Uncle Sam Wants Whom?
The Draft and the Quality of Military Personnel

by

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Abstract

Supposedly, the draft enables the military to attract more able individuals than a volunteer military, increasing welfare. Using a theoretical model, I find a draft cannot increase welfare when the military costlessly and accurately tests individuals, and does not take the lowest quality applicants. Only if testing is relatively costly or imprecise would a draft dominate a volunteer military. With either a low quality volunteer military or imprecise testing, a volunteer military is more likely to be preferable to a draft the larger the size of the military. The opposite is true with either costly testing or deadweight loss from taxation.

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1. Introduction

Economists have suggested a draft might be preferable to a volunteer military when the quality of military personnel is important. Ross (1990, 1994) considered the merits of a draft when the quality of personnel is not homogeneous.\(^1\) He found a draft might be superior to a volunteer military. More recently, Berck and Lipow (2011) suggest a positive relation between civilian and military productivity may mean one of the usual costs associated with the draft---the “wrong” people are inducted---may disappear.

The question of how to staff the military is more important today because the U.S. military has become more skilled over time. The next three paragraphs illustrate the change in military quality in the U.S. in recent decades, particularly since the end of the draft (1973).

The military defines high quality recruits as those who are high school graduates and who score in the top 50% on the Armed Forces Qualification Test (AFQT).\(^2\) From 1981 to 2004, the percentage of high quality recruits went from 34 to 72 (Army), 55 to 66 (Navy), 49 to 69 (Marines), and 60 to 81 (Air Force) (Asch et al., 2005).

In 2006, 91% of U.S. military recruits were high school graduates, compared to 80% for all U.S. residents ages 18-24. During the Viet Nam War era (1964-1973), when the draft was employed, 72% of those who served in the U.S. military were high school graduates (GAO, 1988). Those who score above the 65\(^{th}\) percentile on the AFQT are in the top two AFQT categories. The bottom category includes those from the 9\(^{th}\) percentile and below (CBO, 2007). Currently, the military has nearly 50% of its recruits from the top two AFQT categories, and 1%

\(^1\) Ross (1990) is an unpublished version of Ross (1994). The former has a more detailed discussion of heterogeneous labor quality. I thank Tom Ross for providing me his unpublished paper.

\(^2\) The Armed Forces Qualification Test (AFQT) is based on the Armed Services Vocational Aptitude Battery (ASVAB). The ASVAB has 10 sub-tests. The AFQT is computed by adding the following: the scores on arithmetic reasoning and math knowledge, and double the scores on paragraph comprehension and word knowledge. Thus, the AFQT roughly measures verbal and quantitative skills (Hosek and Mattock, 2003). The top X% on the AFQT means these individuals scored where the top X% of enlisted personnel and officers scored during World War Two. The AFQT has changed over time so comparability between time periods is somewhat questionable (Pirie, 1980).
from the bottom category. During the last two years of the draft (1971-1973), 33% were in the top two AFQT categories, and almost 25% were in the bottom category (Gilroy, 2010).

Technical skills have become more important in the military. In 2001, 18% of U.S. recruits worked in information technology related tasks, and almost 30% of these individuals were considered information technology core positions. ³

A military in which recruit quality is important is much different than the military usually considered in models that compare a draft to a volunteer military. In those models, the lowest opportunity cost individuals are assumed to be inducted with a volunteer military, without regard to the productivity of those individuals in the military. Additionally, there are several limitations with the previous analyses of a draft with heterogeneous labor quality.

First, there are costs other than the wrong individuals being inducted that remain with a draft. Asch et al. (2005) note military personnel quality depends on a taste for the military and individual effort. Becker (1957) also considered the reduced effort to be expected with conscripts relative to volunteers. Second, if the military attempts to draft those who are more able, these individuals will try to appear less able, reducing the number of high quality individuals drafted. ⁴ Third, even if a draft brings in more able individuals, given the opportunity cost of these individuals, welfare may not be improved. ⁵ Fourth, a draft has been compared to a low quality volunteer military, which is not the kind of military in the U.S. today.

