Reconsidering Spence: Signaling and the Allocation of Individuals to Jobs

by

Timothy Perri*

Revised May 28, 2014

Abstract

Critics claim higher education often does not increase productivity. In his classic signaling work, Spence (1974a) considered a variant of his signaling model in which signaling improves the allocation of individuals to jobs. Using results in signaling games since then---the *Riley outcome* (Riley, 1979), the *intuitive criterion* (Cho and Kreps, 1987), and *undefeated equilibrium* (Mailath *et al*., 1993)--- we find the likelihood of equilibria involving efficient signaling, inefficient signaling, and pooling depends on the fraction of more able individuals in the population. With non-trivial gains from job allocation, inefficient signaling does not appear to be the most likely outcome.

JEL category: D82
Key words: signaling, pooling, Riley outcome, intuitive criterion, and undefeated equilibrium

* Department of Economics, Appalachian State University, Boone, NC, 28608. E-mail: perritj@gmail.com.
Acknowledgements....to be added.

1. Introduction

Scholars and pundits continue to debate whether education increases productivity (Leonhardt, 2011). In a study for the Social Science Research Council, Arum *et al*. (2011) find 36% of U.S. college students learn very little after four years. One economist (Caplan, 2011) recently accepted the idea that little is learned in college, higher education is essentially a signal, and wealth would increase if students spent fewer years in school.

In his classic work on market signaling, Spence (1974a) considered a world in which more able individuals signal their ability in order to be paid more. In Spence's basic model, there is only one type of job, and signaling does not directly affect individual productivity. Signaling simply redistributes wealth from the less able to the more able.[1]

Since Spence's initial analysis of signaling, there have been several results in signaling models. These include the idea of the lowest cost signaling equilibrium---the *Riley outcome* (Riley, 1979) and the *intuitive criterion* (Cho and Kreps, 1987)---and when pooling and not signaling may occur---*undefeated equilibrium* (Mailath *et al*., 1993). In those papers, the focus was on determining what equilibria might survive experimentation by agents in the case of one job type when any signaling equilibrium reduces welfare compared to a world with no signaling.

[1] Spence (2002) considered a situation where education is a signal and also increases individual productivity. In that case, all individuals would invest in some education, even with perfect information. If the marginal cost of signaling is significantly larger for less able individuals than it is for the more able, excessive investment in education would not occur. However, with a small enough difference in the marginal cost of signaling, the more able would have to invest in a larger amount of education than they would with perfect information to prevent being mimicked by the less able. Leppämäki and Mustonen (2009) consider a model where the signal may create either a positive or negative product market externality. Perri (2014) has a model where the signal directly affects productivity. There increasing education cost for the less able may increase welfare by reducing over-investment by the more able.

Even without directly increasing productivity, there is a social gain from education as a signal if it allows individuals to be sorted to jobs where their productivity is the highest.[2] Kenneth Arrow was asked about education being wasteful in the signaling model. Arrow responded "It permits the sorting. That in itself may be socially gainful. And Spence's model in its form doesn't fully convey all its implications. It implies that an individual will be productive no matter what."[3]

Spence (1974a) considered a variant of his basic signaling model in which there are two types of jobs, and in which signaling can improve welfare by improving the allocation of individuals to jobs. Spence found signaling may increase or decrease welfare.[4]

Because there are questions about how much education directly increases productivity, it is useful to consider the net social value of education when education does not directly affect productivity, educational signaling may occur, and educational signals may be used to sort people among jobs. When using the *Riley outcome, intuitive criterion*, and *undefeated equilibrium*, it is possible to be more precise than Spence was in determining when signaling would occur, and what the effect of signaling on welfare would be. These results do not appear to have been used previously[5] to consider the Spence model when job allocation matters.[6]

---

[2] Arcidiacono *et al*. (2010) suggest college may directly reveal aspects of individual ability to employers. The analysis herein does not depend on how information is revealed---directly or indirectly as firms draw inferences based on educational attainment.

[3] *The Region* (1995, p.8).

[4] "...since the result of the analysis is, very roughly, that signaling may or may not be worth it, I have placed the analysis in an appendix." Spence (1974a, p.152).

[5] Greenberg (1989) modeled job allocation, and found signaling might result in inefficient allocation of individuals to jobs. His results are based on the possibility of multiple signaling equilibria (ruled out by the *intuitive criterion*), and on not considering when individuals would deviate from a pooling equilibrium (*undefeated equilibrium*). Von Ungern-Sternberg (1979) has a model with two job types in which productivity of the more able grows in skilled jobs but not in unskilled jobs. Absent signaling, all start in unskilled jobs by assumption. In the model herein, with no signaling, all could be in either skilled or unskilled jobs, depending on productivity of individuals in either job, and the fraction of the types of individuals in the population.

Most signaling research has focused on the existence of equilibrium.[7] The goal of this paper is to apply the *Riley outcome*, the *intuitive criterion*, and *undefeated equilibrium* to essentially the same model used by Spence (1974a).[8] We address two important issues in Spence's analysis of the allocation of individuals to jobs.

First, Spence allows for multiple signaling equilibria.[9] He mentions the possibility of firms competing and lowering the required education level as much as possible, but suggests there is not much incentive for them to do so.[10] However, Cho and Kreps (1987) argue only a separating equilibrium with the lowest possible level of the signal, the *Riley outcome*, survives their *intuitive criterion*.[11]

Second, Spence was not clear on when pooling equilibria might occur, arguing we might have such equilibria "...either by fiat, or naturally as an equilibrium..."[12] With *undefeated equilibrium*, Mailath *et al.* (1993) refine the *intuitive criterion*, which rules out all pooling equilibria, and demonstrate pooling should occur when the more able are better off pooling than they would be in a separating equilibrium with the lowest level of the signal. They find pooling occurs when the fraction of more able individuals in the population is sufficiently large.

---

[6] A different job allocation literature was begun by Waldman (1984). In his model, job assignment within a firm is used by potential employers as a signal of an individual's ability. For a model that combines education as a test and job assignment as a signal, see Perri (1994).

[7] Among the large literature on existence of equilibrium in signaling games are Riley (1975, 1979, 1985, 2001, and 2002), Cho and Kreps (1987), Mailath *et al*. (1993), and Cai *et al*. (2007).

[8] Spence (1974b) considered a model of job allocation in which individuals are continuously distributed with respect to ability. The results are similar to those with two types of ability, so the latter is the focus herein.

[9] In a later paper, Spence (1976) considered competition in the credentials dimension which eliminated all but the Pareto dominant signaling equilibrium, but did so only with certain production functions. Thus, multiple signaling equilibria were not ruled out in general. Also, in that paper, the signal directly affects productivity, which is *not* the case herein or in Spence (1974a).

[10] Spence (1974a, p.170).

[11] Riley (1975) argued 1) only the signaling equilibrium with the lowest level of signaling would survive experimentation by buyers, and 2) even that equilibrium might not survive. Riley (1979) suggests different assumptions can result in survival of the signaling equilibrium with the lowest level of signaling, as does the work of Cho and Kreps (1987).

[12] Spence (1974a, p.156).

