The Draft and the Quality of Military Personnel

by

Timothy Perri*

March 09, 2010

Abstract

It has been argued the draft may enable the military to attract more able individuals than a volunteer military, and thus increase welfare. In our theoretical model, we find this may be the case if a volunteer military simply takes the least able individuals. When the military tests individuals and does not take the lowest quality applicants, neither a random draft nor a draft with testing increases welfare, and both usually decrease welfare. Only if testing is relatively costly would a random draft dominate a volunteer military with testing.

* Department of Economics, Appalachian State University, Boone, NC, 28608.
(perritj@gmail.com)
1. Introduction

The U.S. military has become more skilled over time. The military defines high quality recruits as those who are high school graduates and who score in the top 50% on the Armed Forces Qualification Test (AFQT). From 1981 to 2004, the percentage of high quality recruits went from 34 to 72 (Army), 55 to 66 (Navy), 49 to 69 (Marines), and 60 to 81 (Air Force) (Asch et al., 2005). In 2006, 91% of U.S. military recruits were high school graduates, compared to 80% for all U.S. residents ages 18-24. In 2001, 18% of U.S. recruits worked in information technology related tasks, and almost 30% of these individuals were considered information technology core positions (Hosek et al., 2004).

As early as 1990, economists have suggested a draft might be preferable to a volunteer military when the quality of military personnel is important. Ross (1990, 1994) considered the merits of a draft when the quality of personnel is not homogeneous. He finds some reason to believe a draft might be superior to a volunteer military. More recently, Berck and Lipow (2008) suggest a positive relation between civilian and military productivity may mean one of the usual costs associated with the draft---the “wrong” people are inducted---may disappear.

There are several problems with the previous analysis of a draft with heterogeneous labor quality. First, there are costs other than the wrong individuals being inducted that remain with a draft. Asch et al. (2005) note military personnel quality depends on a taste for the military and individual effort. Becker (1957) also considered the reduced effort to be expected with conscripts

---

1 The Armed Forces Qualification Test (AFQT) is based on the Armed Services Vocational Aptitude Battery (ASVAB). The ASVAB has 10 sub-tests. The AFQT is computed by adding the following: the scores on arithmetic reasoning and math knowledge, and double the scores on paragraph comprehension and word knowledge. Thus, the AFQT roughly measures verbal and quantitative skills (Hosek and Mattock, 2003). The top X% on the AFQT means these individuals scored where the top X% of enlisted personnel and officers scored during World War Two. The AFQT has changed over time so comparability between time periods is somewhat questionable (Pirie, 1980).
2 Information technology related positions include navigators and radar operators. Core IT positions include system operators and network analysts (Hosek et al., 2004).
3 Ross (1990) is an unpublished version of Ross (1994). The former has a more detailed discussion of heterogeneous labor quality. I thank Tom Ross for providing me his unpublished paper.
relative to volunteers. Second, if the military attempts to draft the more able, these individuals will try to appear less able, reducing the number of high quality individuals drafted.⁴ Third, even if a draft brings in more able individuals, given the opportunity cost of these individuals, welfare may not be improved.⁵ Finally, a draft has been compared to a low quality volunteer military—which is not the kind of military in the U.S. today.

The main focus of this paper is to consider a theoretical model in which the military may tests individuals, excluding those with low ability. We compare a volunteer military with testing to two kinds of draft militaries, one with testing and one with a random draft (and no testing). We allow for the possibility some individuals may be able to “fail” the test, that is appear to be of less ability. In order to not bias the results against the draft, we assume some who would like to fail the test are not able to do so.

The rest of the paper proceeds as follows. The outline of the model is provided in the next section. Although the main focus is on a military with testing, in Section 3, testing is ignored, and a random draft and a low quality volunteer military are compared. In Section 4, testing is introduced. Costly testing is considered in Section 5. In Section 6, costly deferments and the deadweight loss from taxation are examined because they have been analyzed in a world of homogeneous labor. The paper is summarized in Section 7.

2. Outline of the model

Consider a world in which the number of individuals available for work is normalized to one. An individual’s ability/quality is denoted by $q$, where $q$ is uniformly distributed on $[0,1]$.

---

⁴ Richard Danzig (then assistant secretary of defense) argued, if the military tried to draft the top half of the ability distribution, “You generate a lot of people who are trying to cheat, to appear to be not as good as they really are...” (Danzig, 1982, p.110).

