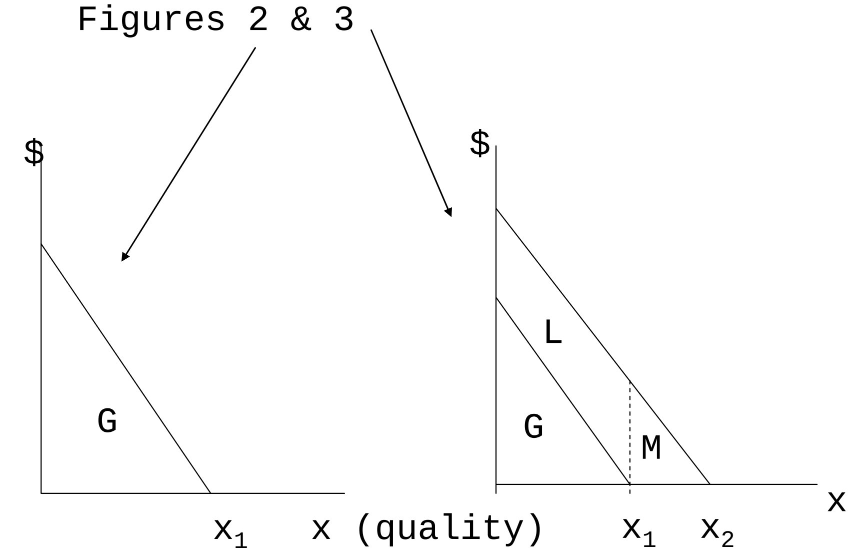
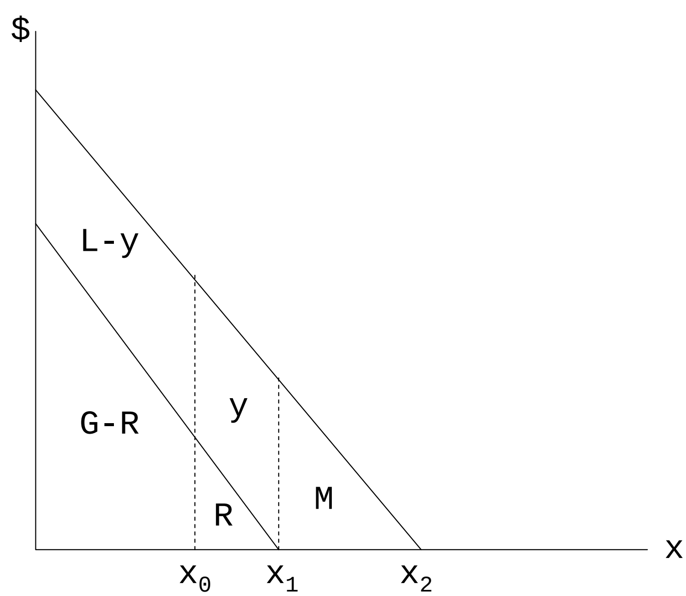


Figure 4



Part 3. **Signaling**

The Spence Signaling Model with Two Jobs

Introduction.

In the basic Spence signaling model (1974), there is only one type of job, and signaling cannot increase social wealth. Signaling rearranges wealth, but at the cost of the resources that are used to signal, so society's wealth decreases. With two different types of jobs, potentially, signaling could improve wealth, and such a problem is considered below. The following variables are used:

- $\alpha =$ the fraction of "good" workers.

- 1- α = the fraction of "bad" workers (employers know these fractions).

- The total number of workers is irrelevant, so set it = 1; thus α = both the fraction & number of good workers, etc.

- *MRP* = marginal revenue product; *MRP* is constant. This is the value of a worker to a firm.

- In *skilled* jobs, $MRP_{good} = 2$ & $MRP_{bad} = 0$.

- In *unskilled* jobs, $MRP_{good} = MRP_{bad} = 1$.

- ω = wealth of society.

- y = level of the signal.

- Total signaling cost is $y/2$ for good workers & y for bad workers.

Without signaling, all workers appear identical, and either all are placed in skilled jobs, or all are placed in unskilled jobs. Since the expected *MRP* if all are placed in skilled jobs is $2\alpha + 0(1\text{-}\alpha) = 2\alpha$, and the expected productivity if all are placed in unskilled jobs is 1, all will be placed in skilled jobs if $\alpha > ½$, & all will be placed in unskilled jobs if $\alpha < ½$.[1]

Signaling.