Hopkins (2012) considered signaling with more than one job. He analyzed a matching model in which the signal identifies individuals' inherent ability, and may increase the value of the match. In the Hopkins model, individuals do not consider the benefit of their signal to potential partners, and may underinvest in the signal. As in the model herein, the quality of firms is observable in Hopkins' model. Thus, signaling in a matching environment may involve externalities that have opposite effects. In a job allocation model, there are no offsetting effects. However, the social and private returns to signaling are not the same, so signaling may be efficient or inefficient.[13]

Hoppe *et al.* (2009) consider a marriage market with symmetric ignorance of the quality of men and women, both of whom signal their quality. Hoppe *et al*. also find both positive and negative externalities.[14] Symmetric ignorance seems less relevant for the job model herein. Prospective employees know about pay and working conditions at a firm, which reveal a good deal about a firm's quality.

Before proceeding, note how the terms welfare, efficiency, and inefficiency are used in this paper. Welfare refers to the value of output minus any signaling cost. With complete information, an individual would costlessly be allocated to the job where his productivity is highest. We consider a world of costly information for firms regarding individual quality. Thus, discussions herein of welfare, or of situations being efficient or inefficient, are in terms of the feasible alternatives, and are not compared to a world of costless information. For example, if we

[13] Wolpin (1977) considers the use of signaling (more precisely, screening) by firms to reduce misallocation between and within firms in a model in which a larger variance in skill of labor inputs reduces expected output. He argues too much or too little screening could occur. There is only one occupation in his model, so there is no gain from assigning individuals to different types of jobs. An individual is equally valuable at any firm, given the skill variance at that firm.

[14] For example, a man who signals imposes a negative externality on lower quality men since the latter have one less high quality woman with whom to match. Conversely, a man who signals presents a positive externality for women by his presence.

say signaling is inefficient, we mean relative to the pooling equilibrium that would result absent signaling.

In the next section, we consider Spence's model (1974a) of job allocation. Using Spence's four basic assumptions, along with the *Riley outcome*, the *intuitive criterion*, and *undefeated equilibrium*, we show the following in Section 3. When the fraction of more able individuals, α, is relatively small, we get an equilibrium with signaling that improves welfare. When α is somewhat larger, we get a signaling equilibrium that lowers welfare. For the largest values of α, we get a pooling equilibrium with higher welfare than with signaling. The likelihood of these different scenarios is considered in Section 4. Related issues are discussed in Section 5, and a summary is contained in Section 6.

2. The Spence Model of Job Allocation

Spence (1974a) apparently did not view the job allocation problem as having great importance because the results were ambiguous.[15] We also find signaling may or may not improve wealth. However, our results are more precise than in Spence in terms of when signaling will occur, and what the effect of signaling on wealth will be. Particularly, we find whether wealth is increased or not depends on the share of more able individuals in the population, α.

[15] See footnote four. In his book, Spence says he considered the problem of job allocation "...largely in response to suggestions and questions raised by Zvi Griliches and George Stigler..." (1974a, p.152). He also devoted only four of thirty-seven pages in a journal article on signaling to job allocation (Spence, 1974b).

The job allocation model we use follows the model in Spence (1974a). Assume two types of individuals, *more able* and *less able*, and two types of jobs, *skilled* and *unskilled*.[16] The (constant) marginal revenue products (MRPs) of individuals in different jobs are listed in Table One. We assume firms pay a wage equal to MRP when MRP is revealed in a signaling equilibrium. In a pooling equilibrium, firms pay a wage equal to expected MRP. Thus, firms earn zero profit. Firms can hire any number of individuals in either skilled or unskilled jobs.

Spence used four basic assumptions, which we also employ. The first three assumptions refer to Table One where MRP is denoted by θ_i, $i = U, L$, or M.

Assumption One. The more able are more productive in the skilled job (θ_M) than are the less able (θ_L).

Assumption Two. The more able are more productive in the skilled job (θ_M) than are the less able in the unskilled job (θ_U).

Assumption Three. The less able are more productive in the unskilled job (θ_U) than they are in the skilled job (θ_L).

Thus, from the first three assumptions, $\theta_M > \theta_U > \theta_L$.

Assumption Four. The marginal cost of signaling is lower for the more able than it is for the less able.

With the level of the signal denoted by y, assume the cost of signaling is y for the less able and y/g for the more able, with $g > 1$.

Assumption One suggests individuals are not homogeneous in productivity in the skilled job. Assumptions Two and Four are necessary for signaling to occur. Note three reasons for Assumption Three. First, it is the same assumption as in Spence (1974a, 1976). Second,

[16] In Spence (1974a), more able individuals are called Group 2, less able individuals are called Group 1, the skilled job is called Job 2, and the unskilled job is called Job 1.

empirical results in Willis and Rosen (1979) suggest individuals who are more productive than others in one occupation would be less productive than the latter in alternative careers. This implies switching the less able from unskilled jobs to skilled jobs, and the more able from the skilled jobs to the unskilled job, would result in lower productivity for both types. Third, if we assume the less able have the same productivity in either job, then we would effectively have a model with one job type. Then pooling would only occur in the skilled sector, there would be no social gain from allocating the less able to unskilled jobs, and the model would be similar to the basic Spencian model where the gain to signaling is private and not social.

The Hoppe *et al*. (2009) marriage model discussed in the previous section assumes complementarity between the quality of men and women. Thus, the higher the quality of a person with whom one matches, the higher the match output. In order to prevent a solution where, say, all women match with the best men, Hoppe *et al*. assume a fixed number of men and women. As did Spence (1974a), we have chosen the alternative approach of Assumption Three (the less able are more productive in the unskilled job than they are in the skilled job).

Alternatively, assume Table One shows *net* productivity (marginal revenue product). Let gross productivity in the skilled job be $\theta_M + j$ for *M*s and $\theta_L + j$ for *L*s, with $\theta_L + j > \theta_U$ and j = fixed cost per worker as in Oi (1962). There is no fixed cost in the unskilled job, so net and gross productivity are the same there. *Gross* productivity is higher for *L*s in the skilled job than in unskilled jobs. If $\theta_M > \theta_U > \theta_L$, as assumed above, *net* productivity is higher for *M*s in the skilled job than in the unskilled job, and net productivity is lower for *L*s in the skilled job than in the unskilled job. Thus, fixed cost per worker in the skilled job permits the existence of gross complementarity, even though the less able are more valuable in the unskilled job given fixed costs per worker in the skilled job.

An additional question is the relative productivity of the two types of individuals in the unskilled job. Spence considered two possibilities: a) the more able have a larger MRP in the unskilled job than the less able, and b) the opposite of a). From Table One, we assume both types have the same productivity in the unskilled job, θ_U.[17] There are three reasons for this assumption. First, it is essentially the average of what Spence considered. Second, it allows us to reduce possible cases to consider. Third, ability should be of less importance in a job requiring less skill. In the next section, we discuss how results would change if more and less able individuals do not have the same productivity in the unskilled job.

Pooling equilibria play an important role in this paper. When pooling occurs, we assume the least cost pooling equilibrium results, that is both types of individuals set $y = 0$. We then have two potential pooling equilibria (Sub-section 3C). In one equilibrium, all individuals are hired in the unskilled job. In the other equilibrium, all go to the skilled job. Which equilibrium occurs depends on where expected productivity of individuals is the highest. In contrast, in the marriage model in Hoppe *et al*. (2009), pooling implies random matching of individuals. In our model, one type of job (skilled or unskilled) is not active if pooling occurs. In the marriage model, only two can enter into one match. Withdrawal of types does not occur because all can be matched with someone, except that the number of one type matching with others is constrained by the number of the other type.