⁵ Berk and Lipow (2008) note a draft may cause the enlistment of too many high quality individuals.
Civilian productivity is simply $q$. One can be employed either in the civilian sector or in the military, and there is no disutility associated with being in either sector. With one caveat, in the military, individuals are either productive, with a marginal revenue product of $k$, or unproductive, with a marginal revenue product of zero. Those who are productive in the military have $q \geq q^*$, so the fraction of individuals who would be productive in the military\(^6\) equals $1-q^*$. Only a fraction of the labor force, $m$, is desired in the military; additional individuals in the military in excess of $m$ have no value (Figure 1).

![Figure 1. Military marginal revenue product for those with $q \geq q^*$.](image)

The assumptions regarding military productivity address two points discussed in Section One. First, those more able in civilian employment are more productive in the military. Second, it may be too costly to have those with the highest civilian productivity in the military. The second point militates against a draft socially dominating a volunteer military. However, we

---

\(^6\) Ross (1990) also assumes a fraction of potential military personnel would be unproductive if enlisted.
will consider the possibility those productive in the military are so scarce it is optimal to have those with the highest civilian productivity in the military.

When we consider the military testing applicants, if individuals desire to be in the military, it will be assumed the test allows the military to determine who would be productive if enlisted. One possible error in testing was discussed previously: those who do no want to serve in the military may fail the test and appear to be unproductive if enlisted. Let $f$ equal the probability an individual who would otherwise pass the test may purposely fail. If the “test” is simply one’s score on the Armed Forces Qualifications Test (AFQT), then $f$ should equal one since anyone can fail if he so desires. However, the military can use as a test some combination of educational attainment and the AFQT score. In this case, even an intentionally low score on the AFQT may not preclude the military recognizing whether one would be productive if enlisted. Thus, we allow for the possibility $f < 1$.  

3. A random draft vs. a low quality volunteer military

Although our main focus is on a military when testing occurs, consider whether a random draft would dominate a low-quality volunteer military (LQVM)---that is when no testing occurs. In order to allow the highest quality with a draft, it is assumed no volunteers are allowed with a draft. In a draft (with no testing), $m$ individuals are called at random, so the expected number of productive individuals in the military is $m(1-q^*)$. With $q$ uniformly distributed between zero and one, the draft can not dominate a volunteer military because more able individuals will all intentionally fail the test if drafted and paid less then their opportunity cost. Thus, we implicitly assume $f$ is non-trivially lower than one, and assume $p$ is low enough it can be ignored.

---

7 We will not consider the possibility one who is unproductive might pass the test, $p$. If only the AFQT were used, then, as argued in the text, $f$ should equal one. In this case we could have $p > 0$. However, the more other measures of ability (such as education) are used, the less likely it is errors will be made, so both $f$ and $p$ should be reduced. If $f$ is close to one, the draft can not dominate a volunteer military because more able individuals will all intentionally fail the test if drafted and paid less then their opportunity cost. Thus, we implicitly assume $f$ is non-trivially lower than one, and assume $p$ is low enough it can be ignored.

8 Beginning in 1918, no volunteers were allowed in World War One (Chambers, 1987).
and one, the $m$ individuals called have an expected civilian output of $\frac{1}{2}$. Thus welfare with a random draft, $\Omega_{random
draft}$, is:

$$\Omega_{random
draft} = m[(1-q^*)k - \frac{1}{2}]. \quad (1)$$

For a random draft to be preferred to no military $(1-q^*)k - \frac{1}{2} > 0$, or

$$k > \frac{1}{2(1-q^*)}. \quad (2)$$

The RHS of ineq.(2) is minimized when $q^* = 0$, so, for there to be any case for a random draft, we must have $k > \frac{1}{2}$. In the rest of this paper, it is assumed ineq.(2) holds.

