Substitution and Superstars

by

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Revised August 22, 2014

Forthcoming in *Economics Letters*

Abstract

The existing superstar model (Rosen, 1981) does not require imperfect substitutes, and the convexity of *total earnings* with respect to talent is due to greater output for those with more talent. Our model explains why *wages* would increase at an increasing rate in talent. Imperfect substitutability between non-superstars and superstars with probabilistic production results in convexity in wage rates.

JEL classifications: D11, D31, and J31

Keywords: Superstars, imperfect substitutes, and convex wages

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1. Introduction

In his seminal paper on superstars (Rosen, 1981), Sherwin Rosen analyzed markets that he argued contain at least one of two features: poor talent is an inadequate substitute for superior talent (superstars), and technology is such that many buyers can be served simultaneously. There is low marginal cost of providing additional units of the service, as with *joint goods*. These features may lead to total earnings for an individual or total revenue for a firm increasing at an increasing rate in talent (convexity in earnings)---because output is positively related to talent---and a few high quality individuals or firms selling a large percentage of market output and reaping a large percentage of market revenue.

However, convexity in *per unit earnings* (wages or prices) cannot be explained by low marginal cost of production. We demonstrate how imperfect substitutability can explain convexity in the wage rate. We do so in a model of probabilistic success, where the combined effort of more than one non-superstar can have the same likelihood of accomplishing a task as a superstar. Thus, we offer an explanation of one superstar phenomenon not explained by Rosen's classic model---convexity in per unit earnings.

Rosen mentioned doctors and lawyers as examples of superstars (Rosen, 1981) where there is poor substitutability of lesser talent for superior talent (Rosen, 1983). A low marginal cost of serving many customers does not characterize the market for lawyers and doctors. Nor does it explain why the real earnings of the highest paid dentists tripled from 1979 to 1989 while average dental earnings barely increased (Frank and Cook, 1995). Convex earnings profiles for surgeons, lawyers, and dentists must be due to the *wage rate* for these individuals increasing rapidly in talent. This contrasts with *media markets* (Borghans and Groot, 1998) such as television, movies, and recorded music, where a few individuals may capture much of a market's output and revenue. There low marginal cost certainly exists.[1]

Another reason for high compensation for superstars, which is somewhat similar to the effect of large output in media markets, is the increased return from activities such as lawsuits. In a University of Chicago Law School Roundtable discussion of superstar effects, Robert Frank (UC, 1999) observed that a lawyer who used to represent one client now might be involved in a class action lawsuit involving a much larger payoff. Sherwin Rosen (UC, 1999) noted how top lawyers work on big claims, and argued that less than the best lawyers could not be used in such cases. Relatedly, Kevin Murphy (UC, 1999) claimed that "congestion" affects superstar earnings. Murphy argued congestion implies one could not use one thousand storefront lawyers in court versus one superstar lawyer, so the former are supposedly not substitutes for the latter.

We show that congestion is not necessary for convexity in earnings with respect to talent. Also, congestion is not necessarily a problem in many activities with superstar effects. It is possible for several less talented lawyers to replace a superstar lawyer. More than one lawyer can conduct arguments in court (although not at once), and multiple lawyers are often used in researching and preparing cases.[2]

The usual view of a superstar is one where "...lesser talent often is a poor substitute for greater talent."[3] In order to consider imperfect substitutes, we follow Rosen (1981) in two ways. First, Rosen's suggestion a 10% more successful surgeon would likely command a wage

[1] Frank and Cook (1995) refer to markets with superstar effects as "winner-take-all markets." In such markets, output is dominated by a few sellers. This contrasts with the competitive model of superstars (Perri, 2013), in which each seller only captures a small share of market output, but revenue is increasing and convex in seller quality since higher quality sellers have a higher demand and thus receive a higher price and sell more. The competitive model can explain why superstar musicians earn much more than lower quality musicians in concerts (Krueger, 2005) where, unlike recorded music, superstars are physically unable to perform in a large percentage of all concerts.

