Substitution and Superstars

by

Timothy Perri*

Revised May 08, 2012

*Department of Economics, Appalachian State University, Boone, NC, 28608. E-mail: perritj@gmail.com. Acknowledgements...to be added.

Substitution and Superstars
Timothy Perri
May 08, 2012

Abstract

The existing superstar model (Rosen 1981) does not require imperfect substitutes, and the convexity of *total earnings* with respect to talent is due to greater output for those with more talent. Our model explains why *wages* would increase at an increasing rate in talent. Imperfect substitution results due to the probabilistic nature of production. If buyers view those with less talent to be worse substitutes for superstars, the former will earn lower wages relative to superstars, but their absolute wage may increase. Costs to consumers from repeated consumption---multiple surgeries for example---are neither necessary nor sufficient for convexity in wages.

JEL classifications: D11, D31, and J31

Keywords: Superstars, imperfect substitutes, and convex earnings

1. Introduction

It has been a little more than thirty years since the publication of Sherwin Rosen's seminal paper on superstars (Rosen, 1981). In that paper, and in a non-technical paper with the same title published two years later (Rosen, 1983), Rosen analyzed markets that he argued contain at least one of two features: poor talent is an inadequate substitute for superior talent (superstars), and technology is such that many buyers can be served simultaneously (there is low marginal cost of providing additional units of the service), as with *joint goods*. These features may lead to total earnings for an individual or total revenue for a firm increasing at an increasing rate in talent (convexity in earnings), and a few high quality individuals or firms selling a large percentage of market output and reaping a large percentage of market revenue.

Rosen (1981) noted that imperfect substitutability can not account for a significant concentration of output among a few sellers, nor is imperfect substitutability necessary to explain convexity in total earnings. He also briefly considered the case (developed in more detail in Perri, forthcoming) in which perfect substitutes are assumed. In that case, convexity of total revenue results because higher quality sellers have both higher prices and output. Marginal cost increases in output, and may even increase in quality, provided it does not increase too rapidly in the latter. Superstars may earn a disproportionate share of market revenue, but do not produce a significant percentage of market output. In order to have large percentages of market output accruing to a few sellers, low marginal cost is required.[1]

Thus, imperfect substitutability is not required for either convexity of total earnings (total revenue for a firm) or a large concentration of market output and earnings among a few sellers. Additionally, convexity in *per unit earnings* (wages or prices) can not be explained by low

[1] Low or even declining marginal cost is not sufficient to ensure a large concentration of total earnings among a few sellers. Such concentration requires either superior talent or network effects, which could be important for sellers in the entertainment industry where consumers of more popular sellers find it easier to find those who also are fans of these sellers (Frank and Nüesch, 2012).

marginal cost of production. Our goal herein is to demonstrate how imperfect substitutability can explain convexity in the wage rate. We do so in a model of probabilistic success, where the combined effort of more than one non-superstar can have almost the same likelihood of accomplishing a task as a superstar. Thus, we offer an explanation of one superstar phenomenon not explained by Rosen's classic model---convexity in per unit earnings. Also, we explicitly derive a measure of substitutability which is not present in the Rosen model.

Rosen mentioned doctors as an example of superstars (Rosen, 1981). Further, he suggested a surgeon who is 10% more likely to save a life should earn much more than a 10% premium. He also used lawyers as an example of poor substitutability of lesser talent for superior talent (Rosen, 1983). A low marginal cost of serving many customers does not characterize the market for lawyers and doctors. Nor does it explain why the real earnings of the highest paid dentists tripled from 1979 to 1989 while average dental earnings barely increased (Frank and Cook, 1995). Convex earnings profiles for surgeons, lawyers, and dentists must be due to the *wage rate* for these individuals increasing rapidly in talent. This contrasts with *media markets* (Borghans and Groot, 1998) such as television, movies, and recorded music, where a few individuals may capture much of a market's output and revenue. There low marginal cost certainly exists.[2]

The outline of the rest of the paper is as follows. In Section 2, we discuss imperfect substitution. In Section 3, we develop the formal model of imperfect substitution for an individual consumer. Since hiring more than one non-superstar may imply a sequence of hires, there could be costs to consumers due to delay in accomplishing the desired task. Such costs are