If no testing is used, a volunteer military will set a wage, $w_{iv}$, equal to $m$, and attract the $m$ individuals with the lowest quality. If $m \leq q^*$, no individual would be productive in the military with a volunteer system, and the draft would dominate a LQVM. If $m > q^*$, then $m-q^*$ individuals will be productive in the military with a LQVM, and, with the average opportunity cost of the individuals enlisted equal to $m/2$, welfare would be:

$$\Omega_{LQVM} = (m-q^*)k - \frac{m^2}{2} \quad (3)$$

Using eqs.(1) and (3), a random draft is socially preferable to a LQVM if:

$$kq^* > \frac{m}{2}. \quad (4)$$
Assuming \( \text{ineq.}(2) \) holds (a random draft is preferable to no military), with \( m < 1 \), if fewer than 50% of the pool of potential military manpower would be productive if enlisted \( (q^* > \frac{1}{2}) \), then a draft is socially preferable to a LQVM. It is not clear whether \( q^* \geq \frac{1}{2} \) because of the uncertain size of the actual military labor pool. Some evidence may help illuminate the issue.

Recent testimony by the Director for Accession Policy for the Department of Defense (Gilroy, 2009) shows, for fiscal year 2009, almost 85% of the 31 million individuals in the U.S. who are ages 17-24 are unfit for military service, but this still leaves almost 5 million individuals, with the number of new enlistees desired less than 200,000 per year. For 2008, for all services, 92% of new recruits were high school graduates, and 69% scored in the top half of the Armed Forces Qualification Test. Also, the population ages 17-24 is expected to grow from 31 million to 35 million by 2025.

Thus, if the entire population of individuals in the relevant age group is the potential military labor pool, then \( q^* > \frac{1}{2} \). However, if we restrict the pool to those who would not be deferred, then \( q^* < \frac{1}{2} \). Also, it appears there are more productive individuals than the military desires, so \( 1-q^* > m \). However, we will consider the case when \( 1-q^* < m \).

From \( \text{ineq.}(4) \), a random draft is more likely to be preferred to a LQVM the larger are \( k \) and \( q^* \) and the smaller is \( m \). With a random draft, the number of productive individuals in the military is \( m(1-q^*) \). Assuming \( m > q^* \), then \( m-q^* \) individuals are productive with a LQVM. More individuals are productive with a random draft than with a LQVM if \( m < 1 \). Therefore, an increase in the value of a productive individual in the military, \( k \), increases welfare with a random draft more than it does with a LQVM. An increase in \( q^* \) reduces the number of productive individuals in the military, but does so one for one with a LQVM and only by \( 1-m \) for a unit increase in \( q^* \) with a random draft. Thus, a random draft is more likely to be socially
preferable to a LQVM as \( q^* \) increases. A decrease in \( m \) decreases the number of productive individuals one for one with a LQVM, but only decreases the number of productive individuals with a random draft by \( 1-q^* \), so a smaller \( m \) implies a random draft is more likely to be preferred to a LQVM.

The result a smaller military implies a random draft is more likely to be preferred to a LQVM is the opposite of what has been found with homogeneous military quality (Johnson, 1990, Lee and McKenzie, 1992, Ross, 1994, Warner and Negrusa, 2005, and Perri, 2009). However, the previous studies all compared the reduced deadweight loss from taxation with a draft (due to a lower military wage) to other costs of a draft, with the former more important the larger the military.

Since a random draft results in more individuals who are productive in the military, a LQVM can only dominate a random draft because the former involves a lower opportunity cost. For example, suppose \( k = 1 \), \( q^* = .2 \), and \( m = .5 \). Thus, a random draft is preferable to no military (ineq.(2)). Now military output equals the number of individuals in the military who are productive (since \( k = 1 \)), and this number is .4 with a random draft and .3 with a LQVM. The opportunity cost of the military equals .25 with a random draft \((m/2)\) and .125 with a LQVM \((m^2/2)\). Thus, welfare is higher with the LQVM than with a random draft---.175 versus .15. Although a random draft means more military output than with a LQVM, it does not necessarily mean higher welfare.

4. Testing: a high quality volunteer military is possible

A. The case of a relatively small military \((m \leq 1-q^*)\)
Consider the possibility of testing individuals to see if they will be productive in the military. Ross (1990) assumes quality is observable at induction centers via tests. Berck and Lipow (2008) argue quality is unobservable prior to enlistment. We assume the “test” is some combination of one’s educational record and one’s score on an entrance exam such as the AFQT. Assume (for now) the test is costless and is accurate except for the possibility an individual who tries to fail is able to do so with a probability of $f$. Thus, with a volunteer military or a draft, the military can costlessly call individuals, test them, and see who passes. For now, we consider the case where there is a sufficient number of productive individuals to satisfy the military’s demand: $m \leq 1-q^*$. 

