Adverse Selection & Signaling
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Timothy Perri
Appalachian State University

I. Adverse selection: the market for lemons

The quality of some products is difficult to judge at purchase. Price will reflect average/expected value to the buyer, but also the highest quality to a seller of a good offered for sale. Thus, the higher quality sellers may not offer their goods for sale.

Suppose a given good has a range of quality: $x_{min} \leq x \leq x_{max}$. Assume x is distributed uniformally. Now x = quality/value to sellers (who know x). A buyer who knew x would pay vx. If $v < 1$, there would be no market even with perfect information. If $v = 1$, there would barely be a market with perfect information since buyers & sellers place the same value on the good. Thus, assume $v > 1$. Also, for a reason explained below, assume $v < 2$.

Now price = 2 things in equilibrium: 1) the expected value, E(vx), to buyers; & 2) the maximum quality (to sellers) sold, x^*. Under *perfect information*, all items are sold ($x^* = x_{\max}$) since $v > 1$. However, since price = P = the expected value to buyers, the higher quality sellers may find $P < x$---they won't offer their goods for sale.

In equilibrium, assuming some but not all goods are traded (the best ones are not traded), E(vx) = x*,

$$\frac{v[x_{min}+x*]}{2} = x^* \qquad (1)$$

Crudely, the left-hand side (LHS) of (1) = demand, & the right-hand side (RHS) of (1) = supply. Now, manipulating (1):

$vx_{min} + vx^* = 2x^*$,

$vx_{min} = (2-v)x^*$,

$$x^* = \frac{vx_{min}}{2-v}. \qquad (2)$$

Note two things using (1) that occur with a uniform distribution, & may occur with other distributions.

1st, if $v > 2$ the LHS of (1) > the RHS of (1). Make the LHS as small as possible: $x_{min} = 0$. If $v > 2$, LHS > x^* for all values of x^* including $x^* = x_{max}$. Suppose $v = 2.1$ & $x_{max} = 10$. Buyers know the expected value of all items to them = 2.1[10/2] = 10.5---all will be offered for sale since $x_{max} = 10$.
Thus, if buyers value items sufficiently more than sellers, the market works as with perfect information---all items are sold.

2^{nd}, if $v < 2$ & $x_{min} = 0$, **no items will be sold**. Now LHS of (1) < x^* There is no possibility LHS = RHS except when $x^* = 0$---no items are sold. Generally, I will assume $x_{min} > 0$ & $v < 2$.

EXAMPLE ONE. $x_{min} = 10$, $x_{max} = 40$, & $v = 1.5$.

Using (2), $\boldsymbol{x^*}$ = 1.5 (10)/.5 = **30 = *P*.**
Items that range in value to sellers from 10 to 30 are sold.
The highest quality seller ($x = x^* = 30$) is just willing to sell.
Those with $30 < x \leq 40$ will NOT sell.
Average x sold = 20.
Average value to buyer = 1.5(20) = 30.

EXAMPLE TWO. Same as Example One <u>except</u> $v = 1.25$. Find $\boldsymbol{x^*}$. Check to make sure that $E(vx) = x^* = P$---the average value to buyers = the maximum value to a seller = the price.

II. Educational Signaling

Signaling occurs when some on the informed side of the market want to communicate information to the uninformed side. Usually sellers (workers in the labor market) are informed. Signaling involves taking an action (the signal) which is costly but which reveals the sender's type. If different types select different signals, they reveal their types.

To be successful, signals must be credible. Credible signals occur if the higher quality/more able sellers have a lower marginal cost (MC) of signaling than do less able sellers.

Conditions for signaling to occur.

If the more able have a lower MC of signaling than the less able, then it is *possible* signaling will occur. Two things are necessary for a *signaling equilibrium* in which the more able choose a higher level of the signal than the less able choose.

1^{st}, the level of the signal, y, must be such that, given signaling cost, a) the more able prefer to be correctly viewed as more able, & b) the less able prefer to be correctly viewed as less able----that is, they do not optimally mimic the more able.

2^{nd}, the net payoff for the more able in a signaling equilibrium must at least equal their payoff in a *pooling equilibrium* in which both types choose the same level of y.

Simplifying assumptions.

In order to not needlessly complicate the analysis, it is assumed the signal, education, units of which are denoted by y, has no direct effect on individual productivity. Thus, in a pooling equilibrium, all would set $y = 0$. Also, it is assumed the signal is obtained immediately, & work occurs for one period (so present discounted value analysis can be ignored).