[2] Frank and Cook (1995) refer to markets with superstar effects as "winner-take-all markets." In such markets, output is dominated by a few sellers. This contrasts to the competitive model of superstars (Perri, forthcoming), in which each seller only captures a small share of market output, but revenue is increasing and convex in seller quality since higher quality sellers have a higher demand and thus receive a higher price and sell more. The competitive model can explain why superstar musicians earn much more than lower quality musicians in concerts (Krueger, 2005), where, unlike recorded music, superstars are physically unable to perform in a large percentage of all concerts.

considered in Section 4. It is shown such costs are neither necessary nor sufficient for convexity in the wage rate. Market equilibrium is considered in Section 5, and Section 6 contains a summary.

2. Substitution

The usual view of a superstar is one where "...lesser talent often is a poor substitute for greater talent."[3] Superstars occur in markets where consumers place "...considerable weight on quality versus quantity."[4] In order to consider imperfect substitutes, we follow Rosen (1981) in two ways. First, Rosen's suggestion a 10% more successful surgeon would likely command a wage premium of more than 10% implies a probabilistic dimension to production which provides a simple way to model substitutability. Second, for sellers arrayed by quality, Rosen argued:

> "Though sellers of different quality are imperfectly substitutable with each other, the extent of substitution decreases with distance. In the limit *very close neighbors are virtually perfect substitutes*" (emphasis added).[5]

We combine both of these points in order to consider the extent of substitutability between individuals, where more talented individuals are more likely to succeed in producing the service desired by consumers.

Consider employing one superstar who has a higher chance of success than that of non-superstars. What is of interest is the probability of success when more than one non-superstar are employed, with the possibility of both simultaneous and sequential use of non-superstars.

---

[3] Rosen, 1981, p.846.
[4] Autor, 2005, p.2.
[5] Rosen, 1981, p.850.

Rosen (1983) mentions, without elaborating, the case of two lawyers, each of whom individually has a 50% chance of winning a case. He suggests employing both lawyers might not elevate the probability of winning much above 50%, and might actually decrease the likelihood of winning. However, unless one lawyer impedes another, the probability of winning should increase as more lawyers are employed. Clients often engage teams of lawyers. Presumably more non-superstars could substitute for some number of superstar lawyers. Surgeons also work in teams, and it is possible to have more than one surgeon present even if only one actually performs the surgery.

One way of considering substitution is to suppose talent means the likelihood one will correctly determine how to proceed with a task---a case for lawyers, an operation for a surgeon, etc. In Rosen's example, one can think of employing two lawyers, *A* and *B*, each of whom has a 50% of succeeding. If one lawyer can not determine how to proceed, the view of the other is immediately considered (that is, there is no delay of any consequence). The probability the two lawyers are successful is then, given independence of success,

{probability of success for *A*} + {probability of failure for *A*}{probability of success for *B*} = $5 + .5^2 = .75$.

The same probability of success might be attainable if the lawyers were hired sequentially in criminal trials: first one trial occurs, and then, if the individual is not acquitted, either a second similar trial (if the first resulted in a hung jury or a mistrial) or an appeal (if the first trial is lost) occurs. For now, we ignore delay costs (possibly lost work time for a second trial, and incarceration awaiting an appeal if convicted in the first trial). It is possible the two trials may not be independent events: losing the first trial might affect the likelihood of winning the second, given lawyer talent. However, assuming independence allows a simple way to consider imperfect substitutes, and may be a reasonable approximation of reality.

Using the idea from Rosen that very close services are essentially perfect substitutes, suppose n non-superstars can produce a likelihood of success equal to λ times the probability a superstar would succeed, with λ possibly very close to one. If an individual would pay v for the services of a superstar, the individual would be willing to pay λv for the *combined services of the n non-superstars*. Further, to emphasize Rosen's point substitutability diminishes with distance, it is assumed a success rate less than λ has no value to the consumer. Individuals decide what value of λ is acceptable to them. In the next section, it will be shown a larger value of λ is naturally interpreted as less substitutability of non-superstars for superstars.