3. A job allocation model

A. Outline of the game

[17] Another variation (Spence, 1981) is when the more able have the same productivity in the unskilled job as the less able have in the skilled job. Spence did not justify this assumption, but it is simply another way of introducing a social return to signaling. It can be shown that none of the results herein would be materially affected if we assumed the more able have the same productivity in the unskilled job as the less able have in the skilled job.

Most details of the model are found in Assumptions One through Four in Section 2. Employers know α of N potential employees are more able. With no signaling, all are paid the same wage, W_{pool}, where W_{pool} is the larger of the expected productivities in the two jobs when all are in one job: $W_{pool} = max\{\theta_U, \alpha\theta_M + (1-\alpha)\theta_L\}$. The Riley outcome (Riley, 1979) is when less able individuals set $y = 0$, and more able individuals set $y = y_{Riley}$---the lowest level of the signal that induces a separating equilibrium.

Individuals move first. If more able individuals do not signal, firms respond and hire individuals in the job where expected productivity is the highest, with pay of $max\{\theta_U, \alpha\theta_M + (1-\alpha)\theta_L\}$. If signaling occurs, more able individuals set $y = y_{Riley}$, and less able individuals set $y = 0$. With signaling, firms respond by hiring those with $y = y_{Riley}$ in the skilled job at a wage of θ_M, and those with $y = 0$ in the unskilled job at a wage of θ_U.

Using the *intuitive criterion* (Cho and Kreps, 1987), signaling only occurs at the *Riley outcome*. Further, the *undefeated equilibrium* refinement (Mailath *et al*., 1993) suggests signaling only occurs if it is preferred by more able individuals to pooling at $y = 0$.[18]

B. Equilibrium refinements

The *intuitive criterion* and *undefeated equilibrium* are explained in the Appendix in a simple example. It is sufficient to discuss how these concepts enable us to reduce the set of equilibria without eliminating reasonable equilibria. The *intuitive criterion* uses out-of-equilibrium beliefs to eliminate all separating equilibria except the Pareto dominant

[18] Cai *et al*. (2007) consider an equilibrium refinement they call the *Local Credibility Test* in a signaling game. They consider multiple types of individuals, with imperfect negative correlation between productivity and the marginal cost of signaling. As with *undefeated equilibrium*, the *Local Credibility Test* is more likely to select the *Riley outcome* over pooling the lower the percentage of more able individuals in the population.

equilibrium---the one with the lowest feasible level of the signal. Unfortunately, all pooling equilibria are eliminated by the *intuitive criterion*. The *undefeated equilibrium* refinement essentially amends the *intuitive criterion* to allow for a pooling equilibrium when pooling implies a larger payoff for more able individuals than would a separating equilibrium with $y = y_{Riley}$. What *undefeated equilibrium* does is assume the less able realize their payoff in a pooling equilibrium will *not* be available if the more able deviate from pooling.[19]

C. When pooling would be in the unskilled job

If $\theta_U > \alpha\theta_M + (1-\alpha)\theta_L$, expected productivity with no signaling is greater in the unskilled job than in the skilled job.[20] This occurs when $\alpha < \alpha^*$:

$$\alpha^* = \frac{\theta_U - \theta_L}{\theta_M - \theta_L} . \qquad (1)$$

With $\theta_M > \theta_U$, $0 < \alpha^* < 1$. Signaling will always occur in this case (see below). In a signaling equilibrium, those who signal will go to the skilled job and be paid θ_M, and those who do not signal will go to the unskilled job and be paid θ_U. For signaling to occur, more able individuals must prefer to be viewed correctly, and the less able must not want to mimic the more able. The basic conditions for signaling to occur are then:

[19] Koufopoulos (2011) has argued the use of equilibrium refinements in signaling games is unnecessary if commitment to contracts by uninformed players is endogenous. However, in effect, *undefeated equilibrium* allows firms to not exogenously commit to a potential pooling equilibrium, so the use of both the *intuitive criterion* and *undefeated equilibrium* yields similar results to those found in Koufopoulos.

[20] Bickhchandani, Hirshleifer, and Riley (2013) consider a model with a primary job and an outside opportunity that pays more than the expected productivity of individuals in the primary job. This is essentially what we have herein when pooling is in the unskilled job.

$$\theta_M - y/g \geq \theta_U, \tag{2}$$

$$\theta_M - y < \theta_U. \tag{3}$$

Ineqs.(2) and (3) yield:

$$\theta_M - \theta_U < y \leq g(\theta_M - \theta_U). \tag{4}$$

The lowest level of the signal that induces a separating/signaling equilibrium is when $y = y_{Riley}$:

$$y_{Riley} \approx \theta_M - \theta_U. \tag{5}$$

The net payoff to more able individuals from signaling is $\theta_M - \theta_U - y_{Riley}/g = (\theta_M - \theta_U)\left(1 - \frac{1}{g}\right) > 0$. This is also the condition for signaling to increase welfare. Because of the assumption all are worth θ_U in the unskilled job, the pooling wage in the unskilled job is the same as the wage one gets in a signaling equilibrium if one does not signal, θ_U. Signaling occurs and increases welfare because signaling results in the reallocation of more able individuals from the unskilled job to the skilled job were they are more productive, and because the output gain exceeds the cost.[21] The private return to signaling equals the social return.

[21] One criticism of signaling models is the assumption signaling cost and productivity are inversely related. Such an assumption is not necessary if individuals receive a “grade” in addition to the level of the signal, and if the more able are more likely to get a good grade (Weiss, 1983). Suppose in our model 1) all have an identical marginal cost of the signal, say equal to one, and 2) more able individuals pass a test with a probability of one, and less able individuals pass a test with a probability of p, $0 < p < 1$. Then $y_{Riley} = p(\theta_M - \theta_U)$, and, in a separating equilibrium,

Note, the assumption all have the same productivity in the unskilled job produces minor differences in our results and in those in Spence (1974a), who considered the possibility either type might be more productive than the other in the unskilled jobs. We will return to this issue in Sub-section F below.

D. When signaling increases welfare with pooling in the skilled job

When $\alpha > \alpha^*$, pooling is in the skilled job. In the next sub-section, we apply the concept of *undefeated equilibrium* to see when more able individuals prefer signaling to pooling. Assume for now signaling occurs. Consider when signaling would increase welfare. Now the social return to signaling comes from moving less able individuals from skilled jobs to unskilled jobs, with an output gain of θ_U - θ_L per less able individual. The total gain to society from signaling is $(\theta_U - \theta_L)(1-\alpha)N$. The social cost of signaling is the amount spent by the more able individuals, which equals $\frac{(\theta_M - \theta_U)\alpha N}{g}$. When pooling is in the skilled job, signaling increases welfare if $\alpha < \alpha^{**}$:

$$\alpha^{**} = \frac{g(\theta_U - \theta_L)}{g(\theta_U - \theta_L) + \theta_M - \theta_U}\,. \tag{6}$$

---

less able individuals set $y = 0$, and more able individuals set $y = y_{Riley}$. Nothing fundamental changes with grades and identical education cost. In order to compare our results with the original model in Spence (1974a), we ignore grades and maintain the assumption the more able have a lower marginal cost of signaling. Regev (2012) has a model in which education cost is the same for all, the productivity gain from education is higher for more able individuals, and employers noisily observe productivity. In her model, a completely separating equilibrium does not exist, but a mixed strategy equilibrium partially reveals ability.