We will compare a high-quality volunteer military (HQVM) to a random draft and to a draft with testing. First, we determine whether a HQVM dominates a LQVM. Welfare with a LQVM is given by eq.(3). With a HQVM, the wage is set to just attract the $m$ productive individuals who have the lowest opportunity cost. Thus the wage will equal $q^*+m$, and only those who are tested and found to be productive are enlisted---those with $q \in [q^*, q^*+m]$. The average opportunity cost of those enlisted is then $q^*+\frac{m}{2}$. Thus, welfare with a HQVM is:

$$\Omega_{HQVM} = m \left( k - q^* - \frac{m}{2} \right).$$

(5)

Using eqs.(3) and (5), a HQVM is preferred to a LQVM if $k > m$. A HQVM dominates a LQVM if the marginal (and average) value of a productive individual in the military exceeds the marginal (and average) value of a productive individual in the military.

---

9 If $k$ is small enough, welfare will be higher if the military sets a lower wage and attracts fewer than $m$ productive individuals. However, the choice of a welfare-maximizing wage does not change the essential results, as shown in the next sub-section.
opportunity cost of the most able (in civilian output) individual enlisted with a LQVM. In order to meaningfully compare a HQVM with a draft, it is assumed \( k > m \).

First, we consider a draft with testing. With, \( w_d \) the wage with a draft, individuals who are called and have \( q > w_d \) will try to fail the test. The number who pass the test who will be productive in the military is then:

\[
max(0, w_d - q^*) + (1-f)(1-w_d). \tag{6}
\]

If \( w_d > q^* \), there are \( w_d-q^* \) individuals who will be productive in the military and who will not try to fail the test. If \( w_d \leq q^* \), all those who would be productive in the military will try to fail the test. Thus, the military should either set \( w_d \) equal to zero, or set \( w_d > q^* \) so \( m \) individuals pass the test. If \( (1-f)(1-q^*) \geq m \), there are enough productive individuals who pass the test if \( w_d = 0 \). For now consider the case when \( (1-f)(1-q^*) < m \), so, assuming \( w_d > 0 \), we use eq.(6), to get \( m \) productive individuals in the military. Solving for \( w_d \):

\[
w_d = 1 - \frac{1-m-q^*}{f}. \tag{7}
\]

Note \( \lim_{f \to 1} w_d = m+q^* \)--the wage with a HQVM--and \( \frac{\partial w_d}{\partial f} > 1 \). If everyone who wishes to fail the test can do so, then \( f = 1 \). As \( f \) is reduced from one, \( w_d \) is reduced because the draft brings in more individuals who would prefer not to serve \( (q > w_d) \) and who are productive. This reduces welfare since higher opportunity cost individuals who are productive serve in the military instead of lower opportunity cost individuals (some with \( q^* \leq q \leq w_d \)).
Given $m \leq 1 - q^*$, with either a draft and testing or a HQVM, all $m$ individuals who are enlisted are productive. With a HQVM, the opportunity cost is $m(q^* + \frac{m}{2})$. With a draft and testing, there are $w_d - q^*$ individuals who willingly are drafted, and who have an average opportunity cost of $\frac{aq^* + w_d}{2}$, and there are $(1 - f)(1 - w_d)$ individuals who try to fail the test but do not, and who have an average opportunity cost of $\frac{1 + w_d}{2}$, so, using eq.(7), the opportunity cost of a draft with testing is given by:

$$OC_{\text{draft/testing}} = \frac{1}{2} \left[ 1 - (q^*)^2 + (1 - m - q^*) \left( \frac{1 - m - q^*}{f} - 2 \right) \right].$$

(8)

It is easy to see $\lim_{f \to 1} OC_{\text{draft/testing}} = m(q^* + \frac{m}{2})$---the opportunity cost with a HQVM---and $\frac{\partial OC_{\text{draft/testing}}}{\partial f} < 0$. If all who want to fail the test do so ($f = 1$), the draft and a HQVM pay the same wage and enlist exactly the same individuals, those with $q \in [q^*, q^* + m]$. If $f < 1$, some higher opportunity cost individuals replace lower opportunity cost individuals with a draft so the draft implies lower welfare than with a HQVM.