For signaling, start by ignoring pooling (when all are viewed the same because no signaling occurs). That is, start by assuming beliefs of employers that those who do <u>not</u>

[1] This assumes a trivial percentage of an individual's output is reaped by firms. Then firms will gain more by taking the same percentage from the highest output, which is 2α if $\alpha > ½$.

signal are bad workers & placed in unskilled jobs. Good workers want to be correctly viewed as good if:

$$2 - y/2 \geq 1, \text{ or } \boldsymbol{y \leq 2}. \quad (1)$$

Bad workers prefer to be correctly viewed as bad if:

$$2 - y < 1, \text{ or } \boldsymbol{y > 1}. \quad (2)$$

Thus, signaling can occur if:

$$\boldsymbol{1 < y \leq 2}. \quad (3)$$

Firms will compete & drive y as low as possible, to what is called the *Riley outcome* (1979), $y = 1 + \varepsilon$, where ε is a small positive number. For simplicity, ignore ε, & assume, for $y = 1$, bad workers will not signal.

Good workers & pooling.

Think of the problem this way. If no one signals, ultimately firms realize this & compete for workers, recognizing α of the workers are good. With pooling, all are paid the same. When $\alpha < ½$, all would be put in unskilled jobs & paid 1. When $\alpha > ½$, all would be put in skilled jobs & paid 2α. When would good workers deviate from a pooling equilibrium? With $y = 1$, the net income of a good worker who signals is $2 - y/2 = 1.5$

- When $\alpha < ½$, the wage with pooling = 1. Thus, good workers always prefer signaling to pooling if $\alpha < ½$.

- When $\alpha > ½$, the wage with pooling = 2α. Good workers would prefer to deviate from a pooling equilibrium by signaling if $1.5 > 2\alpha$, or if:

$$\boldsymbol{\alpha < 3/4}. \quad (4)$$

Wealth & pooling.

With signaling, α of workers are in good jobs (with *MRP* = 2), $1-\alpha$ of workers are in unskilled jobs (with *MRP* = 1), & α of workers spend ½ each signaling. Thus, wealth with signaling is:

$$\omega_{signal} = 2\alpha + 1 - \alpha - \alpha/2 = \mathbf{1 + .5\alpha}.$$

If $\alpha < ½$, welfare with pooling, $\omega_{pool}^{\alpha<1/2}$, is 1: all workers are placed in unskilled jobs. Clearly $\omega_{signal} > \omega_{pool}^{\alpha<1/2}$. With signaling, every good worker who reallocates from the unskilled job to the skilled job increases output/wealth by 1 minus the cost of signaling---1/2. Thus, when all α. good workers signal, wealth increases by .5α.

If $\alpha > ½$, welfare with pooling, $\omega_{pool}^{\alpha>1/2}$, is 2$\alpha$. Thus $\omega_{signal} > \omega_{pool}^{\alpha>1/2}$ if $1 + .5\alpha > 2\alpha$, or if:

$$\boldsymbol{\alpha < 2/3.} \tag{5}$$

The reason (4) & (5) differ is good individuals look at their own benefit from deviating from pooling, how much more they will be paid, which, if $\alpha < ½$, is a private & social benefit since it reflects the larger MRP for good individuals in skilled jobs. However, if $\alpha > ½$, all individuals are in the skilled jobs with pooling. Now the good workers'gain is simply higher earnings for them; there is no more output from them. The social gain from signaling in this case is 1-α: the number of bad workers who are identified (when good workers signal) & are moved from the skilled sector (where their *MRP* = 0) to the unskilled sector (where their *MRP* = 1).

Since, when $\alpha > ½$, good workers look at their private gain from identifying themselves---getting paid 2 at a cost of ½ versus getting paid 2α, and this is not the social gain, we can have inefficient signaling. Indeed, if $2/3 < \alpha. < 3/4$, individuals prefer to signal, but signaling reduces welfare (Figure 1).

We now summarize the results

- If $\alpha < ½$, good workers would signal, & doing so moves them from unskilled jobs to skilled jobs---a wealth increase since output goes up 1 at a cost of $y/2$ = ½ for each good worker.