The main focus of this paper is to consider a theoretical model in which the military may test individuals, excluding those with low ability. I compare a volunteer military with testing to two kinds of draft militaries, one with testing and one with a random draft (and no testing). I

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³ Information technology related positions include navigators and radar operators. Core IT positions include system operators and network analysts (Hosek et al., 2004).

⁴ Richard Danzig (then assistant secretary of defense) argued, if the military tried to draft the top half of the ability distribution, “You generate a lot of people who are trying to cheat, to appear to be not as good as they really are...” (Danzig, 1982, p.110).

⁵ Berk and Lipow (2011) note a draft may cause the enlistment of too many high quality individuals.
allow for the possibility some individuals may be able to “fail” the test, that is appear to be of less ability.

Among the results are the following. When the military costlessly and accurately tests individuals, and does not take the lowest quality applicants, a draft cannot increase welfare. Only if testing is relatively costly or imprecise would a draft dominate a volunteer military. Ignoring deadweight loss from taxation, with either a low quality volunteer military or imprecise testing, a volunteer military is more likely to be preferable to a draft the larger the size of the military. The opposite is true with either costly testing or deadweight loss from taxation.

The rest of the paper proceeds as follows. The outline of the model is in the next section. Although the main focus is on a military with testing, in Section 3, testing is ignored and a random draft and a low quality volunteer military are compared. In Section 4, testing is introduced. Costly testing is considered in Section 5, and imprecise testing is the focus of Section 6. In Section 7, costly deferments and the deadweight loss from taxation are examined since they have been analyzed previously with homogeneous labor. The paper is summarized in Section 8.

2. Outline of the model

The following are the basic assumptions of the model.

- The number of individuals available for work is normalized to one.
- A fixed number of individuals, \( m \), is required in the military in order for the military to produce.
- An individual’s civilian quality/output is denoted by \( q \), where \( q \) is uniformly distributed on \([0,1]\) with a density of one.
• In the military, individuals are either productive, with an individual output of \( k \), or unproductive, with an output of zero.\(^6\) Those who are productive in the military have \( q \geq q^* \), so the fraction of individuals who would be productive in the military\(^7\) equals 1 - \( q^* \).

• Except for in Section 7, lump sum taxation is assumed so there is no deadweight loss from taxation to pay military wages.

If I assumed the military tried to attract \( m \) productive individuals in expectation, in some cases, more than \( m \) individuals would be enlisted. I assume the military is constrained to enlist exactly \( m \) individuals, and will attempt to get as many who are productive given that constraint. Assuming a fixed military productivity, \( k \), for those with \( q \geq q^* \) allows me to focus on which system, a draft or a volunteer military, obtains the largest number of high quality (productive) individuals, and the opportunity cost of those individuals.

These assumptions allow those more able in civilian employment to be more productive in the military. Also, my assumptions imply welfare is higher if those with the lowest civilian output of those with \( q \geq q^* \) are in the military. This is consistent with the idea that the most productive individuals as civilians would be less productive in the military, \( k < 1 \). Even if \( k \geq 1 \), it may be too costly to have those with the highest civilian productivity in the military.

More importantly, my assumptions are not critical to the results herein.\(^8\) The potential gain from a draft is it will enlist people who would not voluntarily enlist. Absent the cases listed below, I find a draft will not be able to increase welfare when the volunteer military is high quality, designed to get, if possible, only those with \( q \geq q^* \). This would be true regardless of the assumptions of the how military output depends on \( q \). I will show that a draft can improve

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\(^6\) I could assume “unproductive” individuals have some positive value in the military that is less than \( k \). To simplify, I assume this value is zero. Individuals with \( q \geq q^* \) have productivity of \( k \) each in the military provided there are not more than \( m \) individuals in total enlisted. With \( m \) productive individuals in the military, additional individuals, regardless of \( q \), have no value.

\(^7\) Ross (1990) also assumes a fraction of potential military personnel would be unproductive if enlisted.