Now suppose $m \leq (1 - f)(1 - q^*)$. Thus, a sufficient number of productive individuals who will not fail the test exist so, with a draft and testing, the military can set $w_d = 0$ and induct at random $m$ of those who pass the test (since it is assumed the military does not observe $q$). The average opportunity cost of those inducted with a draft is $\frac{1 + d^*}{2}$, so welfare with a draft is $m(k - \frac{1 + d^*}{2})$. Using eq.(5), welfare with a draft exceeds that with a HQVM only if $\frac{1 + d^*}{2} < q^* + \frac{m}{2}$, or if $q^* + m > 1$, which is not true in this case. Thus, at least for the case when $m \leq 1 - q^*$, a draft
with testing can not increase welfare in comparison with a HQVM, and will reduce welfare if some individuals who wish to fail military testing can not do so.\textsuperscript{10}

With \( m \leq 1-q^* \), we have seen a HQVM is socially preferable to a draft with testing. We now consider whether a HQVM dominates a random draft when the latter involves no testing. Using \textit{eqs.} (1) and (5), A HQVM dominates a random draft if:

\[
\frac{1-m}{2} > q^*(1-k). \tag{9}
\]

Clearly, if \( k \geq 1 \), a HQVM dominates a random draft. Also, a HQVM is less likely to dominate a random draft the larger is \( m \). Since \( m \leq 1-q^* \), substitute \( 1-q^* \) for \( m \) in \textit{ineq.} (9), and the inequality becomes \( k > \frac{1}{2} \), which, from the previous section, must hold in order for a random draft to be preferred to no military. Therefore, at least for the case of a relatively small military, \( m \leq 1-q^* \), a HQM is socially preferable to either a random draft or a draft with testing.

\textbf{B. The case of a relatively large military (}\( m > 1-q^* \))

If \( m > 1-q^* \), there are not enough productive individuals to satisfy the military’s demand. As suggested in Section 3, this does not appear to be the case in the U.S. today. However, continued increases in the fraction of the U.S. population who are obese or otherwise unfit, accompanied by an unexpected increase in the desired size of the U.S. military, could result in such a situation. Also, for a county like Israel, this case may be relevant today.

With a draft and testing, the number of productive individuals obtained is again given by \textit{eq.} (6). If \( w_d > q^* \), the average opportunity cost of those in the military for the \( w_d-q^* \) individuals

\textsuperscript{10} One advantage of the draft with testing is the lower wage may imply a lower deadweight loss from taxation, which, as noted in Section 1, is assumed away in most of this paper, but is considered in Section 6.
who do not try to fail the test is \( \frac{q*+w_d}{2} \), and the average opportunity cost of the \((1-f)(1-w_d)\)

individuals who try to fail but do not is \( \frac{1+w_d}{2} \). Welfare with draft and testing, \( \Omega_{draft/test} \), is then:

\[
\Omega_{draft/test} = k[w_d - q^* + (1-f)(1-w_d)] - \frac{1}{2}[(w_d - q^*)(w_d + q^*) + (1-f)(1-w_d)(1+w_d)].
\]  

(10)

In this case, the military must choose the wage,\(^1\) given a higher wage will yield more in

the military---because those willing to serve increase one for one with a wage increase, and those

who do not want to serve decrease by \(1-f\) for each unit increase in the wage---but will also

increase the average opportunity cost of both types who are enlisted. The welfare-maximizing

wage is determined by:

\[
\frac{\partial \Omega_{draft/test}}{\partial w_d} = f(k - w_d) = 0.
\]

(11)

From eq. (11), we can conclude \( w_d = \min(1,k) \). If \( k < 1 \), \( w_d = k \), and, if \( k \geq 1 \), \( w_d = 1 \), since that is

the highest wage anyone would earn in the civilian sector (when \( q = 1 \)).\(^2\)

With a HQVM, the number of productive individuals attracted is \( w_v-q^* \), and these

individuals have an average opportunity cost of \( \frac{q^*+w_v}{2} \). Thus, with a volunteer military, the

welfare-maximizing wage is determined by maximizing \( \Omega_{HQVM} \):

\[
\Omega_{HQVM} = \left[(w_v - q^*) \left(k - \frac{w_v+q^*}{2}\right) \right].
\]

(12)

\(^1\) As noted in the previous sub-section, we could have considered the optimal choice of the military wage when

\( m < 1-q^* \). In this sub-section, it is shown the results are similar when the military wage is chosen and when it is

assumed to be set to enlist as many productive individuals as possible in the military, provided the number does not

exceed \( m \).

