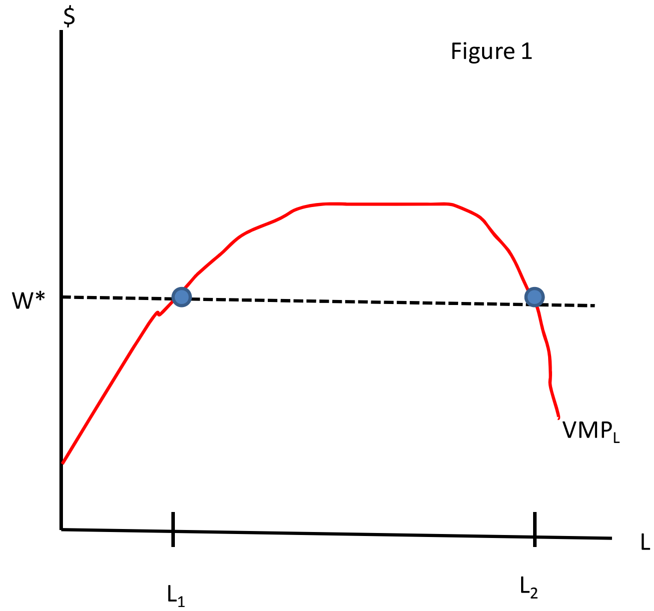
PROBLEM SET TWO--MBA 5110

1. Suppose $MP_K = 100$, $MP_L = 20$, $r = \$50$, & $w = \$20$. Does the combination of K & L now employed minimize the cost of producing the firm's current output? Show what occurs using an isoquant and an isocost line.

2. If $r\uparrow$, show diagrammatically & explain what is now the cost-minimizing way to produce a given q.

3. Using **Figure 1**, explain what is & what is not the profit-maximizing point to hire labor.

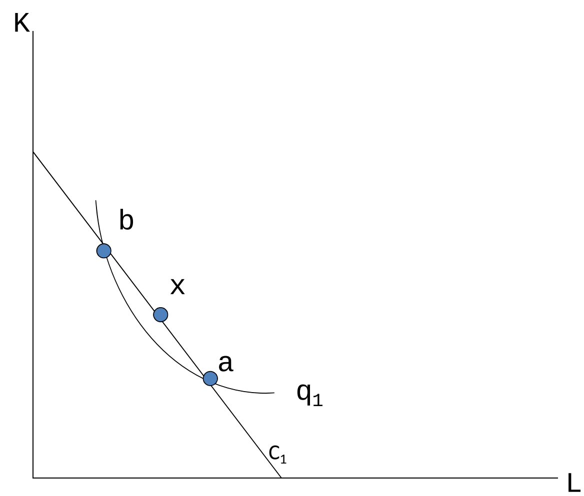


4. Explain costs & benefits with *piece rates* & *output-based pay*. In particular, what are problems with output-based pay for K-12 teachers?

Answers

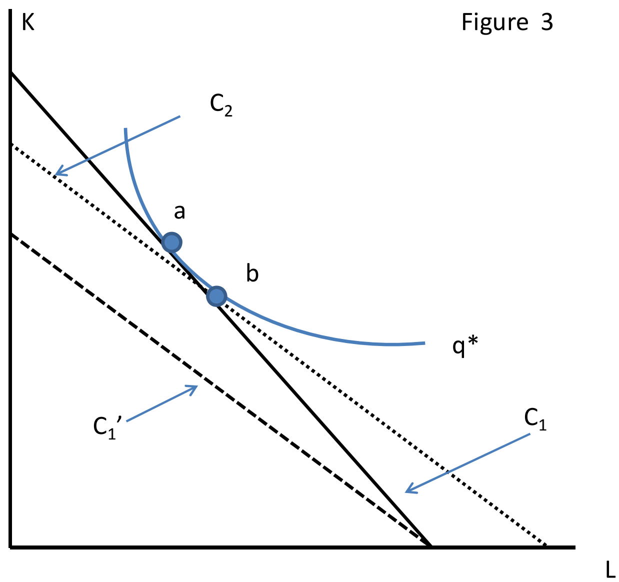
1. $|\text{Slope}_{\text{isoquant}}| = \dfrac{MP_L}{MP_K}$ & $|\text{Slope}_{\text{isocost}}| = \dfrac{w}{r}$, so here $|\text{Slope}_{\text{isoquant}}| = 1/5$ & $|\text{Slope}_{\text{isocost}}| = 2/5$. Thus, the isocost line is steeper than the isoquant, so we are at point a in **Figure 2**. Since $\dfrac{MP_L}{MP_K} < \dfrac{w}{r}$, rearrange & find $\dfrac{MP_L}{w} = \mathbf{1} < \dfrac{MP_K}{r} = \mathbf{2}$: spending \$1 more on K increases output by 2 units, & spending \$1 more on L increases output by **1** unit. Thus, spending \$1 more on K & \$1 less on L $\Rightarrow \Delta q = 1$ & $\Delta C = 0$: we move up C_1 to the left from point a. As we continue to increase K & decrease L, $MP_K\downarrow$ & $MP_L\uparrow$ until $\dfrac{MP_L}{w} = \dfrac{MP_K}{r}$ ---somewhere between points a & b, say at x.

Figure 2



2. Initially, cost is minimized at point a (**Figure 3**); the firm produces q^* at a total cost of C_1. If $r\uparrow$, the same total cost now is represented by C_1'. To find the optimal way to produce q^*, move to an isocost line tangent to C_1' reflecting the new relative input prices, w/r. This is isocost line C_2, which clearly involves higher cost than C_1. Because $w/r\downarrow$, C_2 must be tangent to the isoquant q^* where $|\text{slope}_{\text{isoquant}}|$ is smaller than at a; this occurs at point b. Thus, to produce the same output, q^*, the firm would optimally substitute L for K since the latter is relatively more expensive.

3. The profit-maximizing point to hire labor is where the value of the marginal product of labor ($VMP_L = P \times MP_L$ (product price times the marginal product of labor) equals the wage (w). This occurs when $L = L_2$ in **Figure 1**. When $L = L_1$, profit is at a (local) *minimum*. For $L < L_1$, $w > VMP_L$, so reducing L lowers cost more than it lowers revenue: profit increases. For $L > L_1$, $w < VMP_L$, so increasing L raises cost less than it raises revenue: profit again increases.



4. Output-based pay, of which a piece rate is one example, provides incentives for more work effort, & induces more productive individuals to apply to firms offering such pay. A cost is individuals have an incentive to sacrifice quality. Thus, such schemes must find a way to penalize low quality, which is possible if a firm can identify who produced what (as Safelite Glass could do). Also capital may be abused (as with taxi cabs) if piece rates are used.

Teacher pay based on standardized test scores (STS) must control for prior ability of students. STS induces "teaching to the test," which is less of a problem the broader the knowledge covered by the test. Pay based on STS may induce teachers to focus effort on marginal students---those close to passing---at the expense of worse students, & may reduce teacher cooperation---which is less of a problem the less important is such cooperation.