Spence Revisited:

Signaling and the Allocation of Individuals to Jobs

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- In a study for the Social Science Research Council, Arum *et al*. (2011) find 36% of college students have learned very little after four years.

- Even if education does not directly affect productivity, if it is used as a signal of inherent ability, it may increase wealth.

- Spence (1974, book): more able individuals signal their ability in order to be paid more.

- Spence's John Bates Clark Award (1981):

"Helen of Troy's face launched a thousand ships. Not many Ph.D. theses can be said to have launched even a hundred articles...Spence's certainly comes close." (*AER*, May 1982).

- Signaling has been used in many areas of economics,

but, most of the focus in the literature on the generic Spence model has been on the existence of equilibrium.

Basic Spence model.

- If there is only one type of job, and signaling, say, via education, does not directly affect individual productivity, then wealth is simply redistributed from the less able to the more.

- The signal is a complete social waste!! ☹ ☹

- Private return to the signal is > 0.

- Social return to the signal = 0.

- This *suggests*, if there *is* a private return:

  Private return > social return > 0.

∴ In this case, not all signaling is wasteful, but too much signaling still occurs.

- However, it may be the returns are not additive. There may be a social return which could be $\lessgtr$ than the social return.

- Spence (1974, book) also considered a model in which there are 2 types of jobs.

- Then signaling can increase wealth by improving the allocation of individuals to jobs.

- Spence (1974, book) found signaling may increase or decrease wealth when the allocation of individuals to jobs matters.

- Spence allowed for multiple signaling equilibria & was not clear on when *pooling equilibria* might occur.

- Pooling: no one signals.

- Subsequent developments in *signaling games* allow more precise answers for when signaling or pooling would occur.

- This work focuses on the *existence* of equilibrium.

- Using these results & the 4 basic assumptions in Spence (1974, book), I re-examine the job allocation problem.

RESULTS SINCE SPENCE'S ANALYIS:

- y = level of the signal (say education).

1) *Riley outcome*. More able individuals choose the lowest feasible level of the signal, y_{Riley}. Less able individuals set $y = 0$ (Riley, 1975 & 1979).

2) *Intuitive criterion*. Only the *Riley outcome* survives experimentation by individuals (Cho & Kreps, 1987).

- Problem: Intuitive criterion rules out all pooling.

3) *Undefeated equilibrium*. If the more able have a higher payoff with pooling ($y = 0$), they will not deviate from pooling to the *Riley outcome* (Mailath *et al*., 1993).

- With 2 types of jobs, we have 2 possible pooling equilibria: all

are placed in one job, or all are placed in the other job.

In either case, $y = 0 \; \forall$ individuals.

Table One. MRPs.		
	More Able	Less Able
Skilled Jobs	$e\theta$	θ
Unskilled Jobs	$a\theta$	$a\theta$

Assumption One. The more able are more productive in the skilled job than are the less able. This requires $e > 1$.

Assumption Two. The more able are more productive in skilled jobs than are the less able in unskilled jobs. This requires $e > a$.

Assumption Three. *The less able are more productive in unskilled jobs than they are in skilled jobs.* This requires $a > 1$.

Assumption Four. *The marginal cost of signaling is lower for the more able than it is for the less able.* Assume the cost of signaling is y for the less able & y/g for the more able, $g > 1$.

$\therefore$ We have $e > a > 1$ & $g > 1$.

Assumption Five. I assume more able & less able have =

productivity in the unskilled job.

- Spence had 2 cases: more able more productive than less able in unskilled job, & vice versa.

- If we consider those cases, we will just get more possibilities, but nothing that is significantly different (see below).

MODEL.

- Let α = the fraction of more able individuals.
- Then $W_{pool} = max\{(\alpha e + 1\text{-}\alpha)\theta, a\theta\}$

- If all are in low skilled jobs with pooling, the social return is to move the more able to skilled jobs.

- If all are in more skilled jobs with pooling, the social return is to move the less able to unskilled jobs.

- Pooling is in the unskilled job if:

$$a\theta > (\alpha e + 1\text{-}\alpha)\theta,$$

or when $\alpha < \alpha^*$ with:

$$\alpha^* = \frac{a-1}{e-1}. \qquad \{0 < \alpha^* < 1\}$$

- Consider signaling when pooling would be in unskilled job.

