A Primer on Signaling

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Introduction.

Signaling occurs when the informed side of market wants to communicate information to the uninformed side. Usually sellers (workers in the labor market) are informed. Signaling involves taking an action (the signal) which is costly but which reveals the sender's type. If different types select different signals, they reveal their types.

To be successful, signals must be credible. Credible signals occur if the higher quality/more able sellers have a lower marginal cost (MC) of signaling than do less able sellers.

I. Quality Assuring Price.

Suppose there are two kinds of firms: *premium* & *discount*. There are many potential firms of each type. Each firm can produce only 1 unit so *TC* = *AC* = *MC*. Firms can produce either *high* or *low* quality goods. For a premium firm, the cost of a high quality good = \$2, & the cost of a low quality good = \$1. For a discount firm, the cost of a high quality good = \$2.5, & the cost of a low quality good = \$1.5. All buyers value the low quality item by < \$1, so, under perfect information regarding product quality, low quality items would not be offered for sale (since P_{Low} < \$1, π < 0 for either type of firm selling low quality items). Under perfect information, entry or exit of premium firms would drive $\pi_{High}^{Premium}$ = 0 so P_{High} = \$2.

Asymmetric information. One type of firm. For now focus on premium firms. Suppose firms *could* live forever and have an interest rate = *r*. Now buyers do not immediately know the quality of the item they purchase, but learn quality after one period, & communicate this information to other buyers. Thus, a premium firm has 2 choices: 1) behave as a fly-by-night firm (promising high quality but delivering low quality) & exist for one period (π is received at the end of each period); or 2) deliver the high quality that is promised. In the 2nd case, $\pi_{High}^{Premium} > 0$ is required. Why? Suppose P_{High} = \$2 so $\pi_{High}^{Premium} = 0$. A fly-by-night premium firm will have $\pi_{Low}^{Premium} = \frac{2-1}{1+r} = \frac{1}{1+r} > 0$. All premium firms would promise high quality & produce low quality. The minimum price required to induce premium firms to produce high quality is when $\pi_{High}^{Premium} = \pi_{Low}^{Premium}$. Call this the *quality assuring price*, *P**. *P** comes from the following equation (remember: producing low quality will only last for one period):

$$\frac{P^*-2}{r} = \frac{P^*-1}{1+r},$$

$$(1+r)(P^*-2) = r(P^*-1)$$

$$(1+r)P^* - rP^* = 2(1+r) - r,$$

P* = 2 + r.

Now $\pi_{High}^{Premium} = \frac{2+r-2}{r} = \$1$, & $\pi_{Low}^{Premium} = \frac{2+r-1}{1+r} = \$1$. A price slightly larger than 2 + *r* will make premium firms strictly prefer to produce high quality (rather than promise high quality & produce low quality), but, for simplicity, we will use the price that makes $\pi_{High}^{Premium} = \pi_{Low}^{Premium}$.

Now *P** signals to buyers that such a firm has an incentive to stick around (deliver the promised quality level).

Two types of firms. Now bring back discount firms that have a cost of a high quality good = \$2.5, & a cost of a low quality good = \$1.5. If *P* = 2 + *r*:

$\pi_{High}^{Discount} = \frac{2+r-2.5}{r} = \frac{r-.5}{r}$, & $\pi_{Low}^{Discount} = \frac{2+r-1.5}{1+r} = \frac{.5+r}{1+r}$. Discount firms will clearly prefer fly-by-night behavior if *r* < .5. If *r* = .05, $\pi_{High}^{Discount} = \frac{-.45}{.05} = -\$9 < \pi_{Low}^{Discount} = \frac{.55}{1.05} \approx \$.524$.

Now *P* = 2 + *r* will NOT prevent fly-by-night behavior from discount firms since $\pi_{High}^{Discount} < 0$ if *r* < .5, which it generally will be, & $\pi_{Low}^{Discount} > 0$ for any *r*. If buyers can not tell premium firms from discount firms, they may not buy since they fear purchasing low quality.

How can premium firms *signal* they are not discount firms? Premium firms could invest in sunk cost items (with a cost assumed to occur right now for simplicity so we do not have to discount it) that cost slightly more than \$.524---say \$.525---leaving premium firms with π = 1 - .525 = \$.475. Discount firms will not match the expenditure on these items because to do so would yield them π < 0. The sunk cost items signal a firm is not a higher cost (discount) firm, & has an incentive to deliver the promised quality.

Why must these be sunk cost items (signs, expensive floors, etc.)? Because, if the items had salvage value so the cost was < \$.524 after selling these items, the signal would not be credible---a discount firm could afford it.

II. Educational Signaling
Simplifying assumptions.

In order to not needlessly complicate the analysis, it is assumed the signal has no direct effect on individual productivity. Thus, in a pooling equilibrium, none invest in this education/signal. Also, it is assumed the signal is obtained immediately, & work occurs for one period (so present discounted value analysis can be ignored).

Basic Spence signaling model

This model was developed almost 40 years ago by Michael Spence (2001 Nobel *laureate*).
The simplest model is one in which there are 2 types of individuals, *good* & *bad*.

Productivity is constant & is the value of an individual to any firm.

Let (marginal) productivity = q. This is the value to the firm of hiring someone (assuming product price = 1 for simplicity).

For good individuals, productivity = q_G, &, for bad individuals, productivity = q_B.
$q_G > q_B > 0$.

Let g = the fraction of individuals who are good.

Let C = the cost of obtaining a given education.
$C_B > C_G$---this is what enables signaling to be possible.

Why could $C_B > C_G$?
If good individuals spend less time studying, they have more time for work, lowering the *net foregone earnings* that are part of the cost of education.

1st see what individuals would be paid with no signaling----expected productivity = $\bar{q} = gq_G + (1\text{-}g)q_B$.
Now put pooling aside for the moment.

Spence initially *assumed* we looked for what was necessary for signaling to occur when, if you do not signal, you are viewed as a bad individual.
That is, pooling was ignored.

2nd would good individuals prefer to be sorted from bad individuals when the alternative is to be viewed as bad?
Yes if $q_G - C_G > q_B$. (1)

Would bad individuals choose to *not* mimic good individuals?
Yes if $q_G - C_B < q_B$. (2)

Now put back the possibility no one chooses education---*pooling*.
Then there is a stronger condition than (1) for good individuals.

No good individual would deviate from a pooling e unless:
$q_G - C_G > \bar{q}$. (3)

(3) is stronger than (1) since $\bar{q} > q_B$.

The point is, for signaling, bad individuals must not want to mimic the good individuals if the good individuals invest in the signal (education), which is inequality (2), but the good individuals must actually prefer the equilibrium in which they signal to the one with pooling, which is inequality (3).

Thus, for signaling to occur we use (2) & (3).

$$\boldsymbol{q_G - q_B < C_B} \quad (4)$$

(4) says it is too costly for bad individuals to mimic good individuals if good individuals choose this level of education.

$\boldsymbol{q_G - \bar{q} > C_G}$ (5)

(5) says good individuals are better off signaling & not being lumped in with bad individuals.

Example. $q_G = 20$, $q_B = 6$, & $g = 1/4$.

Now $\bar{q} = (1/4)(20) + (3/4)(6) = 9.5$.

Now #4 is $C_B > 14$.

Now #5 is $C_G < 10.5$.

Thus, the cost of the signal to good individuals must be sufficiently low, and the cost of the signal to bad individuals must be sufficiently high for signaling to work

If #s 4 & 5 do not hold, **pooling** will occur. No one will invest in this education, & all will be paid $\bar{q} = 9.5$.