\(^8\) See Section 8 for a comparison of my results to those in Birchenall and Koch (2012(a)) who use a Roy model of self-selection, and get results similar to what I find in my model with costless and precise testing.
welfare only if 1) the volunteer military is low quality—takes those with the lowest civilian output, regardless of their military output; 2) testing is very costly; 3) testing is relatively imprecise; or 4) there is deadweight loss from taxation required to finance the military.

In Section 4, I consider a costless and precise test except for the one possible error in testing discussed previously: those who do not want to serve in the military may fail the test and appear to be unproductive. Such behavior occurs only with a draft and testing. Let $f$ equal the probability an individual who would otherwise pass the test may purposely fail. If the “test” is simply one’s score on the Armed Forces Qualifications Test (AFQT), then $f$ should equal one since anyone can fail if he so desires. However, the military can use as a test some combination of educational attainment and the AFQT score. In this case, even an intentionally low score on the AFQT may not preclude the military recognizing whether one would be productive if enlisted. Thus, I allow for the possibility $f < 1$.\(^9\)

3. A random draft vs. a low quality volunteer military

Although my main focus is on a military when testing occurs, consider when a random draft would dominate a low-quality volunteer military (LQVM) when no testing occurs. In order to allow the highest quality with a draft, it is assumed no volunteers are allowed when a draft is used.\(^{10}\) In a draft (with no testing), $m$ individuals are called at random, so the expected number of productive individuals in the military is $m(1-q^*)$. With $q$ uniformly distributed between zero and one, the individuals drafted have an expected civilian output of $\frac{1}{2}$. Thus, welfare with a random draft, $\Omega_{RD}$, is:

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\(^9\) I will not consider the possibility one who is unproductive might pass the test, $p$. If only the AFQT were used, then, as argued in the text, $f$ should equal one. In that case, one might expect $p > 0$. However, the more other measures of ability (such as education) are used, the less likely it is errors will be made, so both $f$ and $p$ should be reduced. If $f$ is close to one, the draft cannot dominate a volunteer military because more able individuals will all intentionally fail the test if drafted and paid less than their opportunity cost. Thus, I implicitly assume $f$ is non-trivially lower than one, and assume $p$ is low enough it can be ignored.

\(^{10}\) Beginning in 1918, no volunteers were allowed in World War One (Chambers, 1987).
\[ \Omega_{RD} = m[(1-q*)k - \frac{1}{2}] . \]  

(1)

For a random draft to be preferred to no military, \((1-q*)k - \frac{1}{2} > 0\), or

\[ k > \frac{1}{2(1-q*)} . \]  

(2)

The RHS of ineq.(2) is minimized and equals \(\frac{1}{2}\) when \(q^* = 0\), so, for there to be any case for a random draft, we must have \(k > \frac{1}{2}\). In the rest of this paper, it is assumed ineq.(2) holds.

If no testing is used, a volunteer military will set a wage equal to \(m\) and attract the \(m\) individuals with the lowest quality. If \(m \leq q^*\), no individual would be productive in the military with a volunteer system, and the draft would dominate a LQVM. If \(m > q^*\), then \(m-q^*\) individuals will be productive in the military with a LQVM, and, with the average opportunity cost of the individuals enlisted equal to \(m/2\), welfare would be:

\[ \Omega_{LQVM} = (m-q^*)k - \frac{m^2}{2} . \]  

(3)

Using eqs.(1) and (3), a random draft is socially preferable to a LQVM if:

\[ kq^* > \frac{m}{2} . \]  

(4)

Assuming ineq.(2) holds, with \(m < 1\), if fewer than 50% of the pool of potential military manpower would be productive if enlisted, \(q^* > \frac{1}{2}\), then a draft is socially preferable to a LQVM.
since $k > 1$ to satisfy ineq.(2) if $q^* > \frac{1}{2}$. It is not clear whether $q^* \geq \frac{1}{2}$ because of the uncertain size of the actual military labor pool. Some evidence may help illuminate the issue.