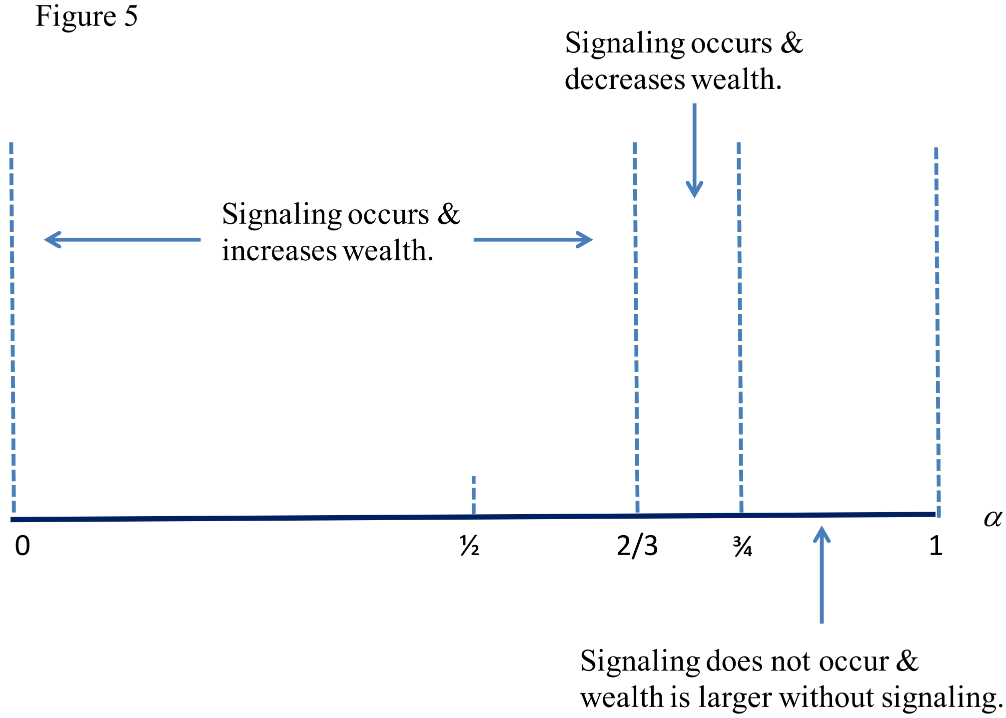
- If $½ < \alpha < 2/3$, signaling occurs & increases wealth. Bad workers move from skilled to unskilled jobs; this increases wealth by 1-α (the number of bad workers multiplied by the difference in their output in unskilled & skilled jobs, = 1). The total cost of signaling by good workers is $\alpha/2$. This increases wealth if $1-\alpha > \alpha/2$, or if $2 - 2\alpha > \alpha$, or if $\alpha < 2/3$---which is true.

- If $2/3 < \alpha < 3/4$, signaling occurs & decreases wealth. The gain is again 1-α , & the cost is again $\alpha/2$, but $\alpha > 2/3$, so wealth is reduced by signaling.

- If $3/4 < \alpha$, good workers will <u>not</u> signal, & wealth is higher with pooling because $\alpha > 2/3$.

Thus signaling can increase or decrease wealth, depending on the size of α.

Figure 5



References

Riley, John G. "Informational Equilibrium." *Econometrica* 47 (March 1979): 331-359.

Spence, A. Michael. *Market Signaling*. Harvard University Press, 1974.

[redacted] PRESENT VALUE OF A FUTURE $1.00: WHAT A DOLLAR AT END OF SPECIFIED FUTURE YEAR IS WORTH TODAY AT ALTERNATIVE INTEREST RATES [redacted]