- The more able prefer to signal (& not be viewed as less able), when those who are viewed as less able would be in unskilled jobs, if:

$$e\theta - y/g \geq a\theta.$$

- The less able prefer to not signal if:

$$e\theta - y < a\theta.$$

- ∴ Signaling occurs if:

$$(e\text{-}a)\theta < y \leq (e\text{-}a)\theta g.$$

$$\therefore y_{Riley} \approx (e\text{-}a)\theta.$$

- The net payoff to a more able individual from signaling is the wage gain minus signaling cost:

$$(e\text{-}a)\theta - y_{Riley}/g.$$

- However, $(e\text{-}a)\theta$ = the output gain from reallocating more able from unskilled jobs to skilled jobs.

∴ The social & private gains from signaling are the same.

- Signaling increases wealth because it allocates more able individuals to jobs where they are more productive by the amount $(e\text{-}a)\theta$. ☺☺

If $\alpha < \alpha^* = \frac{a-1}{e-1}$, signaling occurs & welfare is increased.

- What if prod is ≠ for the 2 types in unskilled job?

- We could have too much signaling (if the more able are more productive in unskilled job than the less able)---wage gain from signaling is > productivity gain.

- We could have too little signaling (if the more able are less productive in unskilled job than the less able)---wage gain from signaling is < productivity gain.

- I view the problem as: there is some skill, of which the more able have more, but which is of no value in the unskilled sector, so both types have the same productivity there.

When $\alpha > \alpha^*$, pooling would be in skilled jobs.

- 2 questions. 1) when will pooling occur, & 2) is welfare

higher with signaling or pooling?

- Now signaling involves a social gain---the less able are moved to unskilled sector (where they are more productive), & a private gain---the wage gain to the more able (who are then not pooled with the less able).

- Social gain ≠ private gain.

- In the basic Spence model, the social gain = 0.

- Signaling increases wealth if $\alpha < \alpha^{**}$:

$$\alpha^{**} = \frac{g(a-1)}{g(a-1)+e-a}. \qquad \{0 < \alpha^{*} < \alpha^{**} < 1\}$$

- Why is wealth reduced with signaling for $\alpha > \alpha^{**}$?

- Social gain from signaling (when pooling is in skilled jobs)

results from the # of less able who then work in unskilled jobs

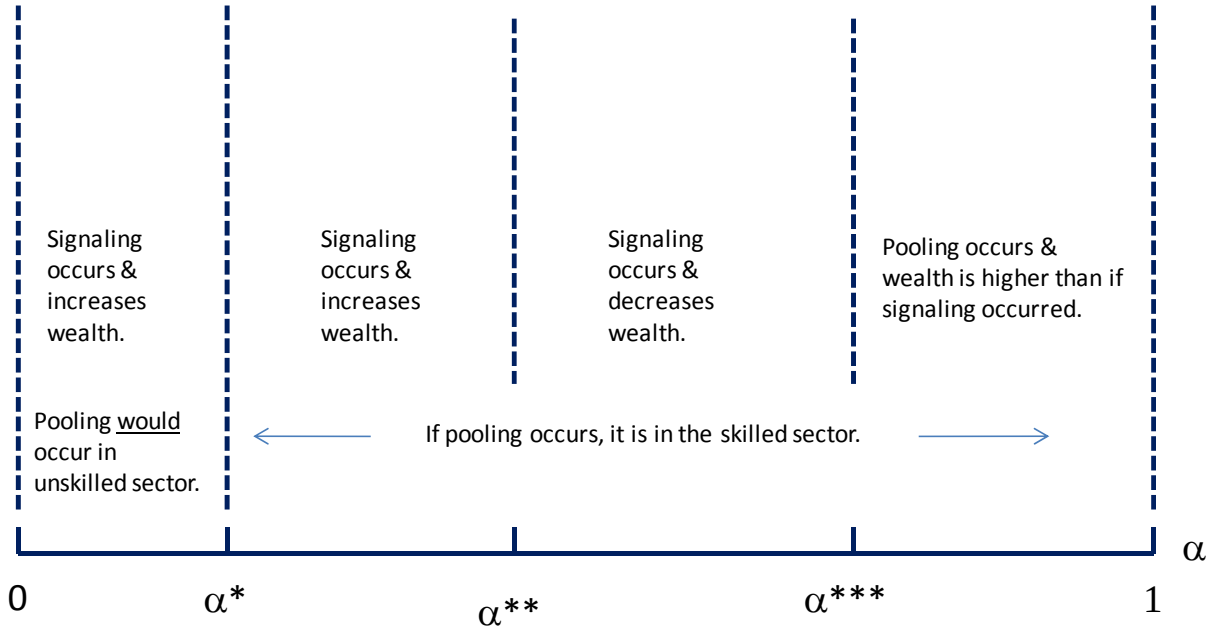
(vs. skilled jobs) where they are more productive.