Recent testimony by the Director for Accession Policy for the Department of Defense (Gilroy, 2009) shows, for fiscal year 2009, almost 85% of the 31 million individuals in the U.S. who were ages 17-24 were unfit for military service. However, this still left almost 5 million individuals, with the number of new enlistees desired less than 200,000 per year. For 2008, for all services, 92% of new recruits were high school graduates, and 69% scored in the top half of the Armed Forces Qualification Test. Also, the population aged 17-24 is expected to grow from 31 million to 35 million by 2025.

Thus, if the entire population of individuals in the relevant age group is the potential military labor pool, then $q^* > \frac{1}{2}$. However, if the pool is restricted to those who would not be deferred, then a majority may be productive in the military, so $q^* < \frac{1}{2}$. Also, it appears there are more productive individuals than the military desires, so $m \leq 1-q^*$. I will consider the case when $m > 1-q^*$ in the Appendix.

From ineq.(4), a random draft is more likely to be preferred to a LQVM the larger are $k$ and $q^*$ and the smaller is $m$. With a random draft, the number of productive individuals in the military is $m(1-q^*)$. Assuming $m > q^*$, then $m-q^*$ individuals are productive with a LQVM. More individuals are productive with a random draft than with a LQVM if $m < 1$. Therefore, an increase in the value of a productive individual in the military, $k$, increases welfare with a random draft more than it does with a LQVM. Note, a wartime situation may suggest an increase in $k$. If that is true, a random draft is more likely to be socially preferable to a LQVM in wartime.

An increase in $q^*$ reduces the number of productive individuals in the military, but does so one for one with a LQVM and only by $1-m$ for a unit increase in $q^*$ with a random draft.
Thus, a random draft is more likely to be socially preferable to a LQVM as $q^*$ increases. A decrease in $m$ decreases the number of productive individuals one for one with a LQVM, but only decreases the number of productive individuals with a random draft by $1-q^*$, so a smaller $m$ implies a random draft is more likely to be preferred to a LQVM.

The result a larger military implies a LQVM is more likely to be preferred to a random draft is the opposite of what has been found with homogeneous military quality (Johnson, 1990, Lee and McKenzie, 1992, Ross, 1994, Warner and Negrusa, 2005, and Perri, 2010). However, the previous studies all compared the reduced deadweight loss from taxation with a draft (due to a lower military wage) to other costs of a draft, with the former more important the larger the military.

Since a random draft results in more individuals who are productive in the military, a LQVM can only dominate a random draft because the former involves a lower opportunity cost. For example, suppose $k = 1$, $q^* = .2$, and $m = .5$. Now military output equals the number of individuals in the military who are productive (since $k = 1$), and this number is $m(1-q^*) = .4$ with a random draft and $m- q^* = .3$ with a LQVM. The opportunity cost of the military equals .25 with a random draft ($m/2$) and .125 with a LQVM ($m^2/2$). Thus, welfare is higher with the LQVM than with a random draft---.175 versus .15. Although a random draft means more military output than with a LQVM, it does not necessarily mean higher welfare.

4. Testing: a high quality volunteer military is possible

Consider the possibility of testing individuals to see if they will be productive in the military. Ross (1990) assumes quality is observable at induction centers via tests. Berck and Lipow (2011) argue quality is unobservable prior to enlistment. I assume the “test” is some
combination of one’s educational record and one’s score on an entrance exam such as the AFQT. Assume (for now) the test is costless and is accurate except for the possibility an individual who tries to fail is able to do so with a probability of $f$, with $f$ independent of $q$. Thus, with a volunteer military or a draft, the military can costlessly call individuals, test them, and see who passes. Assume there is a sufficient number of productive individuals to satisfy the military’s demand, or $m \leq 1-q^*$. The possibility $m > 1-q^*$ is considered in the Appendix.