\(^2\) We assume \( k > q^* \). If this is not true, then no productive individuals would be enlisted with a HQVM.
\[ \frac{\partial \Omega_{HQVM}}{\partial w_v} = k - w_v. \] (13)

Thus, \( w_v = w_d = \text{min}(1,k) \). With a draft and testing, if \( w_d \) is set above \( q^* \), so some productive individuals will not try to fail the test, a draft and a volunteer military set identical wages.

Letting \( w_d = w_v = k \), with \( k > q^* \) (or else no productive individuals would be enlisted with a HQVM), substitute in eqs. (10) and (12) for \( w_d \) and \( w_v \). We find a HQVM is preferred to a draft with testing if:

\[(1 - f)(1 + k^2) > 2k(1 - f) + 2q^*(q^* - k).\] (14)

With \( k > q^* \), the second term on RHS(14) is negative. Thus, if LHS(14) exceeds the first term on RHS(14), ineq. (14) holds. This simplifies to \( k^2 - 2k + 1 \equiv z > 0 \). Now \( \frac{\partial z}{\partial k} = 2(k-1) \), and \( \frac{\partial^2 z}{\partial k^2} = 2 \). The minimum value of \( z \) occurs when \( k = 1 \) and \( z = 0 \). Thus, \( z \geq 0 \), with the strict inequality holding when \( k \neq 1 \). Since the second term on the RHS(14) is negative, clearly ineq.(14) holds and the HQVM dominates the draft in this case.

With \( w_d = w_v = k < 1 \), a HQVM enlists \( k - q^* \) productive individuals, and (using eq.(6)) a draft with testing enlists \( k - q^* + (1 - f)(1 - k) \) individuals. However, the draft brings in higher opportunity cost individuals for whom it is not socially optimal to be enlisted. Although a military with the draft is assumed to choose the wage that maximizes welfare, this maximization occurs given reluctant individuals who do not fail the test will be inducted. A volunteer military accounts for the opportunity cost of all individuals inducted when the wage is chosen.
If \( k \geq 1 \), \( w_d = w_v = 1 \). In this case, it is optimal to have all \( 1-q^* \) productive individuals in the military, and a draft and a HQVM would be identical.

Now, with a draft and testing, if \( w_d = 0 \), so no productive individual wishes to be in the military, the number of productive individuals inducted equals \( (1-f)(1-q^*) \), and the average opportunity cost of those attracted is \( \frac{1+q^*}{2} \). Thus, using eq.(12) if \( w_v = 1 \), it is easy to see the draft has the same average opportunity cost as a HQVM, but attracts fewer productive individuals, and thus is dominated by a HQVM. When, \( w_v = k \), it is also true a HQVM dominates a draft with testing (see the Appendix).

Finally, we consider whether a HQVM is preferred to a random draft when no testing is used in the latter. Consider the case where the volunteer wage equals one \( (k \geq 1) \). Using eq.(12), welfare with the HQVM is then \((1-q^*)(k-\frac{1+q^*}{2})\), and welfare from a random draft is given by eq.(1). The HQVM is preferred to a random draft if:

\[
(1-q^*)(1-m)k > \frac{1}{2}[(1-q^*)(1+q^*)-m].
\] (15)

Both sides of ineq.(15) are linear and decreasing in \( m \). If \( m = 1 \), \( LHS_{(15)} = 0 \), and \( RHS_{(15)} = -\frac{1}{2}[q^*]^2 \). Since \( m > 1-q^* \) in this case, the smallest value of \( m \) is slightly larger than \( 1-q^* \). Substituting \( 1-q^* \) in ineq.(15) for \( m \) yields \( LHS_{(15)} = kq^*(1-q^*) \) and \( RHS_{(15)} = \frac{1}{2}q^*(1-q^*) \). Since we must have \( k > \frac{1}{2} \), the HQVM dominates a random draft.