- A larger $\alpha \Rightarrow$ fewer less able individuals.

- Signaling is preferred by the more able to pooling (at skilled jobs) if $\alpha < \alpha^{***}$:

$$\alpha^{***} = \frac{g(e-1)+a-e}{g(e-1)}. \qquad \{0 < \alpha^{*} < \alpha^{**} < \alpha^{***} < 1\}$$

Figure One


Signaling occurs & increases wealth.
Signaling occurs & increases wealth.
Signaling occurs & decreases wealth.
Pooling occurs & wealth is higher than if signaling occurred.
Pooling would occur in unskilled sector.
If pooling occurs, it is in the skilled sector.
α
0
α*
α**
α***
1

$\therefore$

Efficient signaling occurs if $\alpha < \alpha^{**}$. ☺ ☺ ☺

Inefficient signaling occurs if $\alpha^{**} < \alpha < \alpha^{***}$. ☹ ☹ ☹

Efficient pooling occurs if $\alpha^{***} < \alpha$. ☺ ☺ ☺

Table Two. Values for α^*, α^{**}, and α^{***}.					
e	*a*	*g*	α^*	α^{**}	α^{***}
2	1.01	2	.01	.02	.51
2	1.1	2	.1	.18	.55
2	1.1	1.5	.1	.14	.4
2	1.25	2	.25	.4	.625
2	1.25	1.5	.25	.33	.5
2	1.5	1.5	.5	.6	.67
2	1.5	1.25	.5	.56	.6
2	1.1	1.1	.1	.11	.18
1.5	1.01	1.5	.02	.03	.35
1.5	1.1	1.5	.2	.27	.47
1.5	1.25	1.5	.5	.6	.67

Approximately the basic Spence model with 1 job.

Basic Spence model but with 2 jobs.

- If $a \approx 1$, there is essentially NO gain from job allocation.

1) Pooling is almost always in skilled job (where the gain from job allocation is in moving the less able to unskilled jobs).

2) There is almost no difference in output moving a less able individual from a skilled job to an unskilled job--- $(a\text{-}1)\theta$.

- With a non-trivial gain from job assignment, $a > 1$, the result is much different.
- Say $a = 1.25$ & $e = 2$. Now the less able are 25% more productive in unskilled jobs than in skilled jobs, which is ¼ the advantage the more able hold over the less able in skilled jobs.
- Line 4 of Table 2.

• The efficient signaling range is almost double that of the inefficient signaling range.

• In general, less likely to have inefficient signaling as $g\downarrow$, $e\downarrow$, & $a\uparrow$ (if $g < 2$).

• $y_{Riley} \approx (e\text{-}a)\theta$, so $e\downarrow$ & $a\uparrow \Rightarrow$ lower signaling cost, but $g\downarrow \Rightarrow$ higher signaling cost.

- As $g\downarrow$, the range for efficient signaling falls.

- However, $g\downarrow$ results in a larger pooling range

- The latter effect dominates if $g < 2$, so it is less likely we have inefficient signaling.

Future Work

Consider multiple types of individuals.

