PROBLEM SET FOUR --MBA 5110

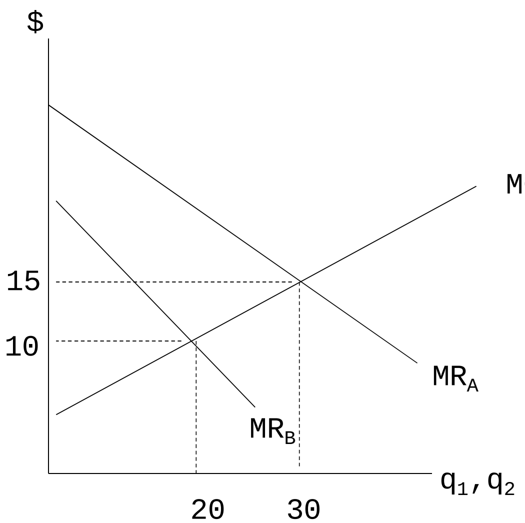
1. A monopolist can segment its market into two sub-markets, call them 1 & 2. C = \$200 + 5Q, with $Q = q_1 + q_2$. The demand in the sub-markets is:

$$P_1 = 20 - q_1/2 \ \& \ P_2 = 35 - q_2.$$

a) Find the profit-maximizing q_1, q_2, P_1, P_2, and find π, and E_p^D in each sub-market at the profit-maximizing P & q.
b) Which sub-market gets the lowest P? Why?
c) What happens if there is a capacity constraint $Q \leq 20$?

2. In Figure 1, is π maximized when $q_A = 30$ & $q_B = 20$? Explain.

Figure 1



3. Stars have a value of \$60 & lemons have a value of \$30 to firms. Firms are unable to cheaply identify who is a star. Education, y, is cheaper for stars than for lemons because stars exert less effort than lemons. For a star, education costs $y/2$, &, for a lemon, education costs $2y/3$. Let y be a continuous variable (that is, it can be a non-integer).
a) Show algebraically & explain the lowest & highest values for y for which signaling could occur.
b) Assuming $y = y_{Riley}$, if the fraction of stars in the population is known to equal s, when will stars prefer signaling to pooling?

4. Suppose utility = $U = 10\sqrt{I}$, where I = income. I = \$100 (probability = .25) or \$900 (probability = .75).

a) Find E(I) & E(U).

b) Find the risk premium (RP).

Answers

1. a) & b) MC = \$5. $TR_1 = 20q_1 - \frac{q_1^2}{2}$, & $TR_2 = 35q_2 - q_2^2$. Thus $MR_1 = 20 - q_1$ & $MR_2 = 35 - 2q_2$.

Set $MR_1 = MC$ & $MR_2 = MC$: $20 - q_1 = 5$ & $35 - 2q_2 = 5$, so $\boldsymbol{q_1 = q_2 = 15}$.
Insert q_1 into the demand for sub-market 1 & do likewise for sub-market 2 & get P_1 & P_2:
$\boldsymbol{P_1}$ **= \$12.5** & $\boldsymbol{P_2}$ **= \$20**.
$\pi = P_1q_1 + P_2q_2 - 200 - 5(q_1 + q_2)$ = **\$137.5**.

$E_p^D = \frac{1}{slope}\frac{P}{q}$, so $E_p^{D_1} = -2(12.5)/15 \cong$ **-1.67** & $E_p^{D_2} = -20/15 \cong$ **-1.33**.

The sub-market with the highest $\left|E_P^D\right|$ gets the lowest P---sub-market 1.

c) If $Q \leq 20$, let $q_2 = 20 - q_1$ (or you could let $q_1 - 20 - q_2$) & set MRs equal:

$20 - q_1 = 35 - 2(20 - q_1)$,

$25 = 3q_1$, **8.33** $\cong q_1$ & **11.67** $\cong q_2$ (unless the qs must be integers)

$P_1 \cong$ **\$15.83** & $P_2 \cong$ **\$23.33**; both Ps↑ & both qs↓ due to the capacity constraint.

2. The relevant MC is MC for $Q = 50$, which is clearly > \$15. Thus, the π-maximizing $Q < 50$. Given the firm sells $Q = 50$, it should sell more in sub-market A & less in sub-market B since, with $q_A = 30$ & $q_B = 20$, MR_A = \$15 & MR_B = \$10. If the firm sells 1 more unit in sub-market A & 1 less unit in sub-market B, ΔR = \$5 (\$15-\$10) , & $\Delta C = 0$, so $\Delta\pi$ = \$5. The firm should continue to sell more in sub-market A & less in sub-market B until $MR_A = MR_B$, which $\Rightarrow q_A > 30$, $q_B < 20$, & \$10 < $MR_A = MR_B$ < \$15.

3. a) If employers believe those with $y \geq y^*$ are stars, then the conditions for a star to signal & a lemon to *not* mimic a star (given those who signal will be paid 60, & others will be paid 30) are:

$$60 - y/2 \geq 30,$$

$$\mathbf{60} \geq y. \tag{1}$$

$$60 - 2y/3 < 30,$$

$$\mathbf{45} < y. \tag{2}$$

$\therefore$ $45 < y \leq 60$.

Thus, $45 < y^* \leq 60$. Competition by firms for workers will drive $y^* \rightarrow 45 \equiv y_{Riley}$. Technically, y^* must be slightly greater than 45 for lemons *not* to mimic stars, but we can use $y^* = 45$.

b) If all set $y = 0$ (pooling), then the pooling wage is $W_{Pool} = 60s + 30(1\text{-}s) = 30(1+s)$. The payoff to a star from signaling = $60 - y_{Riley}/2 = 37.5$. Stars prefer signaling to pooling if $37.5 > 30(1+s)$, or $s < .25$.

4. $E(I)$ = [probability I = \$100][\$100] + [probability I = \$900][\$900] = .25[\$100] + .75[\$900] = **\$700**.

$E(U)$ = [probability I = \$100][U(\$100)] + [probability I = \$900][U(\$900)] =

$[.25][10\sqrt{100}] + [.75][10\sqrt{900}] = 25 + 225 =$ **250**.

To find RP, find the certain I that yields $U = 250$:

$10\sqrt{I} = 250$

$\sqrt{I} = 25$

$I = 25^2 =$ **625**.

Thus RP = **\$75**.