Clearly $\alpha^{**} < 1$. Also, $\alpha^{**} > \alpha^{*}$ if $g > 1$, which is true. Below we will see signaling will occur if $\alpha < \alpha^{***}$, with $\alpha^{**} < \alpha^{***}$. Thus, if $\alpha^{*} < \alpha < \alpha^{**}$, signaling occurs and increases welfare.[22]

When pooling would be in the skilled job, the private gain to more able individuals from signaling is the increased pay that results versus pooling. The social gain from signaling results from reallocating the less able from the skilled job to the unskilled job. The effect of a higher α on the total private gain from signaling is ambiguous, but the individual return falls. As α increases, the pooling wage rises, so there is less of a return to signaling per more able individual, but there are more of these individuals. An increase in α lowers the social gain from signaling because there are fewer less able individuals to reallocate to unskilled jobs. The social cost of signaling rises as α increases because more signal. For $\alpha > \alpha^{**}$, welfare decreases if signaling occurs.

E. When pooling is preferred to signaling

To determine when pooling will occur when it involves all going to the skilled job, we use *undefeated equilibrium* to refine the *intuitive criterion*. *Undefeated equilibrium* implies more able individuals will deviate from pooling when they are better off by signaling with $y = y_{Riley}$.

---

[22] Suppose θ_U equals productivity only of *L*s in the unskilled job. Productivity of *M*s in the unskilled job equals $\hat{\theta} < \theta_M$. Herein, we assumed $\hat{\theta} = \theta_U$. The value for when expected productivity is the same in the skilled and unskilled job is $\hat{\alpha}^{*}$, $\hat{\alpha}^{*} = \frac{\theta_U - \theta_L}{\theta_M - \hat{\theta} + \theta_U - \theta_L}$. Note $\hat{\alpha}^{*}$ is positively related to $\hat{\theta}$. Since α^{**} and α^{***} do not depend on *M*s' productivity in the unskilled job, they are unchanged. There is no fundamental change from what was derived in the text as long as $\hat{\alpha}^{*} < \alpha^{**}$, so there is a range in which signaling occurs and increases welfare, given that pooling would be in the skilled job. We have $\hat{\alpha}^{*} < \alpha^{**}$ if $g > \frac{\theta_M - \theta_U}{\theta_M - \hat{\theta}} = \hat{g}$. If $\hat{\theta} \leq \theta_U$, $\hat{g} \leq 1$. With $g > 1$ for signaling to occur, we then essentially have the results as in the text. If $\hat{\theta} > \theta_U$, $\hat{g} > 1$. In that case, we require g to be sufficiently larger than one in order to have $\hat{\alpha}^{*} < \alpha^{**}$. Spence (1976) assumed $\hat{\theta} \geq \theta_U$ (in our notation).

When pooling would be in the skilled job, *M*s prefer signaling to pooling if net earnings with signaling exceed those with pooling (with $y = 0$ with pooling), or if

$\theta_M - \frac{(\theta_m - \theta_U)}{g} > \alpha\theta_M + (1-\alpha)\theta_L$. Thus, signaling occurs if $\alpha < \alpha^{***}$.

$$\alpha^{***} = \frac{\theta_m - \theta_L - \frac{(\theta_m - \theta_U)}{g}}{\theta_m - \theta_L} \,. \tag{7}$$

Now $\alpha^{***} < 1$ since $\theta_M > \theta_U$. Also, $\alpha^{***} > \alpha^{**}$ if $g > 1$, which is true. Thus, we have $0 < \alpha^* < \alpha^{**} < \alpha^{***} < 1$. The possible results are shown in Figure One. The width of the ranges shown in Figure One depends on θ_U, θ_L, θ_M, and g, as will be discussed below.

Summarizing, for low enough values of α, $\alpha < \alpha^*$, signaling occurs and increases welfare when the alternative is pooling in the unskilled job. The output gain from signaling involves reallocating more able individuals from the unskilled job to the skilled job. The private return to signaling---the wage increase for the more able---equals the social return.

For somewhat higher values of α, $\alpha^* < \alpha$, pooling would be in the skilled job. There is a social gain from signaling---the reallocation of the less able from the skilled job to the unskilled job---which differs from the private gain---increased earnings for the more able.

For $\alpha^* < \alpha < \alpha^{**}$, the social gain from signaling exceeds the cost, but, for higher values of α, $\alpha^{**} < \alpha$, when there are relatively few less able individuals, the social gain from signaling is less than the cost. For even higher values for α, $\alpha^{***} < \alpha$, the more able will not deviate from pooling, and welfare is higher than if signaling occurred. Thus, only if $\alpha^{**} < \alpha < \alpha^{***}$ will inefficient signaling occur.

F. Comparing the results to those in Spence (1974a).

i. Pooling would be in the unskilled job, and the more able are less productive than the less able in that job.

As discussed above, we assumed *M*s and *L*s have the same productivity in the unskilled job, θ_U. Spence (1974a) considered the possibility individuals differ in productivity in the unskilled job. Thus, let the expected MRP in the unskilled job equal $\overline{MRP}_{unskilled}$, which equals the pooling wage when pooling is in the unskilled job. Suppose $\hat{\theta}$ is an *M*'s productivity in the unskilled job, θ_U is an *L*'s productivity in the unskilled job, and $\hat{\theta} < \theta_U$. Thus, $\hat{\theta} < \overline{MRP}_{unskilled} < \theta_U$. In this case, Spence found both types benefit from signaling. We would have the same result if we had assumed *M*s were less productive than *L*s in the unskilled job. The earnings gain to an *M* from signaling is $\theta_M - \overline{MRP}_{unskilled}$, which is less than the output gain from reallocating *M*s to the skilled job, $\theta_M - \hat{\theta}$. With $\theta_U > \overline{MRP}_{unskilled}$, *L*s gain when they are no longer pooled with those less productive than them in the unskilled job. By assuming $\hat{\theta} = \theta_U$, we differ from Spence (1974a) only in that we find an *L*'s earnings are unchanged if signaling and not pooling occurs. With $\hat{\theta} \leq \theta_U$, signaling increases welfare because *M*s signal when their increase in earnings exceeds signaling cost, and the output gain exceeds the earnings gain to *M*s.

ii. Pooling would be in the unskilled job, and the more able are more productive than the less able in that job.

Now $\theta_U < \overline{MRP}_{unskilled} < \hat{\theta}$. Thus, *L*s are hurt with signaling since they are no longer pooled with those more able them in unskilled jobs. In this case, Spence found that signaling may cause *M*s to be worse off, and welfare may be reduced. The latter result is from assuming that $\hat{\theta} > \theta_U$, as opposed to our assumption that $\hat{\theta} = \theta_U$. The earnings gain to an *M* from signaling,

θ_M - $\overline{MRP}_{unskilled}$, exceeds the output gain, θ_M - $\hat{\theta}$. Thus, *M*s may signal when it is inefficient to do so.

However, the result, that *M*s might be worse off with signaling is *not* because of the assumption $\hat{\theta} > \theta_U$. Using the *intuitive criterion*, we assume the signal is at its lowest possible level, y_{Riley}. Also, using the further refinement *undefeated equilibrium*, the more able would not deviate from a pooling equilibrium in which they are better off than they would be with signaling.

iii. Pooling would be in the skilled job.