We will compare a high-quality volunteer military (HQVM) to a random draft and to a draft with testing. First, I determine whether a HQVM dominates a LQVM. Welfare with a LQVM is given by eq. (3). With a HQVM, the wage is set to just attract the $m$ productive individuals who have the lowest opportunity cost. Thus the wage will equal $q^*+m$, and only those who are tested and found to be productive are enlisted---those with $q \in [q^*, q^*+m]$. The average opportunity cost of those enlisted is then $q^*+\frac{m}{2}$. Welfare with a HQVM is:

$$ \Omega_{HQVM} = m \left( k - q^* - \frac{m}{2} \right), $$

(5)

Using eqs. (3) and (5), a HQVM is preferred to a LQVM if $k > m$. A HQVM dominates a LQVM if the marginal (and average) value of a productive individual in the military exceeds the opportunity cost of the most able (in civilian output) individual enlisted with a LQVM. In order to meaningfully compare a HQVM with a draft, it is assumed $k > m$.  

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11 If $k$ is small enough, welfare will be higher if the military sets a lower wage and attracts fewer than $m$ productive individuals. However, the choice of a welfare-maximizing wage does not change the essential results, as shown in the Appendix.

12 From the condition for a random draft to be preferred to military (ineq.(2)), $k > \frac{1}{3}$. Thus, if the military requires less than $\frac{1}{3}$ of the eligible population, $k > m$. 

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First, I consider a draft with testing. With \( w_d \) the wage with a draft, individuals who are called and have \( q > w_d \) will try to fail the test. The number who pass the test who will be productive in the military is then:

\[
\max(0, w_d - q^*) + [1-f][1- \max(w_d, q^*)].
\]  

(6)

If \( w_d > q^* \), there are \( w_d - q^* \) individuals who will be productive in the military and who will not try to fail the test. If \( w_d \leq q^* \), all those who would be productive in the military will try to fail the test. Thus, if the military places even a trivial on keeping its payroll cost as low as possible, it should either set \( w_d \) equal to zero or set \( w_d > q^* \) so \( m \) individuals pass the test. If \( m \leq (1-f)(1-q^*) \), there are enough productive individuals who pass the test if \( w_d = 0 \). For now consider the case when \( m > (1-f)(1-q^*) \). Assuming \( w_d > q^* \), use eq.(6), to get \( m \) productive individuals in the military. Solving for \( w_d \):

\[
w_d = 1 - \frac{1-m-q^*}{f}.
\]  

(7)

Note \( \lim_{f \to 1} w_d = m+q^* \)--the wage with a HQVM---and \( \frac{\partial w_d}{\partial f} > 0 \). If everyone who wishes to fail the test can do so, then \( f = 1 \). As \( f \) is reduced from one, \( w_d \) is reduced because the draft brings in more individuals who would prefer not to serve \( (q > w_d) \) and who are productive. Note that a lower \( f \) reduces welfare since higher opportunity cost individuals who are productive serve in the military instead of lower opportunity cost individuals (the latter some of those with \( q^* \leq q \leq w_d \)).
Given \( m \leq 1-q^* \), with either a draft and testing or a HQVM, all \( m \) individuals who are enlisted are productive. With a HQVM, the opportunity cost of enlistees is \( m(q^* + \frac{m}{2}) \). With a draft and testing, there are \( w_d - q^* \) individuals who willingly are drafted and who have an average opportunity cost of \( \frac{q^* + w_d}{2} \), and there are \((1-f)(1-w_d)\) individuals who try to fail the test but do not, and who have an average opportunity cost of \( \frac{1+w_d}{2} \), so, using eq.(7) and simplifying, the opportunity cost of a draft with testing is given by:

\[
OC_{draft/testing} = \frac{1}{2} \left[ 1 - (q^*)^2 + (1 - m - q^*)(\frac{1+m-q^*}{f} - 2) \right].
\]  

(8)

It is easy to see \( \lim_{f \to 1} OC_{draft/testing} = m(q^* + \frac{m}{2}) \)--the opportunity cost with a HQVM--and \( \frac{\partial OC_{draft/testing}}{\partial f} < 0 \). If all who want to fail the test do so \( f = 1 \), the draft and a HQVM pay the same wage and enlist exactly the same individuals, those with \( q \in [q^*, q^*+m] \). If \( f < 1 \), some higher opportunity cost individuals replace lower opportunity cost individuals with a draft, so the draft implies lower welfare than with a HQVM.