---

13 When \( w_v = k \), the HQVM dominates a random draft if \( (k-q^*)^2 > m[2(1-q^*)k-1] \). This is least likely to hold the larger is \( m \), so let \( m = 1 \). The inequality then becomes \( k^2 + [q^*]^2 > 2k - 1 \). Since \( \frac{1}{2} < k < 1 \) in this case, let \( k = \frac{1}{2} + \varepsilon \), \( 0 < \varepsilon < \frac{1}{2} \). Then the inequality becomes \( \frac{1}{4} + [q^*]^2 > \varepsilon(1-\varepsilon) \). The RHS of the inequality is maximized when \( \varepsilon = \frac{1}{2} \), yielding a RHS = \( \frac{1}{4} \). Thus, the HQVM dominates a random draft in this case. Note, we assumed previously \( k > m \) (in order for a HQVM to dominate a LQVM), but that is irrelevant for the proof in this footnote since we simply needed to show a HQVM dominates a random draft when \( w_v = k \).
In this section, we found neither a draft with testing nor a random draft ever dominates a HQVM when testing is costless. When the military is relatively small \((m \leq 1-q^*\)) the draft attracts the same number of productive individuals as does a HQVM, but at a higher opportunity cost. With a larger military \((m > 1-q^*)\), the draft may result in more of the productive individuals enlisted, but at an opportunity cost sufficiently high so welfare is reduced.

5. Costly testing

We now consider costs of testing individuals. Suppose testing costs \(c\) per individual for the military, where \(c\) is assumed to be both a social and private cost. The interesting case is when we compare a HQVM to a military with a random draft (no testing). For brevity, we will consider only the case when \(m < 1-q^*\). Now welfare with a HQVM is:

\[
\Omega_{HQVM} = m \left( k - q^* - \frac{m}{2} \right) - (q^* + m)c, \tag{16}
\]

since the military sets \(w_v = q^* + m\), and that is the number who apply and are tested. Using eq.(1), a HQVM is preferred to a random draft if:

\[
\frac{1-m}{2} > q^*(1-k) + (q^* + m)c. \tag{17}
\]

From ineq.(17), a HQVM is less likely to dominate a random draft the larger is \(m\). Since \(m \leq 1-q^*\), in order to have the least chance a HQVM dominates a random draft, substitute \(1-q^*\) for \(m\) in ineq.(17), and we have:
\[ c < q^*(k - \frac{1}{2}). \] \hspace{1cm} (18)

To appreciate the magnitudes involved, even with \( m \) as large as possible (so a HQVM is least likely to dominate a random draft), suppose \( q^* = \frac{1}{2} \), so 50\% of potential inductees would be productive. From the discussion in Section 3, an expansive view of the potential military labor pool would suggest \( q^* \) is relatively high. Also, from \textit{ineq.}(2), if \( q^* = \frac{1}{2}, k \geq 1 \). Now, if \( k = 1 \), as long as the cost of testing is less than \( \frac{1}{2} \) the civilian output of the least able individual who would be productive in the military (one with \( q = q^* \)), a HQVM would dominate a draft. Thus, without very high testing cost, it is unlikely a random draft would be preferred to a HQVM.

6. Deferments and deadweight loss from taxation

Until now we have ignored aspects of the draft versus a volunteer military that have previously been considered. Johnson (1990), Lee and McKenzie (1992) and Ross (1994) have argued the deadweight loss (DWL) from taxation may result in a lower social cost for the draft than with a volunteer military. In response, Warner and Asch (1996) noted the reduced productivity of draftees relative to volunteers, Warner and Negrusa (2005) compared the costs of draft evasion and DWL from taxation, and Perri (2009) considered the tradeoff between the DWL of taxation and the social cost of deferments when individuals can incur costs to attain a deferred status. Although these issues are independent of the problem of the quality of military personnel and have been considered elsewhere, we will now briefly consider DWL and deferments.
Suppose no one can fail the test \((f = 0)\), but all can obtain a deferment at a cost (social and private) of \(D\).\(^{14}\) The DWL from taxation = \(t\)\{military wage payments\}, where \(t\) is assumed to be constant and is the DWL rate per dollar of military payroll. We also assume: \(k = 1\), so all are more valuable in the military, provided the military enlists \(n\) individuals and \(n \leq m\); \(m+q^* < 1\), so the military can attract \(m\) productive individuals; and testing is costless.