When pooling results in all placed in the skilled job, Spence found, when signaling benefits the less able, it also benefits the more able, so welfare clearly increases. First, consider when *L*s could be better off in the unskilled job with signaling than in the skilled job with pooling. Again, assume *M*s and *L*s have productivity of $\hat{\theta}$ and θ_U respectively in the unskilled job. Then *L*s have higher earnings with signaling if:

$$\theta_U > (1-\alpha)\theta_L + \alpha\theta_M. \quad (8)$$

Pooling is in the skilled job if expected productivity is higher there than in the unskilled job:

$$(1-\alpha)\theta_L + \alpha\theta_M > (1-\alpha)\theta_U + \alpha\hat{\theta}. \quad (9)$$

If $\hat{\theta} \geq \theta_U$, *ineqs*. (8) and (9) cannot both hold, and *Ls* cannot prefer signaling to pooling in the skilled job. As noted in footnote twenty-two, Spence (1976) assumed $\hat{\theta} \geq \theta_U$. If *M*s are sufficiently less productive than *L*s in the unskilled job---$\hat{\theta}$ is sufficiently smaller than θ_U----then it is possible for the two inequalities to both hold. In that case, *L*s gain if signaling occurs instead of pooling. Herein, it is assumed that $\hat{\theta} = \theta_U$, so when pooling is in the skilled job, signaling implies lower earnings for *L*s.

Second, as explained in the previous sub-section, *undefeated equilibrium* suggests the more able always gain if signaling occurs. Spence focused on when both types of individuals would be better off with signaling, since signaling clearly is efficient then. What we have shown previously (Sub-Section D above), is the efficiency question comes down to whether the output gain from reallocating the less able from the skilled job to the unskilled job exceeds the cost of more able individuals signaling. As discussed above, signaling can increase welfare if there are relatively few of the more able---α is not too large. As α increases, there are more *M*s to signal, and fewer *L*s to reallocate. Thus, it becomes less likely that signaling increases welfare.

Thus, if we assumed *M*s and *L*s differ in productivity in the unskilled job, there would be some minor changes in our results. However, the main differences between our results and those in Spence (1974a) stem for our use of the *Riley outcome,* the *intuitive criterion*, and *undefeated equilibrium*, none of which was available to Spence.[23]

4. The likelihood of efficient signaling, inefficient signaling, and pooling

[23] As noted in footnote nine, in a later paper, Spence (1976) did consider the equivalent of the *Riley outcome* and *intuitive criterion*, but did so in a model in which productivity is directly affected by the signal. Spence's earlier work (1974a) considers the problem we address herein: the signal does not directly affect productivity, but may increase welfare by allocating individuals to jobs where they are most productive.

We have shown that the relative number of different types of individuals is crucial to the welfare effects of signaling and whether signaling occurs. With α the fraction of more able individuals in the population, there are three possible results: efficient signaling (welfare is higher than with pooling), inefficient signaling (welfare is lower than with pooling), and efficient pooling (welfare is higher than with signaling). Consider what occurs over the range of α. The efficient signaling range, r_e equals α**. The range in which inefficient signaling occurs is $r_i = \alpha^{***} - \alpha^{**}$. The range when pooling (efficiently) occurs is $r_p = 1 - \alpha^{***}$.

Table Two contains some numerical examples. Note that, in the basic Spence (1974a) model with *one type of job*, $\theta_M = 2\theta_U$, $\frac{\theta_L}{\theta_U} = 1$, and $g = 2$. Thus, the first example in Table Two is virtually the same as the basic Spence signaling model with one type of job. In this example, and in all four of the examples shown in bold print, it is assumed $\frac{\theta_L}{\theta_U} = .99$. The less able individuals, *L*s, have (net) productivity in the skilled job that is only 1% lower than their productivity in the unskilled job. In these four examples, even with almost no output gain from reallocating *L*s from the skilled job to the unskilled job, the range of α for which inefficient signaling occurs is never as large as the range when either efficient signaling or pooling occurs (the last column in Table Two), with pooling involving higher welfare than with signaling. Even with little chance of efficient signaling, using *undefeated equilibrium*, there is a large range of α for which pooling (efficiently) occurs.

The other examples considered in Table Two involve a non-trivial output gain (either 11% or 25%) when moving *L*s from the skilled job to the unskilled job. In these examples, the range of α for which inefficient signaling occurs is much smaller than the combined range for efficient signaling and pooling.

Since we wish to know when welfare would be lower with signaling, we focus on $r_i = \alpha^{***} - \alpha^{**}$. Simplifying, we have:

$$r_i = \frac{(\theta_M - \theta_U)^2\left(\frac{g-1}{g}\right)}{[\theta_M - \theta_L][g(\theta_U - \theta_L) + \theta_M - \theta_U]} \,. \tag{10}$$

Consider the impact of g, θ_M, θ_U, and θ_L on r_i. Using *eq*.(10), we see how g affects r_i:

$$\frac{\partial r_i}{\partial g} = \{+\}\{\theta_M - \theta_U + g(2\text{-}g)(\theta_U - \theta_L)\}. \tag{11}$$

Now $\frac{\partial r_i}{\partial g} > 0$ if $g < 2$, and is still positive for some $g > 2$. If g increases, the marginal cost of signaling for more able individuals decreases. Since $\frac{\partial \alpha^{***}}{\partial g} > 0$, the pooling range, r_p, falls as g increases and more able individuals find signaling preferable to pooling for larger values of α. The range of efficient signaling increases since signaling is less costly--- $\frac{\partial \alpha^{**}}{\partial g} > 0$. The residual, the range of inefficient signaling, could increase or decrease as g increases. For low enough values of g, a lower marginal cost of signaling (an increase in g) increases the range of α for which inefficient signaling occurs.

The effect of θ_M on r_i is:

$$\frac{\partial r_i}{\partial \theta_M} = \{+\}\{g[\theta_M(\theta_U - \theta_L) + \theta_U^2 - 2\theta_L^2 - 3\theta_U\theta_L] + (\theta_M - \theta_U)\,(\theta_U - \theta_L)\}. \tag{12}$$

The sign of $\frac{\partial r_i}{\partial \theta_M}$ is ambiguous. An increase in θ_M increases y_{Riley}, thus lowering α^{**}, the range for which efficient signaling occurs: $\frac{\partial \alpha^{**}}{\partial \theta_M} = \{+\}\frac{1}{g}(\theta_L - \theta_U) < 0$. The pooling range, which equals 1-α^{***}, increases because pooling is more likely to be preferred to signaling by *M*s as y_{Riley} increases.

If $\theta_U \approx \theta_L \equiv x$, we have $\frac{\partial r_i}{\partial \theta_M} = \{+\}\{-4gx^2\} < 0$. Thus, at least for the case when there is little to be gained by reallocating *L*s from the skilled job to the unskilled job, a greater productivity for more able individuals in the skilled job implies less likelihood inefficient signaling occurs. If $\theta_U \approx \theta_L$, $\frac{\partial \alpha^{**}}{\partial \theta_M} = \{+\}\frac{1}{g}(\theta_L - \theta_U) \approx 0$: the range of efficient signaling essentially stays the same as θ_M increases. Since the pooling range increases as θ_M increases (as explained above), the likelihood of inefficient signaling falls as θ_M increases if $\theta_U \approx \theta_L$.