Rosen's (1981) method of explicitly deriving imperfect substitution involved a fixed cost of consumption per unit of quality. However, he recognized fixed consumption costs were not necessary for his results. As discussed previously, even perfect substitution can result in total earnings convex in talent and large shares of market output and earnings accruing to a few superstar sellers. We also will consider a cost of consumption which results from delay due to hiring a sequence of non-superstars. However, we will show such a cost is neither necessary nor sufficient for convexity of the wage rate with respect to talent. Convexity results from the probabilistic nature of production of services.

3. A model with substitution

Let λ be the probability of success when n non-superstars are employed (with or without delay). For simplicity, it is assumed the probability of success for a superstar is one. Each non-superstar has a success probability of p, $0 < p < 1$, and the success of one is independent of the success of others. Consider sellers for whom $p < \lambda$. Employing n such non-superstars, the probability of success is given by:

$$\lambda = p + (1\text{-}p)p + (1\text{-}p)^2 p + \ldots + (1\text{-}p)^{n-1} p = 1 - (1\text{-}p)^n \qquad (1)$$

Solving *eq*.(1) for n:

$$n = \frac{ln(1-\lambda)}{ln(1-p)} \qquad (2)$$

Note $|ln(1\text{-}\lambda)| > |ln(1\text{-}p)|$ for $\lambda > p$. The minimum value for λ is p, in which case $n = 1$. Suppose a superstar's value to a consumer is v. Consumers choose λ, which may be close to one. Thus, consumers decide at what level of success for a set of non-superstars, relative to that for superstars, the combined effort for the non-superstars is a perfect substitute for that of a superstar. If $\lambda \leq p$, we have perfect substitutes: a consumer is indifferent to hiring one superstar at a wage of v or one non-superstar at a wage of pv. If $\lambda > p$, as λ increases, superstars and non-superstars become worse substitutes. With $\lambda > p$, $n > 1$.

As suggested by Rosen (1981), the extent of substitution decreases as the outcome with non-superstars is farther from that of superstars. Herein, it is assumed the decrease in substitutability is extreme. If n non-superstars yield an expected probability of success of λ, a consumer would pay λv *in total* for the n non-superstars. However, given the desired level of λ,[6] it is assumed combinations of non-superstars that lead to a probability of success less than λ are

[6] Differences in λ among consumers are considered in Section 5.

not substitutes for superstars, and would not be hired. Again, given $\lambda > p$, consumers would only purchase the services of either a superstar, or of n non-superstars, with n determined by *eq*.(2).

To see the impact of λ and p on n, differentiate n:

$$\frac{\partial n}{\partial p} = \frac{\ln(1-\lambda)}{(1-p)[ln(1-p)]^2} < 0, \tag{3}$$

$$\frac{\partial n}{\partial \lambda} = \frac{-1}{(1-\lambda)ln(1-p)} > 0. \tag{4}$$

As one would expect, as p increases or λ decreases, fewer non-superstars are required to replace a superstar. Consumers decide how close to the success of a superstar they require in order to treat the effort by n individuals as being equivalent to λv. In Section 5, we will consider how the wage of a superstar, v, is determined by market supply and demand. For now, v is fixed so we can set $v = 1$. Then each non-superstar is valued by w:

$$w = \lambda/n = \frac{\lambda ln(1-p)}{ln(1-\lambda)}. \tag{5}$$

Eq.(5) gives the wage for those not considered to be perfect substitutes for superstars (since they have $p < \lambda$) relative to the wage of superstars ($p = 1$). An increase in λ directly increases w because the probability of success of the non-superstars is higher. However, a larger λ requires a larger n, given p, which lowers w. We have:

$$\frac{\partial w}{\partial \lambda} = \frac{ln(1-p)}{[ln(1-\lambda)]^2}\left[ln(1-\lambda) + \frac{\lambda}{1-\lambda}\right]. \tag{6}$$