[2] The model herein does not apply to situations where there is a constraint on how many individuals may be employed. For example, a sports team can have only so many players, and there can be no more than *x* players participating for a team at any one time (eleven for soccer and American football, five for basketball, etc.).

[3] Rosen, 1981, p.846.

premium of more than 10% implies a probabilistic dimension to production that provides a simple way to model substitutability. Second, for sellers arrayed by quality, Rosen argued:

> "Though sellers of different quality are imperfectly substitutable with each other, the extent of substitution decreases with distance. In the limit *very close neighbors are virtually perfect substitutes*" (emphasis added).[4]

We assume buyers place the same value on the services of individual sellers or teams of sellers when the probability of success is the same.

2. A model with substitution

Consider a market for services. Buyers are individual consumers. Sellers are either individual superstars or teams of non-superstars.[5] Let λ be the probability of success for a superstar. Non-superstars each have a success rate of p, with $p < \lambda$. The success of one non-superstar is independent of the success of others. A team of n non-superstars has a success rate of ρ. Following Rosen (1981), we assume producers that are virtually identical are essentially perfect substitutes. Further, we assume imperfect substitution is extreme in that consumers will only hire a team of non-superstars if $\rho = \lambda$.

A team has success if any member succeeds. The probability that no one succeeds is $(1-p)^n$, so $\rho = 1-(1-p)^n$. Setting $\rho = \lambda$ and solving for n:

$$n = \frac{ln(1-\lambda)}{ln(1-p)} \qquad (1)$$

[4] Rosen, 1981, p.850.
[5] Internal difficulties within teams due to strategic interactions are ignored herein since they are not fundamental to the analysis of superstar versus non-superstar wages.

Note $|ln(1-\lambda)| > |ln(1-p)|$ for $p < \lambda$, so $n > 1$. To see the impact of λ and p on n, differentiate n:

$$\frac{\partial n}{\partial p} = \frac{\ln(1-\lambda)}{(1-p)[ln(1-p)]^2} < 0, \tag{2}$$

$$\frac{\partial n}{\partial \lambda} = \frac{-1}{(1-\lambda)ln(1-p)} > 0. \tag{3}$$

An increase in p implies fewer non-superstars are required to attain a success rate of $\rho = \lambda$. A decrease in the level of success for superstars, λ, means fewer non-superstars are required to reach the lower desired success rate.

Let the wage of superstars be given by $W_{SS} = W_{SS}(\theta,\lambda)$, with $\frac{\partial W_{SS}}{\partial \lambda} > 0$ and $\frac{\partial W_{SS}}{\partial \theta} = 1$. A higher success rate for superstars yields them a higher wage. Also, θ is a shift parameter. An increase in θ would occur if either the demand for superstars increased, or the supply of superstars decreased, the former *not* due to superstars having a higher success rate ($d\lambda > 0$). A team of n non-superstars whose success rate equals that of a superstar is of equal value to a superstar for consumers. The wage of a non-superstar, W_{NS}, is then $W_{NS} = W_{SS}/n$:

$$W_{NS} = W_{SS} \frac{ln(1-p)}{ln(1-\lambda)}. \tag{4}$$

The first and second derivatives of W_{NS} with respect to p yield:

$$\frac{\partial W_{NS}}{\partial p} = \frac{-W_{SS}}{(1-p)ln(1-\lambda)} > 0, \tag{5}$$

$$\frac{\partial^2 W_{NS}}{\partial p^2} = \frac{-W_{SS}}{(1-p)^2 ln(1-\lambda)} > 0. \tag{6}$$

The wage of non-superstars is increasing and convex in their individual success probability due to imperfect substitution. Since the wage represents per unit compensation, nothing herein depends on superstars selling more units of their service.

Additionally, an exogenous change in W_{SS} affects W_{NS}:

$$0 < \frac{\partial W_{NS}}{\partial \theta} = \frac{ln(1-p)}{ln(1-\lambda)} < \frac{\partial W_{SS}}{\partial \theta} = 1. \tag{7}$$

As discussed in Section 1, Robert Frank and Sherwin Rosen (UC, 1999) suggest an increased return for lawsuits over time has raised the earnings of superstar lawyers. Although not necessary for convexity of the wage, from *ineq*.(7), we see that more lucrative returns for activities in which superstars are engaged will increase the wage for non-superstars less than the amount by which the superstar wage increases.