Now suppose \( m \leq (1-f)(1-q^*) \). Thus, a sufficient number of productive individuals who will not fail the test exist so, with a draft and testing, the military can set \( w_d = 0 \) and induct at random \( m \) of those who pass the test (since it is assumed the military does not observe civilian productivity, \( q \)). The average opportunity cost of those inducted with a draft is \( \frac{1+q^*}{2} \), so welfare with a draft is \( m(k - \frac{1+q^*}{2}) \). Using eq.(5), welfare with a draft exceeds that with a HQVM only if \( \frac{1+q^*}{2} < q^* + \frac{m}{2} \), or if \( m > 1 - q^* \), which is not true in this case. Thus, at least for the case when
\[ m \leq 1-q^*, \text{ a draft with testing cannot increase welfare in comparison with a HQVM, and will reduce welfare if some individuals who wish to fail military testing cannot do so } (f<1). \]

With \( m \leq 1-q^* \), we have seen a HQVM is socially preferable to a draft with testing. I now consider whether a HQVM dominates a random draft when the latter involves no testing. Using eqs.(1) and (5), A HQVM dominates a random draft if:

\[ \frac{1-m}{2} > q^*(1-k). \]  \hspace{1cm} (9)

Clearly, if \( k \geq 1 \), a HQVM dominates a random draft. A random draft yields fewer productive individuals than does a draft with testing----\( m(1-q^*) \) versus \( m \)---but may involve a lower opportunity cost. Also, a HQVM is less likely to dominate a random draft the larger is \( m \).

Since \( m \leq 1-q^* \), substitute \( 1-q^* \) for \( m \) in \text{ineq.}(9), and the inequality becomes \( k > \frac{1}{2} \), which, from the previous section, must hold in order for a random draft to be preferred to no military. Therefore, at least for the case of a relatively small military, \( m \leq 1-q^* \), a HQM is socially preferable to either a random draft or a draft with testing.

As shown in the Appendix, with a large military, \( m > 1-q^* \), we also find higher welfare with a HQVM than with either a random draft or a draft with testing. Thus, absent a) costly or imprecise testing (Sections 5 and 6 herein), b) a military forced to enlist the lowest quality individuals, or c) deadweight loss from taxation (Section 7 herein), a draft cannot improve and generally reduces welfare relative to a volunteer military. The intuitive reason for this is simple. As long as a volunteer military enlists only the high quality individuals with the lowest opportunity cost, welfare cannot be increased with an alternative enlistment method.

\[ \text{---13 One advantage of the draft with testing is the lower wage may imply a lower deadweight loss from taxation, which, as noted in Section 1, is assumed away in most of this paper, but is considered in Section 7.} \]
5. Costly testing

Now consider costs of testing individuals. Suppose testing costs \( c \) per individual for the military, where \( c \) is assumed to be both a social and private cost. The interesting case is in the comparison of a HQVM to a military with a random draft (no testing). For brevity, I will consider only the case when \( m < 1-q^* \). Now welfare with a HQVM is:

\[
\Omega_{HQVM} = m \left( k - q^* - \frac{m}{2} \right) - (q^* + m)c,
\]  

(10)

since the military sets \( w_v = q^* + m \), and that is the number who apply and are tested. Using eqs. (1) and (10), a HQVM is preferred to a random draft if:

\[
\frac{1-m}{2} > q^*(1-k) + (q^* + m)c.
\]  

(11)

From ineq.(11), a HQVM is less likely to dominate a random draft the larger is \( m \). Since \( m < 1-q^* \), in order to have the least chance a HQVM dominates a random draft, substitute \( 1-q^* \) for \( m \) in ineq.(11), and we have:

\[
q^*(k - \frac{1}{2}) > c.
\]  

(12)

A lower value for \( k \) implies a HQVM is less likely to dominate a random draft. The smallest possible value for \( k \) comes from ineq.(2), which must hold if a random draft is to be preferred to no military. Using ineq.(2), let \( k \to \frac{1}{2(1-q^*)} \). If \( q^* = \frac{1}{4} \), so 75% of the population would be productive in the military, the lowest value of \( k \) is \( 2/3 \), so \( k - \frac{1}{2} = 1/6 \). Thus, from
ineq.(12), as long as the per individual cost of testing is less than $1/6$ the civilian output of the least able individual who would be productive in the military (one with $q = q^*$), a HQVM would dominate a draft.