Now $\frac{\partial w}{\partial \lambda} < 0$ if $-ln(1-\lambda) < \frac{\lambda}{1-\lambda}$. When $\lambda = 0$, $-ln(1-\lambda) = \frac{\lambda}{1-\lambda} = 0$. Also,

$\frac{\partial\left(\frac{\lambda}{1-\lambda}\right)}{\partial \lambda} = \frac{1}{(1-\lambda)^2} > \frac{\partial[-ln(1-\lambda)]}{\partial \lambda} = \frac{1}{1-\lambda}$. Thus, $\frac{\lambda}{1-\lambda} > -ln(1-\lambda)$ for all $\lambda > 0$, and $\frac{\partial w}{\partial \lambda} < 0$. When non-superstars become worse substitutes for superstars, that is, when λ increases, the wage of non-superstars declines (given the wage of superstars).

To see if the wage increases at an increasing rate in ability, *p*, differentiate *w*:

$$\frac{\partial w}{\partial p} = \frac{-\lambda}{(1-p)ln(1-\lambda)} > 0, \tag{7}$$

$$\frac{\partial^2 w}{\partial p^2} = \frac{-\lambda}{(1-p)^2 ln(1-\lambda)} > 0. \tag{8}$$

We find the wage is increasing and convex in ability due to imperfect substitution. Since the wage represents per unit compensation, nothing herein depends on those with more ability (a larger *p*) selling more units of their service. Nor does convexity require the delay costs mentioned above and considered in the next section.

Further, we can see how λ affects the slope and convexity of the wage as a function of *p*:

$$\frac{\partial^2 w}{\partial p \partial \lambda} = \frac{-\left[ln(1-\lambda)+\frac{\lambda}{1-\lambda}\right]}{(1-p)[\ln(1-\lambda)]^2} < 0, \tag{9}$$

$$\frac{\partial^3 w}{\partial p^2 \partial \lambda} = \frac{-\left[ln(1-\lambda)+\frac{\lambda}{1-\lambda}\right]}{(1-p)^2[\ln(1-\lambda)]^2} < 0. \tag{10}$$

Although we must have $\lambda > p$ for there to be imperfect substitution, if superstars and non-superstars are worse substitutes ($d\lambda > 0$), the slope and convexity of *w* with respect to *p* decline. This is because $\frac{\partial w}{\partial \lambda}$ is negative.[7]

Table One shows values for *n* and *w* for cases when superstars and non-superstars are poor substitutes (λ is close to 1). To illustrate the convexity of per unit earnings (*w*) in talent (*p*), consider comparable percentage increases in *p*. For example, if $\lambda = .95$, an increase in *p* from .5 to .7 (a 40% increase in *p*) causes *w* to increase by 73% (from .22 to .38). A further increase in *p* from .7 to 1 (a 43% increase in *p*) results an increase in *w* of 163% (from .38 to 1). For an illustration of Rosen's (1983) point that a surgeon who is 10% more successful in saving lives should be paid a good deal more a than 10% premium, compare superstars and those with $p = .9$. A superstar's success rate is about 11% higher (1 versus .9) than that of one with $p = .9$, but, if $\lambda = .99$, a superstar would be paid twice (1 versus .5) that of one with $p = .9$. With $\lambda = .95$, a superstar would still earn about 37% more (1 versus .73) than the a non-superstar who has $p = .9$.

4. Delay cost

As discussed in section one, if $n > 1$ implies a sequence of hiring of non-stars, there may be costs of delay. These are costs in addition to the total amount paid in wages, *wn*. Denote such costs as $c(n)$. Delay may be important for some activities and trivial for others. For example, if one hires a non-superstar lawn service to kill weeds, several visits by the service for a few weeks simply costs one a little longer time with an unsightly lawn. Alternatively, if one must have surgery, and *n* means more than one operation (versus one performed by a superstar) and not *n*

[7] In Section 5, we consider market determination of the wage of superstars, *v*. There we find a positive effect of λ on *v*, and thus on *w*, in addition to the negative effect of λ on *w* found in this section. The results in this section involve the wage of those who are not perfect substitutes for superstars relative to the wage of superstars.

surgeons performing one operation, delay cost would at least imply additional pain and suffering, and could be substantial.