An increase in the productivity of less able individuals in the skilled job, θ_L, unambiguously increases the range of inefficient signaling:

$$\frac{\partial r_i}{\partial \theta_L} = \{+\}\{g(\theta_M + \theta_U - 2\theta_L) + \theta_M - \theta_U\} > 0. \qquad (13)$$

A larger θ_L reduces the gain to society from signaling, so signaling is less likely to be efficient. The pooling range increases as θ_L increases, since $\frac{\partial \alpha^{***}}{\partial \theta_L} = \{-\}\frac{1}{g}(\theta_M - \theta_U) < 0$. However, the reduction in the efficient range of signaling ---$\frac{\partial r_e}{\partial \theta_L} = \frac{\partial \alpha^{**}}{\partial \theta_L} = \{-\}g(\theta_M - \theta_U) < 0$---dominates, and r_i increases as θ_L increases.

An increase in the productivity of individuals in the unskilled job, θ_U, unambiguously decreases the range of inefficient signaling:

$$\frac{\partial r_i}{\partial \theta_U} = -\{+\}\{2[g(\theta_U - \theta_L) + \theta_M - \theta_U] + (g\text{-}1)(\theta_M - \theta_U)\} < 0. \quad (14)$$

An increase in θ_U decreases y_{Riley}. Consequently, the pooling range, 1-α***, falls as *M*s find signaling preferable to pooling for larger values of α. However, the range of efficient signaling, $r_e = \alpha$**, increases as θ_U increases due to a lower y_{Riley}. The increase in r_e dominates the decrease in r_p, and r_i decreases as θ_U increases.

Summarizing, we find inefficient signaling is *more* likely the smaller the marginal cost of signaling for the more able (unless the marginal cost of signaling is sufficiently small [*g* is sufficiently large]), the higher the productivity in the skilled job for the less able, and the lower the productivity of all in the unskilled job. A higher productivity of the more able in the skilled job has an ambiguous effect on the likelihood of inefficient signaling unless there is little productivity difference for the less able in skilled and unskilled jobs. In that case, a higher productivity of the more able in the skilled job reduces the likelihood of inefficient signaling almost entirely because of an increase in the likelihood pooling occurs.

5. Related Issues

A. Alternatives to signaling

We have demonstrated that educational signaling could easily improve welfare if signaling improves the allocation of individuals to jobs. There remains the question whether potentially cheaper screening mechanisms might exist. Gary Becker (1993) suggested “Surely a year on the job or a systematic interview and applicant-testing program must be a much cheaper and effective way to screen."[24]

In the European Union, applicant test responses are personal data, and some countries (Greece and France in particular) make employment tests difficult (Perritt, 2008). In the U.S., limits to the use of applicant testing are the U.S. Supreme Court decision *Griggs v. Duke Power* (1971), and subsequent court rulings and legislation. O'Keefe and Vedder (2008) argue employers now focus on very narrow subjects directly related to jobs in applicant testing. After Griggs, there has been less use of general aptitude and intelligence tests that might help employers determine an individual's ability to learn and advance at a firm. O'Keefe and Vedder argue educational signaling has substituted for employment testing since nothing prevents individuals from obtaining educational credentials to try to demonstrate ability to firms.[25]

Also, although not modeled herein, knowledge of an individual's ability to receive job training (Riley, 1981) may not be revealed by the narrow, job-specific tests now commonly used by firms.[26] Given current laws and court rulings, good alternatives to educational signaling may not be available.

B. Employer learning

[24] Becker (1993, p.8).

[25] At least one National Football League team, the Eagles, asks potential draftees many questions about their college academics (why a player chose his major, what was his hardest class, etc.). Further, the team believes that college graduation is more than proof of intelligence, and is a signal that an individual is committed to achieving goals (Clark, 2014).

[26] A related problem involves using educational signals to at least partially reveal ability when productivity depends both on ability and the particular job match. Habermalz (2006) finds evidence consistent with educational signaling in such a problem.

Some claim evidence of fairly rapid employer learning of skills implies a low value of educational signaling (Altonji and Pierret, 1997, and Lange, 2007). However, Habermalz (2011) shows such a conclusion is unwarranted. Habermalz demonstrates signaling is not necessary with very rapid employer learning. With very slow employer learning, signaling would not likely be worthwhile due to the short period for which ability would be revealed. With an intermediate length of employer learning, the value of the signal increases with the speed of learning. Estimated returns to education are consistent with theoretical results that imply signaling occurs.[27]

6. Summary

A social return to educational signaling occurs if signaling enables the optimal assignment of individuals to jobs. Using results for equilibrium refinements in signaling games developed since Spence (1974a) considered a model of job allocation, it is possible to be more precise than Spence was in determining when signaling would occur and what the effect of signaling on welfare would be. We find signaling that lowers welfare (is inefficient) is more likely the 1) the higher the productivity in the skilled job for the less able, 2) the lower the productivity of all in the unskilled job, and 3) the lower the marginal cost of signaling for the more able (unless marginal cost is very low). With the less able more productive in the unskilled jobs than in the skilled job, the first two points imply a lower productivity difference for the less able between different jobs. Thus, there is less social gain to reallocating the less able to where they are most productive.

[27] Alós-Ferrer and Prat (2012) develop a model in which an individual's value to an employer is revealed over time. Of importance for the analysis herein, they find pooling should occur (with both types setting the signal equal to zero) when it Pareto dominates the separating equilibrium with the *Riley outcome*.

The third point appears to be puzzling: lower marginal cost for the more able directly increases welfare. However, at high enough values for the fraction of the more able in the population, signaling benefits the more able but lowers welfare. Lower signaling cost increases the likelihood the more able will prefer signaling to pooling in the skilled job, so it is more likely signaling occurs that lowers welfare.

In contrast to Spence's earlier results, we find a clear effect of the fraction of more able individuals in the population, α, on the equilibrium. Small values for α are likely to be associated with efficient signaling, somewhat larger values for α would imply inefficient signaling, and the highest values of α would result in a pooling equilibrium that is efficient.[28] Unless there is little gain from reallocating less able individuals from the skilled job to the unskilled job, the range of α for which inefficient signaling occurs does not appear to be large.

The results herein suggest that higher education might serve a valuable signaling function----even if little human capital is acquired in universities. Except for the original work by Spence (1974a, 1974b), there has been little focus on the social gain from education when the educational signal can improve the allocation of individuals to jobs. For example, in his comparison of human capital and signaling explanations of wages, Weiss (1995) suggests signaling models of education are resisted by some economists because these models imply inefficient equilibria. Our results suggest claims of little human capital accumulation in higher education should be considered given other potential social benefits of higher education, including its role in improving job allocation.

[28] As noted in Section One, the terms efficient and inefficient refer to a world in which optimal job allocation cannot be obtained without costly signaling. Thus, for example, efficient signaling means welfare is higher than it would be with pooling, and not that welfare is higher than it would be with costless allocation of the more able to skilled jobs and the less able to unskilled jobs.

References

Alós-Ferrer, Carlos, and Prat, Julien. “Job Market Signaling and Employer Learning.” *Journal of Economic Theory* 147 (September 2012): 1787-1817.