A greater success rate for superstars, λ, also affects the wage of non-superstars:

$$\frac{\partial W_{NS}}{\partial \lambda} = \frac{ln(1-p)}{ln(1-\lambda)} \left[\frac{\partial W_{SS}}{\partial \lambda} + \frac{W_{SS}}{(1-\lambda) ln(1-\lambda)}\right] = \frac{ln(1-p)}{ln(1-\lambda)} \frac{W_{SS}}{\lambda} \left[\xi_{SS,\lambda} + \frac{\lambda}{(1-\lambda) ln(1-\lambda)}\right], \tag{8}$$

where $\xi_{SS,\lambda}$ is the elasticity of W_{SS} with respect to λ. An increase in λ lowers W_{NS} because it takes more non-superstars to have the same success rate as a superstar. However, an increase in λ raises W_{NS} because $\frac{\partial W_{SS}}{\partial \lambda}$ is positive and W_{NS} is directly related to W_{SS}. Unless $\xi_{SS,\lambda}$ is relatively large, $\frac{\partial W_{NS}}{\partial \lambda}$ will be negative. For example, if $\lambda = .9$, $\frac{\partial W_{NS}}{\partial \lambda} > 0$ only if $\xi_{SS,\lambda} > 3.91$. If $\lambda = .8$,

$\frac{\partial W_{NS}}{\partial \lambda} > 0$ only if $\xi_{SS,\lambda} > 2.49$.

To see the effect of imperfect substitution on the wage of non-superstars, suppose $W_{SS} = \lambda k$, where k is a positive constant. Then $W_{NS} = \lambda k \frac{ln(1-p)}{ln(1-\lambda)}$. Table One shows some examples. Consider two of the cases in Table One. If $\lambda = .9$ and $p = .6$, $\frac{p}{\lambda} = .667$, but $\frac{W_{NS}}{W_{SS}} = .398$: one with about 67% of the probability of success of a superstar would be paid about 40% of what a superstar would earn. If $\lambda = .9$ and $p = .8$, $\frac{p}{\lambda} = .889$, but $\frac{W_{NS}}{W_{SS}} = .669$: one with about 89% of the probability of success of a superstar would be paid about 70% of a superstar's wage. Put differently, recall Rosen's (1981) conjecture a 10% more successful surgeon would likely have a wage premium of more than 10%. Using Table one, a superstar with $\lambda = .9$ has a 12.5% greater probability of success than a non-superstar who has $p = .8$, yet the former earns 43% more (.9k versus .629k) than the latter.

3. Summary

The seminal paper by Rosen (1981) well explains several phenomena in superstar markets: total individual or firm earnings increasing with talent at an increasing rate, and market output and revenue highly concentrated among a few sellers. Rosen's model does not require imperfect substitutes, and the convexity of *total earnings* with respect to talent is due to greater output for those with more talent. Our model explains why *wages* would increase at an increasing rate in talent due to imperfect substitutability between non-superstars and superstars when production is uncertain. If the convexity of wages in talent identified here is combined with significantly lower marginal cost for superstars relative to non-superstars, then the convexity of *total earnings* is even more pronounced.

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Table One. Wages and relative wages of non-superstars and superstars.					
λ	p	$\frac{p}{\lambda}$	W_{SS}	W_{NS}	$\frac{W_{NS}}{W_{SS}}$
.9	.8	.889	$.9k$	$.629k$	.699
.9	.7	.778	$.9k$	$.471k$	.523
..9	.6	.667	$.9k$	$.358k$	.398
.9	.5	.556	$.9k$	$.271k$	.301
.8	.7	.875	$.8k$	$.598k$	.748
.8	.6	.75	$.8k$	$.455k$	.569
.8	.5	.625	$.8k$	$.345k$	.431