Table One. The wage (w) and number (n) of non-superstars (with the wage of superstars = 1).

λ	p	n	w
.999	.9	3	.333
.999	.8	4.29	.233
.999	.7	5.74	.174
.999	.6	7.54	.132
.999	.5	9.97	.1
.99	.9	2	.5
.99	.8	2.86	.355
.99	.7	3.82	.26
.99	.6	5.03	.2
.99	.5	6.64	.15
.95	.9	1.3	.73
.95	.8	1.86	.51
.95	.7	2.49	.38
.95	.6	3.27	.29
.95	.5	4.32	.22

Delay cost causes the wage for non-superstars to be even lower than what results due to imperfect substitution ($\lambda > p$). Let $\delta = c(n)/n$. Thus, δ is the average cost of delay (per non-superstar hired). With primes denoting partial derivatives, we have:

$$\frac{\partial \delta}{\partial n} = \frac{nc' - c}{n^2} = \frac{c}{n^2}\left(\xi_{c,n} - 1\right), \tag{11}$$

where $\xi_{c,n}$ is the elasticity of *c* with respect to *n*. Thus, the average cost of delay is a positive function of *n* if delay cost is elastic in *n*. Note if $c = k(n-1)^{\theta}$, $k > 0$, and $\theta > 0$, then $\frac{\partial \delta}{\partial n} = \frac{k(n-1)^{\theta-1}}{n^2}[n\theta - (n-1)]$. Thus, $\frac{\partial \delta}{\partial n} > 0$ if $\theta > \frac{n-1}{n}$. If $\theta = 1$, so $c'' = 0$, $\xi_{c,n} = \frac{n}{n-1} > 1$ and $\frac{\partial \delta}{\partial n} > 0$. Even if $c'' < 0$ ($\theta < 1$), we can still have $\xi_{c,n} > 1$ if $\theta > \frac{n-1}{n}$.

With delay cost, $w = \frac{\lambda ln(1-p)}{ln(1-\lambda)} - \delta$. We then have:

$$\frac{\partial w}{\partial p} = \frac{-\lambda}{(1-p)ln(1-\lambda)} - \frac{\partial \delta}{\partial n}\frac{\partial n}{\partial p}. \tag{12}$$

The first term on the RHS of *eq*.(12) is positive, and $\frac{\partial n}{\partial p}$ is negative. Delay cost increases $\frac{\partial w}{\partial p}$ if $\frac{\partial \delta}{\partial n} > 0$---that is, if $\xi_{c,n} > 1$. To see how delay cost affects the convexity of *w*, the second derivative of *w* with respect to *p* is now:

$$\frac{\partial^2 w}{\partial p^2} = \frac{-\lambda}{(1-p)^2 ln(1-\lambda)} - \left\{\frac{\partial^2 \delta}{\partial n^2}\left[\frac{\partial n}{\partial p}\right]^2 + \frac{\partial \delta}{\partial n}\frac{\partial^2 n}{\partial p^2}\right\}. \tag{13}$$

The first term on the RHS of *eq*.(13) is positive. Let the {•} term in *eq*.(13) be noted by *J*. If $J < 0$, delay cost makes *w* more convex in *p*.

$$J = \left[\frac{ln(1-\lambda)}{(1-p)[ln(1-p)]^2}\right]^2 [n^3 c'' - 2n(nc' - c)]\frac{1}{n^4}$$
$$+ \frac{(nc'-c)}{n^2(1-p)^2[ln(1-p)]^3} ln(1-\lambda)[ln(1-p) + 2]. \tag{14}$$

If c'' < 0, J is more likely to be negative, but, if c'' > 0, the term in J involving c'' is positive. If $c'' \approx 0$, we have:

$$J = \frac{c ln(1-\lambda)\left(\xi_{c,n}-1\right)}{n^2(1-p)^2[ln(1-p)]^3}\left[ln(1-p) + 2 - \frac{2ln(1-\lambda)}{n ln(1-p)}\right]. \qquad (15)$$