Altonji, Joseph G., and Pierret, Charles. “Employer Learning and the Signaling Value of Education.” In *Internal Labor Markets, Incentives, and Employment*, edited by Isao Ohashi and Toshiaki Tachibanaki. London: Macmillan Press Ltd., 1998: 159-195.

Arcidiacono, Peter, Bayer, Patrick, and Hizmo, Aurel. “Beyond Signaling and Human Capital: Education and the Revelation of Ability.” *American Economic Journal*: *Applied Economics* 2 (October 2010): 76-104.

Arum, Richard, Roksa, Josipa, and Cho, Esther. “Improving Undergraduate Learning: Findings and Policy Recommendations from the SSRC-CLA Longitudinal Project.” Social Science Research Council, 2011.

Becker, Gary S. *Human Capital*, 3rd ed. Chicago, IL: University of Chicago Press, 1993.

Bickhchandani, Sushil, Hirshleifer, Jack, and Riley, John G. *The Analytics of Uncertainty and Information*. 2nd edition, Cambridge, UK: Cambridge University Press, 2013.

Cai, Hongbin, Riley, John, and Ye, Lixin. “Imperfect Signaling and the Local Credibility Test.” Working paper, UCLA, September 2007.

Caplan, Bryan. “The Magic of Education.” *Library of Economics and Liberty* (November 28, 2011), http://econlog.econlib.org/archives/2011/11/the_magic_of_ed.html.

Cho, In-Koo, and Kreps, David M. “Signaling Games and Stable Equilibria.” *Quarterly Journal of Economics* 102 (May 1987): 179-221.

Clark, Kevin. “The Philadelphia Eagles Personnel Strategy: Targeting College Grads.” *Wall Street Journal* (May 20, 2014). http://online.wsj.com/news/articles/SB10001424052702304422704579572050338722302

Greenberg, Joseph. “Job Matching Through the Signaling Mechanism.” *European Journal of Political Economy* 5 (Issues 2-3, 1989): 297-315.

Habermalz, Steffen. “More Detail on the Pattern of Returns to Educational Signals.” *Southern Economic Journal* 73 (July 2006): 125-135.

________________ . “The Speed of Employer Learning and Job Market Signaling Revisited.” *Applied Economics Letters* 18 (May-June 2011): 607-610.

Hopkins, Ed. “Job Market Signaling of Relative Position, or Becker Married to Spence.” *Journal of the European Economic Association* 10 (April 2012): 290-322.

Hoppe, Heidrun C., Moldovanu, Benny, and Sela, Abner. “The Theory of Assortative Matching Based on Costly Signals.” *The Review of Economic Studies* 76 (January 2009): 253-281.

Koufopoulos, Kostas. “Endogenous Commitment and Nash Equilibria in Competitive Markets with Adverse Selection.” Working paper, University of Warwick, September 2011.

Lange, Fabian. “The Speed of Employer Learning.” *Journal of Labor Economics* 25 (January 2007): 1-35.

Leonhardt, David. “Even for Cashiers, College Pays Off.” *New York Times*, June 25, 2011, http://www.nytimes.com/2011//06/26/sunday-review/26leonhradt.html.

Leppämäki, Mikko, and Mustonen, Mikko. “Skill Signaling with Product Market Externality.” *The Economic Journal* 119 (July 2009): 1130-1142.

Mailath, George J., Okuno-Fujiwara, Masahiro, and Postlewaite, Andrew. “Belief-Based Refinements in Signalling Games.” *Journal of Economic Theory* 60 (August 1993): 241-276.

Oi, Walter Y. “Labor as a Quasi-Fixed Factor.” *Journal of Political Economy* 70 (December 1962): 538-555.

O’Keefe, Brian, and Vedder, Richard. “Griggs v. Duke Power: Implications for College Credentialing.” Working paper, Pope Center for Higher Education Policy, 2008.

Perritt, Henry H., Jr. *2008 Employment Law Update*. Chicago: Aspen Publishers, 2008.

Perri, Timothy J. “Testing for Ability When Job Assignment is a Signal. *Labour Economics* 1 (September 1994): 365-381.

______. “Education Cost, Signaling, and Human Capital.” Working paper, Appalachian State University, May 2014.

Regev, Tali. “Education Signaling with Uncertain Returns.” *The B.E. Journal of Theoretical Economics: Contributions* 12 (Issue 1, Article 27, 2012): 1-29.

Riley, John G. “Competitive Signalling.” *Journal of Economic Theory* 10 (April 1975): 174-186.

_______. "Informational Equilibrium." *Econometrica* 47 (March 1979): 331-359.

_______. “Comment.” In *Studies in Labor Markets*, edited by Sherwin Rosen. Chicago: University of Chicago Press, 1981: 371-376.

_______ . “Competition with Hidden Knowledge.” *Journal of Political Economy* 93 (October 1985): 958-976.

_______. “Silver Signals: Twenty-Five Years of Screening and Signaling.” *Journal of Economic Literature* 39 (June 2001): 432-478.

_______. “Weak and Strong Signals.” *Scandinavian Journal of Economics* 104 (June 2002): 213-236.

Spence, Michael. *Market Signaling*. Cambridge, MA: Harvard University Press, 1974(a).

_______. “Competitive and Optimal Responses to Signals: An Analysis of Efficiency and Distribution.” *Journal of Economic Theory* 7 (March 1974(b)): 296-332.

_______. “Competition in Salaries, Credentials, and Signaling Prerequisites for Jobs.” *Quarterly Journal of Economics* 90 (February 1976): 51-74.

_______. “Signaling, Screening, and Information.” In *Studies in Labor Markets*, edited by Sherwin Rosen. Chicago: University of Chicago Press, 1981: 319-357.

_______. “Signaling in Retrospect and the Informational Structure of Markets.” *American Economic Review* 92 (June 2002): 434-459.

The Region, Minneapolis Federal Reserve Bank. Interview with Kenneth Arrow (December 1995). www.minneapolisfed.org/publications_papers_display.cfm?id=3675.

Von Ungern-Sternberg, Thomas. “Education as a Signal.” *Economics Letters* 4 (Number 3, 1979): 235-237.

Waldman, Michael. “Job Assignments, Signaling, and Efficiency.” *Rand Journal of Economics* 15 (Summer 1984): 255-267.

Weiss, Andrew. “A Sorting-cum-Learning Model of Education.” *Journal of Political Economy* 91 (June 1983): 420-442.

_______. “Human Capital vs. Signalling Explanations of Wages.” *Journal of Economic Perspectives* 9 (Fall 1995): 133-154.

Willis, Robert J., and Rosen, Sherwin. “Education and Self-Selection.” *Journal of Political Economy* 87 (October 1979): S7-S36.

Wolpin, Kenneth I. “Education and Screening.” *American Economic Review* 67 (December 1977): 949-958.