Assumptions about individuals.

Let Smart individuals have productivity = 40 & Dumb individuals have productivity = 10. The total cost of obtaining the signal is $y/2$ for Smart individuals & y for Dumb individuals. Why could Smart individuals have lower educational cost than Dumb individuals? One reason is that Smart individuals require less study time, meaning they have more time for work. Thus, *net foregone earnings* (a significant percentage of education cost) are lower for Smart individuals than for Dumb individuals.

Employer beliefs.

To analyze signaling, 1st ignore the possibility of pooling. Assume employers believe those with at least some level of the signal are more able (call them *Smart*), & those with lower (possibly zero levels) of y are less able (call them *Dumb* workers). Thus, firms offer a wage of 40 to anyone with a sufficiently large level of y, and a wage of 10 to others. Firms will ultimately learn individual productivity, & thus will see if their beliefs have been confirmed.

A signaling equilibrium.

Since y does not affect productivity, in a signaling equilibrium, Smart individuals set $y > 0$, & Dumb individuals set $y = 0$.

We must determine what levels of y will work, that is, levels that Smart individuals would choose, & that Dumb individuals would not choose.

Smart individuals must prefer to be viewed as Smart versus being viewed as Dumb:

$$40 - y/2 > 10, \text{ or } \boldsymbol{y < 60}. \qquad (1)$$

Dumb workers must prefer to not mimic Smart workers, so:

$$40 - y < 10, \text{ or } \boldsymbol{y > 30}. \qquad (2)$$

Thus, signaling will occur if:

$$\boldsymbol{30 < y < 60}. \qquad (3)$$

Thus any level of y that exceeds 30 & is less than 60 will work in that, if such a level is believed by firms to mean the individual is Smart, then employers' beliefs will be confirmed.

Competition by firms for workers.

Since any level of y consistent with (3) works, employers will compete for individuals by driving y as low as possible. For simplicity, say $y = 30.1$ (always add .1 in such a problem). Again, with y not affecting productivity, in a signaling equilibrium, there is no reason for Smart individuals to set y greater than the minimum level that will work (in that Dumb individuals will not mimic Smart individuals & choose this level of education). Thus, in a signaling equilibrium, Smart individuals set y = 30.1, & Dumb individuals set y = 0.

Pooling.

Now, for the moment, forget about the possibility of signaling. What would happen if all set y = 0? This is a *pooling equilibrium*. Suppose, absent signaling, firms only know (through learning over time) the expected fraction of each type of individual. Further, suppose s is the fraction of Smart individuals in the population. Then, with pooling, firms will compete & drive the wage, W_{Pool}, to the level of expected productivity:

$$W_{Pool} = 40s + 10(1\text{-}s) = 10 + 30s = \mathbf{10(1+3}\boldsymbol{s}\mathbf{).} \tag{4}$$

Thus, if s = 0 (all individuals are Dumb), W_{Pool} = 10, &, if s = 1 (all individuals are Smart), W_{Pool} = 40.

What will the equilibrium be?

If employers offer to pay 40 for those who set $y \geq 30.1$, what if no one chooses $y > 0$? Then employers will adjust their beliefs (because their initial beliefs will not be confirmed), & will compete for workers by offering to pay W_{Pool}.

Using the results from above, in a signaling equilibrium, with y = 30.1 the net payoff to a Smart individual is $40 - \frac{30.1}{2}$ = **24.95**. Thus Smart individuals will only deviate from a pooling equilibrium by setting y = 30.1 if their net payoff at least equals W_{Pool}:

$$24.95 \geq 10(1+3s),$$

$$2.495 \geq 1+3s,$$

$$1.495/3 \approx \mathbf{.498} \geq \boldsymbol{s}\mathbf{.} \tag{5}$$

In this case, if fewer than approximately ½ of individuals are Smart, it pays the Smart individuals to deviate from a pooling equilibrium. Conversely, if there are enough Smart individuals in the population ($s > .498$), despite the lower pay Smart individuals receive when not distinguished from Dumb individuals (W_{Pool} versus 40), it is not worth it for Smart individuals to expend y = 30.1 = to distinguish themselves & get paid more.

Examples.

❶ Suppose s = .25. Then W_{Pool} = 17.5 < 24.95: Smart individuals prefer signaling to pooling.

❷ Suppose s = .75. Then W_{Pool} = 32.5 > 24.95: Smart individuals prefer pooling to signaling.