Using $n = \frac{ln(1-\lambda)}{ln(1-p)}$, *eq*.(15) becomes

$$J = \frac{c ln(1-\lambda)\left(\xi_{c,n}-1\right)}{n^2(1-p)^2[ln(1-p)]^2}, \qquad (15')$$

which is < 0 if $\xi_{c,n} > 1$. Thus, cost elastic in n and $c'' \approx 0$ (which, as shown above, are not inconsistent) ensure delay cost increases the magnitude of $\frac{\partial w}{\partial p}$ and the convexity of w in p. Although delay cost lowers the wage, given p, such cost is not necessary for w to be convex in p, nor does delay cost unambiguously increase either the first or second derivative of w with respect to p. Imperfect substitution is all that is necessary for the wage to increase at an increasing rate in ability (p).

5. Market equilibrium

A. Introduction

Our previous analysis considered individual demand for non-superstars. We now consider labor market equilibrium when there are 1) differences in consumer values for superstars, and 2) differences in λ among consumers. In Section 3, we considered a fixed wage for superstars, v, which was normalized to one. Now v is determined by market supply and demand, so the wage equation for non-superstars with $\lambda > p$ is *eq*.(5) with v included:

$$w = \frac{\lambda v ln(1-p)}{ln(1-\lambda)}. \qquad (5')$$

As discussed in Section 3, for a buyer with a particular value of λ, sellers with $\lambda \leq p$ are perfect substitutes for superstars, and such a buyer values the individual's service by pv. Buyers view sellers with $\lambda > p$ as imperfect substitutes; such sellers receive a wage given by *eq*.(5').

To illustrate how the wage of a superstar, v, is determined, consider three types of sellers: superstars with $p = 1$, type one individuals (*T1*s) with $p = p_1$, and type two individuals (*T2*s) with $p = p_2$. Assume $0 < p_1 < p_2 < 1$.

B. Buyers have different values for superstars, but identical requirements for "success" (identical λs)

In order to consider different consumer values for superstars, suppose for now all buyers have an identical requirement for success, that is, have an identical value for λ, with $p_1 < \lambda < p_2$. We consider the market for effective superstar services (ESS). T2s are perfect substitutes for superstars; each T2 provides p_2 units of service, and there are N_2 such individuals, so these individuals provide a total of p_2N_2 units of ESS. Each superstar provides one unit of ESS (since $p = 1$ for them); with N_s superstars, these individuals provide N_s units of ESS. There are N_1 T1s, and it takes n of them to provide λ units of ESS, with $n = \frac{ln(1-\lambda)}{ln(1-p_1)}$ (*eq*.(2)). Thus, T1s provide $\lambda N_1/n = \frac{N_1 \lambda ln(1-p_1)}{ln(1-\lambda)}$ units of ESS.

The total supply of ESS is then independent of the price of a superstar, v, and equals N:

$$N = N_s + p_2 N_2 + \frac{N_1 \lambda ln(1-p_1)}{ln(1-\lambda)}. \qquad (16)$$

With consumers differing in their value for superstars, the demand for ESS will slope down. The supply of and demand for ESS determine v. Each superstar is paid v, each T2 is paid vp_2, and each T1 is paid $\frac{\lambda v ln(1-p_1)}{ln(1-\lambda)}$ (*eq*.(5')).

An interesting feature of the market for ESS is that, although λ comes from consumers, it does not affect the demand for ESS. Since λ determines how many imperfect substitutes ($p < \lambda$) are required to produce a perfect substitute (worth λv), λ affects the supply of ESS. For example, an increase in λ means there is a smaller supply of ESS from those who are not perfect substitutes for superstars (T1s in this example) because it takes more of these individuals to produce a likelihood of success equal to λ. However, the supply of ESS is directly increased as λ increases. To find the net effect of λ on the supply of ESS, we have:

$$\frac{\partial N}{\partial \lambda} = \frac{N_1 ln(1-p_1)\left[\ln(1-\lambda)+\frac{\lambda}{1-\lambda}\right]}{[\ln(1-\lambda)]^2} < 0, \qquad (17)$$