Figure One

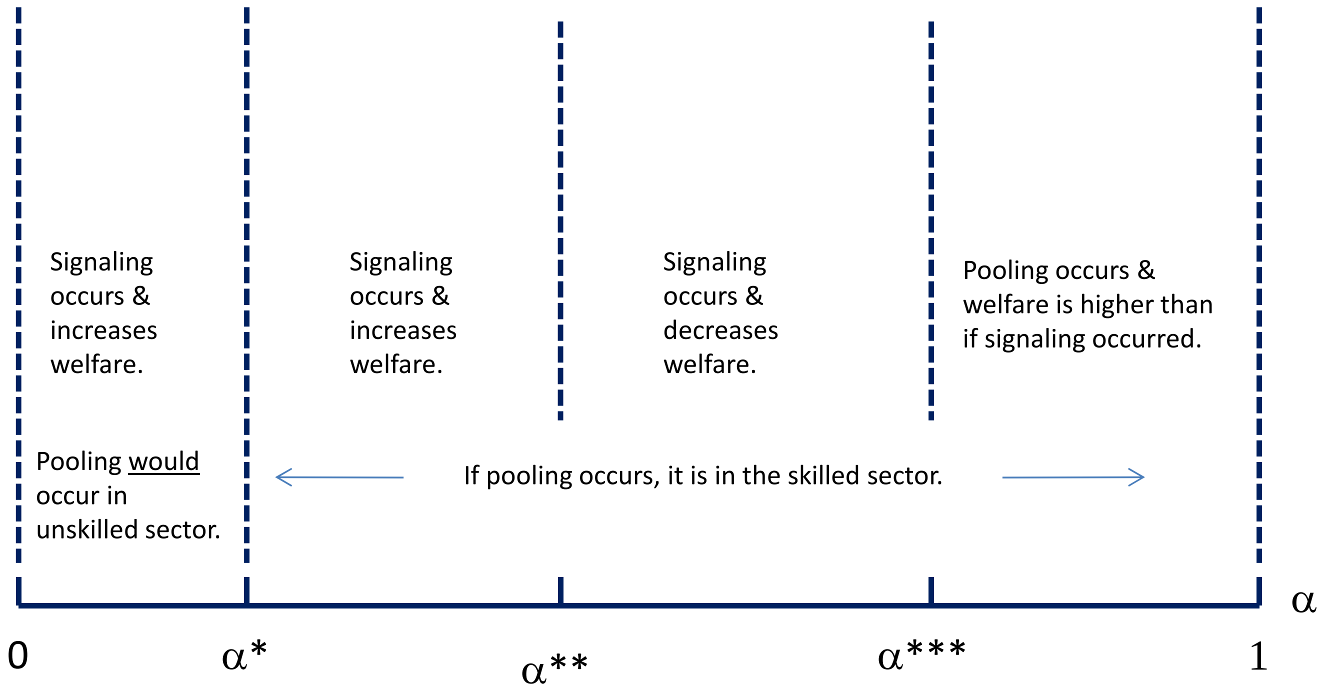
Signaling occurs & increases welfare.
Signaling occurs & increases welfare.
Signaling occurs & decreases welfare.
Pooling occurs & welfare is higher than if signaling occurred.
Pooling would occur in unskilled sector.
If pooling occurs, it is in the skilled sector.
α
0
α*
α**
α***
1

Table One. Marginal revenue products (MRPs).		
	More Able	Less Able
Skilled Job	θ_M	θ_L
Unskilled Job	θ_U	θ_U

Table Two. Values of α when efficient signaling (r_e), inefficient signaling (r_i), and pooling (r_p) occur.						
$\frac{\theta_M}{\theta_U}$	$\frac{\theta_L}{\theta_U}$	g	r_e	r_i	r_p	$\frac{r_e+r_p}{r_i}$
2	**.99**	**2**	**.02**	**.48**	**.5**	**1.08**
2	.9	2	.17	.38	.45	1.63
2	.8	2	.29	.29	.42	2.45
2	**.99**	**1.5**	**.01**	**.33**	**.66**	**2.03**
2	.9	1.5	.13	.26	.61	2.85
2	.8	1.5	.23	.21	.56	3.76
1.5	**.99**	**2**	**.03**	**.47**	**.49**	**1.1**1
1.5	.9	2	.29	.3	.42	2.37
1.5	.8	2	.44	.2	.36	4
1.5	**.99**	**1.5**	**.03**	**.32**	**.65**	**2.13**
1.5	.9	1.5	.23	.21	.56	3.76
1.5	.8	1.5	.38	.15	.48	5.73

Appendix

The Intuitive Criterion and Undefeated Equilibria.

Consider the case with only one job. This is sufficient to explain how *undefeated equilibria* modifies the *intuitive criterion*. More able individuals have productivity equal to 2 and a cost of signaling of $y/2$. Less able individuals have productivity equal to 1 and signaling cost of y. The fraction of more able individuals in the population is α. The pooling wage is $W_{pool} = 2\alpha + 1 - \alpha = 1 + \alpha$. Suppose $\alpha = .75$. From Figure A1, if one assumes (as with the *intuitive criterion*) the less able believe pooling remains available to them even if the more able signal to separate themselves, then the shaded area contains separating equilibria preferred by the more able to pooling, but not preferred by the less able. However, if (as assumed with *undefeated equilibrium*) the less able realize signaling by the more able implies a wage = 1 for those who do not signal, then the relevant level of utility for the less able is given by $U^{separate}_{less\ able}$. Thus, the less able would prefer to signal and be paid 2 as long as $y < 1 = y_{Riley}$, point I_3. Since point I_3 is below $U^{pool}_{more\ able}$, more able individuals will not deviate from the pooling equilibrium with $y = 0$ and a wage of 1.75. Pooling survives the *undefeated equilibrium* refinement if y_{Riley} is not contained in the area in which the more able will deviate from pooling when the less able would not deviate if they believed pooling remained available to them. In Figure A2, with $\alpha = .5$, y_{Riley} (point J_2) is contained in the shaded area. In essence, a pooling equilibrium does not survive if the more able are better off signaling with $y = y_{Riley}$ than they are with pooling and
$y = 0$.

Figure A1. $\alpha = .75$

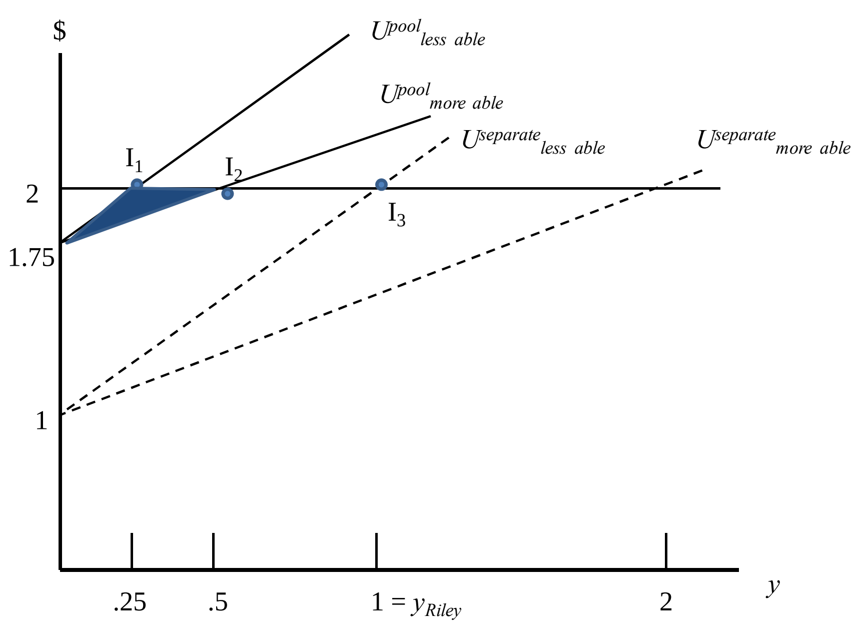

Figure A2. $\alpha = .5$