Thus, we have the HQVM from Section 4A: \(w_v = q^*+m\), and individuals with \(q \in [q^*, q^*+m]\) would be enlisted. With a draft and testing, assume the military sets the wage to just attract the same individuals as with a HQVM (recall the assumption no one can fail the test), so there is no misallocation from the “wrong” individuals being in the military. The tradeoff here is between lower DWL with a draft and the cost of deferments. Those with \(q-D > w_d\) will “purchase” a deferment\(^{15}\). Thus, if the military sets \(w_d = q^*+m-D\),\(^{16}\) those with \(q > q^*+m\) will obtain deferments, those with \(q < q^*\) will be discharged after testing, and those with \(q \in [q^*, q^*+m]\) will be enlisted.

The reduced DWL from a draft is the DWL rate \((t)\) multiplied by the lower wage \((D)\) and the number enlisted \((m)\). The cost of deferments is \(D\) times the number who defer \((1-m-q^*)\).

Thus a HQVM is preferred to a draft if:

\[
    t < \frac{1-m-q^*}{m} \equiv \bar{t}. 
\]

\(^{(19)}\) Ineq. (19) is similar to the result in Perri (2009), who finds, in a model with homogeneous labor, a fairly large military force relative to the military labor pool would be required for the

---

\(^{14}\) Examples of deferments are given in Perri (2009).

\(^{15}\) Deferments act like buyouts that were available in the U.S. Civil War, except the latter involved no social costs. For analysis of the U.S. Civil War draft, see Perri (2008).

\(^{16}\) Paying a lower wage with a draft would result in more deferring and the military attracting fewer than \(m\) productive individuals.
DWL from taxation to be sufficiently large so as to offset the cost of deferments. It is possible World War Two involved a large enough demand for military personnel the draft might have been comparable to a volunteer military in social cost.\(^{17}\)

Using ineq. (19), \(\frac{\partial \xi}{\partial m}\) and \(\frac{\partial \xi}{\partial q^*}\) are both negative. Thus, the greater the demand for military personnel \((d_m > 0)\), and the fewer individuals who would be productive in the military \((d_q^* > 0)\), the less likely is a HQVM to be preferred to a random draft because increases in either \(m\) or \(q^*\) imply fewer individuals will choose to incur the cost to be deferred.

7. Summary

It has been argued the draft may enable the military to attract more able individuals than a volunteer military, and thus increase welfare. In our theoretical model, we find this may be the case if a volunteer military simply takes the least able individuals. However, when the military tests individuals and does not take the lowest quality applicants, neither a random draft nor a draft with testing increases welfare, and both usually decrease welfare. When there is a sufficient number of productive individuals available to the military, and testing would be used by both a volunteer military and a draft, the same number of productive individuals are attracted with either system, but a draft involves a higher opportunity cost.

When the military can not attract the desired number of productive individuals, a draft with testing may attract more individuals who are productive than would a volunteer military, but the higher opportunity cost of the former leads to lower welfare with a draft versus a volunteer military.

\(^{17}\) As is the case herein, Perri (2009) ignores some costs of the draft relative to a volunteer military such as higher turnover and reduced effort.
Finally, only if testing is relatively costly would a random draft dominate a volunteer military with testing. Thus, when recruit quality is important for the military, a draft would not likely be preferable to a volunteer military.
Appendix

Proof a high quality volunteer military dominates a draft when the draft wage is zero. This was demonstrated in the text for the case when the volunteer wage, $w_v$, equals one. We now show this result holds when $w_v = k$. Welfare with a high quality volunteer military (HQVM) is given by eq.(12) with $w_v = k$, and equals $\frac{1}{2}(k-q^*)^2$. With a draft, $(1-f)(1-q^*)$ productive individuals are inducted, with an average opportunity cost of $\frac{1}{2}(k-q^*)^2$, so welfare with a draft is $(1-f)(1-q^*)(k - \frac{1+q^*}{2})$. The HQVM is preferred to the draft if (simplifying terms):

$$k^2 - 2kq^* + [q^*]^2 > (1-f)(1-q^*)(2k - 1 - q^*).$$  \hspace{1cm} (A1)

If $LHS_{(A1)} \geq (1-q^*)(2k-1-q^*)$, then $LHS_{(A1)} > RHS_{(A1)}$. This occurs if $k^2 - 2k + 1 \geq 0$, which was shown to be true in the proof following ineq.(14) in the text. \~
References