$\frac{\partial r_e}{\partial g} = \{+\}(e\text{-}a)(a\text{-}1) > 0,$

$\frac{\partial r_i}{\partial g} = \{+\}[g(a\text{-}1)(2\text{-}g) + e - a],$

$\frac{\partial r_p}{\partial g} = -\{+\}(e\text{-}1)(e\text{-}a) < 0.$

$\frac{\partial r_i}{\partial g} > 0$ if $g < 2.$

$$\frac{\partial r_e}{\partial e} = -\{+\}g(a\text{-}1) < 0,$$

$$\frac{\partial r_i}{\partial e} = \{+\}(e\text{-}a)(a\text{-}1)[g(e + a - 2) + e - a] > 0,$$

$$\frac{\partial r_p}{\partial e} = \{+\}g(a\text{-}1) > 0.$$

$$\frac{\partial r_e}{\partial a} = \{+\}g(e\text{-}a) > 0,$$

$$\frac{\partial r_i}{\partial a} = -\{+\}(e\text{-}a)[2(e\text{-}1)g(a\text{-}1) + 3(e\text{-}1)(e\text{-}a) + g(e\text{-}a)(a\text{-}1) + (e\text{-}a)^2]$$

$$< 0,$$

$$\frac{\partial r_p}{\partial a} = -\{+\} < 0.$$

Figure A1. $\alpha = .75$

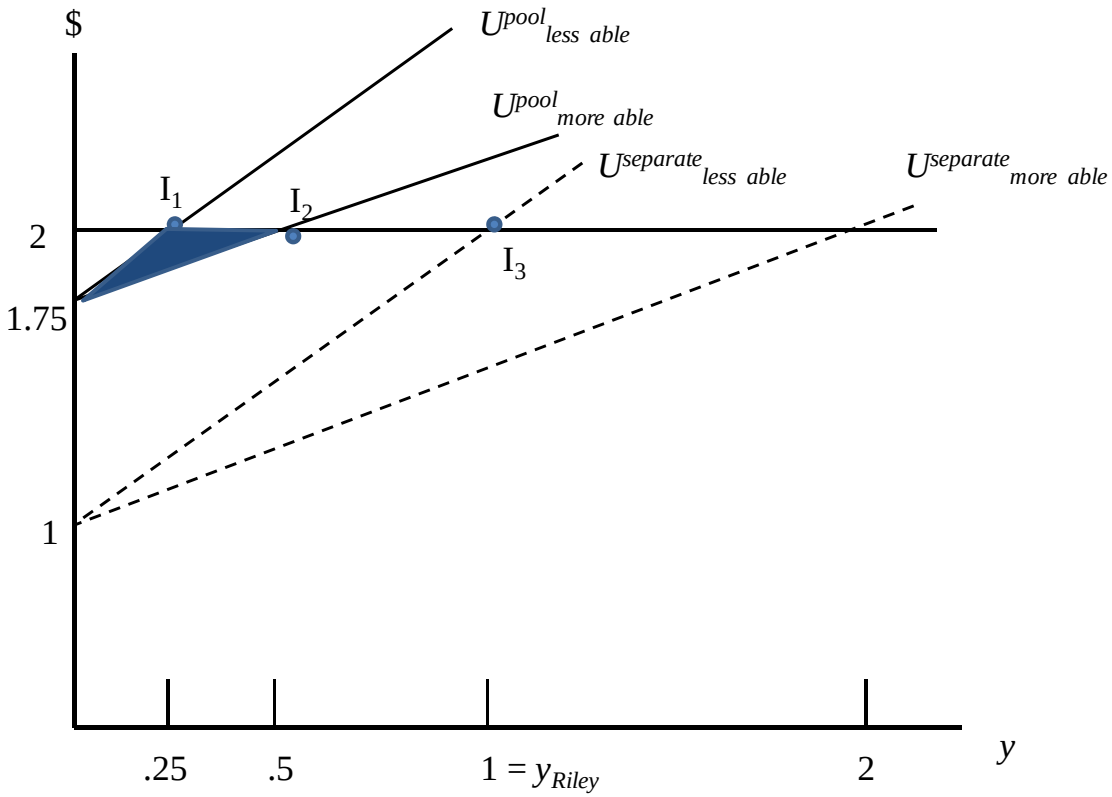


Figure A2. $\alpha = .5$