since, as shown before, $\frac{\lambda}{1-\lambda} > -ln(1-\lambda)$ for all $\lambda > 0$. Thus, an increase in λ---consumers demand a higher level of success for sellers to be perfect substitutes for superstars---will reduce the supply of ESS as long as any sellers have $p < \lambda$. This means a larger λ results in a larger wage for superstars, v, or $\frac{\partial v}{\partial \lambda} > 0$. Sellers who are superstars and sellers who are not superstars but who are perfect substitutes for superstars ($p \geq \lambda$), will receive higher wages as λ increases. For those sellers with $p < \lambda$ we use *eq*.(5'):

$$\frac{\partial w}{\partial \lambda} = \frac{v ln(1-p)}{[ln(1-\lambda)]^2}\left[ln(1-\lambda) + \frac{\lambda}{1-\lambda}\right] + \frac{\lambda ln(1-p)}{ln(1-\lambda)}\frac{\partial v}{\partial \lambda}. \tag{18}$$

The second term on the RHS of *eq*.(18) is positive because $\frac{\partial v}{\partial \lambda} > 0$; the wage of superstars rises as the supply of ESS decreases when λ increases. However, the first term on the RHS of *eq*.(18) is negative (as shown before) because it reflects the relative wage effect of λ for those who are not perfect substitutes for superstars. Such individuals are less valuable, relative to superstars, as λ increases. Thus, for those with $p < \lambda$, the effect of them being poorer substitutes for superstars (λ increasing) on their wages is uncertain, but they will be paid less relative to superstars.

C. Buyers have identical values for superstars but different views towards substitutes (different λs)

To consider the effect of differences in consumers in λ, suppose all consumers value superstars by the same amount. Thus, the demand for effective superstar services (ESS) is horizontal, and the wage a superstar would receive, v, is completely demand determined, and is fixed in this case.

Consider a type one (T1) individual with a success probability of p_1. For buyers with $\lambda \leq p_1$, a T1 is a perfect substitute for a superstar, and would be paid vp_1 if hired only by such buyers. For buyers with $\lambda > p_1$, a T1is not a perfect substitute for superstars, and would be paid

$\frac{\lambda v ln(1-p_1)}{ln(1-\lambda)}$ (*eq*.(5')), which is less than they would be paid by buyers with $\lambda \leq p_1$ since $\frac{\partial w}{\partial \lambda} < 0$ when v is fixed.

Thus, sellers will try to match with buyers who view them as perfect substitutes for superstars. If there are more T1s than would be hired by buyers with $\lambda \leq p_1$, then some T1s will be hired by buyers with $\lambda > p_1$. Suppose the highest λ for any buyer who hires a T1 is $\tilde{\lambda}$, with $\tilde{\lambda} > p_1$. Thus, T1s would receive a wage equal to $\frac{\tilde{\lambda} v ln(1-p_1)}{ln(1-\tilde{\lambda})}$, which will then be the wage all those who hire pay for a T1. Since, given v, $\frac{\tilde{\lambda} v ln(1-p_1)}{ln(1-\tilde{\lambda})} \equiv \tilde{w}$ is inversely related to λ, those with $\lambda < \tilde{\lambda}$, who hire T1s at a wage of $\tilde{w}$ receive consumer surplus. A large enough supply of a particular type of individual will lower the wage for these individuals because some of them will be hired where they are not viewed as perfect substitutes.

Again consider two types of sellers, T1s and T2s, with T2s having $p = p_2 > p_1$, and $p_2 > \tilde{\lambda}$. As just discussed, if there are more T1s than would be hired by buyers with $\lambda \leq p_1$, then the wage for T1s will equal $\tilde{w} < vp_1$. Buyers with $\lambda \leq p_1$ prefer to pay $\tilde{w}$ for T1s than vp_2 for T2s[8]---who are perfect substitutes for superstars for these buyers. Buyers with $p_1 < \lambda < \tilde{\lambda}$ also prefer to pay $\tilde{w}$ for T1s than vp_2 for T2s since $\frac{\lambda v ln(1-p_1)}{ln(1-\lambda)} > \frac{\tilde{\lambda} v ln(1-p_1)}{ln(1-\tilde{\lambda})}$ for $\lambda < \tilde{\lambda}$. A wage of $\frac{\lambda v ln(1-p_1)}{ln(1-\lambda)}$ would make such buyers indifferent to hiring T1s or hiringT2s at a wage of vp_2. For buyers with $\tilde{\lambda} < \lambda < p_2$, the wage required to make them indifferent to hiring a T1 and a T2 is less than $\tilde{w}$. Therefore, they prefer to hire T2s.