Year	3%	4%	5%	6%	7%	8%	10%	12%	15%	20%	Year
1	.971	.962	.952	.943	.935	.926	.909	.893	.870	.833	1
2	.943	.925	.907	.890	.873	.857	.826	.797	.756	.694	2
3	.915	.889	.864	.840	.816	.794	.751	.711	.658	.578	3
4	.888	.855	.823	.792	.763	.735	.683	.636	.572	.482	4
5	.863	.822	.784	.747	.713	.681	.620	.567	.497	.402	5
6	.837	.790	.746	.705	.666	.630	.564	.507	.432	.335	6
7	.813	.760	.711	.665	.623	.583	.513	.452	.376	.279	7
8	.789	.731	.677	.627	.582	.540	.466	.404	.326	.233	8
9	.766	.703	.645	.592	.544	.500	.424	.360	.284	.194	9
10	.744	.676	.614	.558	.508	.463	.385	.322	.247	.162	10
11	.722	.650	.585	.527	.475	.429	.350	.287	.215	.134	11
12	.701	.625	.557	.497	.444	.397	.318	.257	.187	.112	12
13	.681	.601	.530	.469	.415	.368	.289	.229	.162	.0935	13
14	.661	.577	.505	.442	.388	.340	.263	.204	.141	.0779	14
15	.642	.555	.481	.417	.362	.315	.239	.183	.122	.0649	15
16	.623	.534	.458	.394	.339	.292	.217	.163	.107	.0541	16
17	.605	.513	.436	.371	.317	.270	.197	.146	.093	.0451	17
18	.587	.494	.416	.501	.296	.250	.179	.130	.0808	.0376	18
19	.570	.475	.396	.331	.277	.232	.163	.116	.0703	.0313	19
20	.554	.456	.377	.312	.258	.215	.148	.104	.0611	.0261	20
25	.478	.375	.295	.233	.184	.146	.0923	.0588	.0304	.0105	25
30	.412	.308	.231	.174	.131	.0994	.0573	.0334	.0151	.00421	30
40	.307	.208	.142	.0972	.067	.0460	.0221	.0107	.00373	.000680	40
50	.228	.141	.087	.0543	.034	.0213	.00852	.00346	.000922	.000109	50

[redacted] PRESENT CAPITAL VALUE (PRICE) OF ANNUITY OF $1.00, RECEIVED AT END OF EACH YEAR [redacted]

Year	3%	4%	5%	6%	7%	8%	10%	12%	15%	20%	Year
1	0.971	0.960	0.952	0.943	0.935	0.926	0.909	0.890	0.870	0.833	1
2	1.91	1.89	1.86	1.83	1.81	1.78	1.73	1.69	1.63	1.53	2
3	2.83	2.78	2.72	2.67	2.62	2.58	2.48	2.40	2.28	2.11	3
4	3.72	3.63	3.55	3.47	3.39	3.31	3.16	3.04	2.86	2.59	4
5	4.58	4.45	4.33	4.21	4.10	3.99	3.79	3.60	3.35	2.99	5
6	5.42	5.24	5.08	4.92	4.77	4.62	4.35	4.11	3.78	3.33	6
7	6.23	6.00	5.79	5.58	5.39	5.21	4.86	4.56	4.16	3.60	7
8	7.02	6.73	6.46	6.21	5.97	5.75	5.33	4.97	4.49	3.84	8
9	7.79	7.44	7.11	6.80	6.52	6.25	5.75	5.33	4.78	4.03	9
10	8.53	8.11	7.72	7.36	7.02	6.71	6.14	5.65	5.02	4.19	10
11	9.25	8.76	8.31	7.89	7.50	7.14	6.49	5.94	5.23	4.33	11
12	9.95	9.39	8.86	8.38	7.94	7.54	6.81	6.19	5.41	4.44	12
13	10.6	9.99	9.39	8.85	8.36	7.90	7.10	6.42	5.65	4.53	13
14	11.2	10.6	9.90	9.29	8.75	8.24	7.36	6.63	5.76	4.61	14
15	11.9	11.1	10.4	9.72	9.11	8.56	7.61	6.81	5.87	4.68	15
16	12.6	11.6	10.8	10.1	9.45	8.85	7.82	6.97	5.96	4.73	16
17	13.2	12.2	11.3	10.5	9.76	9.12	8.02	7.12	6.03	4.77	17
18	13.8	12.7	11.7	10.8	10.1	9.37	8.20	7.25	6.10	4.81	18
19	14.3	13.1	12.1	11.2	10.3	9.60	8.36	7.37	6.17	4.84	19
20	14.9	13.6	12.5	11.5	10.6	9.82	8.51	7.47	6.37	4.87	20
25	17.4	15.6	14.1	12.8	11.7	10.7	9.08	7.84	6.46	4.95	25
30	19.6	17.3	15.4	13.8	12.4	11.3	9.43	8.06	6.57	4.98	30
40	23.1	19.8	17.2	15.0	13.3	11.9	9.78	8.24	6.64	5.00	40
50	25.7	21.5	18.3	15.8	13.8	12.2	9.91	8.25	6.66	5.00	50