Thus, sellers will try to match with buyers who view the seller as a perfect substitute for superstars ($\lambda \leq p$), and buyers with low values of λ will try to match with sellers with low values

[8] Buyers with $\lambda \leq p_1$ are indifferent to hiring T1s at a wage of vp_1 and T2s at a wage of vp_2 (or superstars at a wage of v).

of p, so positive assortative matching between buyers (ranked by λ) and sellers (ranked by p) will tend to occur.

6. Summary

The seminal paper by Rosen (1981) well explains several phenomena in superstar markets: total individual or firm earnings increasing with talent at an increasing rate, and market output and revenue highly concentrated among a few sellers. These results depend on superstars producing a much higher output than that for non-superstars. However, significantly larger output is not always optimal for superstars due to rising marginal cost. Yet some superstars have much higher per unit compensation than others, with such compensation apparently convex in talent.

The model herein explains convexity in wage rates due to imperfect substitutability between non-superstars and superstars when production is not certain. A virtue of the model is that it allows for a natural measure of the degree of substitutability between superstars and those with lesser talent. Consumers decide how close to the rate of success of a superstar a *set of non-superstars* must be in order for the combined effort of the latter to be essentially a perfect substitute for a superstar--- although *individual non-superstars* are imperfect substitutes (except when $\lambda \leq p$).

If non-superstars become less perfect substitutes for buyers (λ increases), the effective supply of superstar services decreases, which, for this reason, raises the wage of all sellers. However, the wage of those not viewed as perfect substitutes for superstars falls for another reason: it now takes more of them to provide a success rate of λ. Thus, the net effect of an increase in λ on the wage for those who are not perfect substitutes for superstars is uncertain, but their wage relative to superstars will decrease.

One possible source of imperfect substitutability involves costs to consumers from delay when hiring non-superstars implies repeated attempts to produce the desired service. Although delay costs could be substantial, they are neither necessary nor sufficient for the wage increasing and convex with respect to talent.

With λ the success rate for a set of non-superstars relative to that for a superstar, what is not determined in the model herein is why λ might be relatively high (close to one, implying most sellers are very poor substitutes for superstars) or low (close to the probability of success of low talent non-superstars, implying virtually perfect substitutes). An explanation for why λ is relatively high or low is a topic for future research.

References

Autor, David. “MIT 14.662 Graduate Labor Economics II Spring 2005 Lecture Note 4: Market Structure and Earnings Inequality (w/a focus on Superstars).” Department of Economics, MIT, February 2005.

Borghans, Lex, and Groot, Loek. “Superstardom and Monopolistic Power: Why Media Stars Earn More Than Their Marginal Contributions to Welfare.” *Journal of Institutional and Theoretical Economics* 154 (September 1998): 546-571.

Frank, Robert H., and Cook, Philip J. *The Winner-Take-All Society*. New York: Free Press, 1995.

Frank, Egon, and Nüesch, Stephan. “Talent and/or Popularity: What Does it Take to be a Superstar?” *Economic Inquiry* 50 (January 2012): 202-216.

Krueger, Alan B. “The Economics of Real Superstars: The Market for Rock Concerts in the Material World.” *Journal of Labor Economics* 23 (January 2005): 1-30.

Perri, Timothy J. “A Competitive Model of (Super)stars.” *Eastern Economic Journal*, forthcoming.

Rosen, Sherwin. “The Economics of Superstars.” *American Economic Review* 71 (December 1981): 845- 858.

____. “The Economics of Superstars.” *American Scholar* 52 (Autumn 1983): 449-460.