If $q^* = ½$, the minimum value for $k = 1$, so $k - ½ = ½$. In this case, if the cost of testing is less than $½$ the civilian output of the least able individual who would be productive in the military, a HQVM dominates a random draft.

Since the analysis in the preceding two paragraphs used values of $m$ and $k$ that minimized the chance a HQVM with testing dominates a random draft, it appears that, without relatively high testing cost, it is unlikely a random draft would be preferred to a HQVM.

6. Imperfect testing

I now consider a test that is costless but imperfect. Suppose, as before, there are enough individuals in the population who would be productive in the military so $m < 1-q^*$. The test is imperfect because all those with $q \geq \bar{q}$ pass and $0 \leq \bar{q} \leq q^*$. Also, as before, assume the military must have $m$ individuals. Further, assume the military is constrained by congress to offer the lowest wage that will attract $m$ individuals who pass, knowing not all of them will be productive. Note, if we allowed for a higher wage, we would get excess supply of applicants who pass the test. Choosing them at random would imply more who are productive (have $q \geq q^*$), and also a higher opportunity cost.

With the wage, $w$, set to just attract $m$ who pass the test, $w = \bar{q} + m$. The number enlisted who are productive in the military is then $\bar{q} + m - q^*$. For this number to be positive, regardless of how small $\bar{q}$ is, that is, even if $\bar{q} = 0$ (all pass), $m > q^*$. Combined with the assumption
$m < 1-q^*$, we then must have $q^* < ½$: more than ½ of the population would be productive in the military. In the rest of this section, it is assumed $q^* < ½$.

With a volunteer military, those with civilian productivity from $\bar{q}$ to $\bar{q} + m$ are enlisted, so their average opportunity cost is $\bar{q} + \frac{m}{2}$. Welfare with a HQVM is then:

$$\Omega_{HQVM}^{imperf.test} = k(\bar{q} + m - q^*) - m(\bar{q} + \frac{m}{2}).$$  \hspace{1cm} (13)

Welfare with a random draft, $\Omega_{RD}$, was found before (eq. (1)), and equals

$$\Omega_{RD} = m[(1-q^*)k - \frac{1}{2}].$$ Thus, a HQVM is preferred to a random draft if:

$$k[\bar{q} - q^*(1-m)] > m[\bar{q} - \frac{1}{2}(1-m)]. \hspace{1cm} (14)$$

Call the left and right hand sides of ineq.(14) LHS$_{14}$ and RHS$_{14}$. Now LHS$_{14}$ is the value of military output with a HQVM minus the same thing with a random draft, and RHS$_{14}$ is the opportunity cost of a HQVM minus the opportunity cost of a random draft. There are four possibilities, depending on the value of $\bar{q}$.

**Case 1. $\bar{q} = 0.$**

This is the same as when we have a low-quality volunteer military, LQVM, and, as seen before, a LQVM dominates a random draft only if $k < \frac{m}{2\bar{q}}$. Note, as discussed in Section 3, a larger military implies it is more likely a LQVM is preferable to a random draft.

**Case 2. $0 < \bar{q} < q^*(1-m).$**
A HQVM has higher welfare than a random draft if, using ineq.(14):

$$k < \frac{m[\bar{q} - \frac{1}{2}(1-m)]}{\bar{q} - q^*(1-m)}.$$  \hspace{1cm} (15)

Note, with $q^* < \frac{1}{2}$, both the numerator and the denominator of the RHS of ineq.(15) are negative since $\bar{q} < q^*(1-m)$. Now LHS$_{14}$ and RHS$_{14}$ are both negative. There are more individuals who are productive in the military with a random draft than with a HQVM, but the latter has a lower opportunity cost. As in Case 1, only if the value of a productive individual in the military, $k$, is low enough would a HQVM have higher welfare than a random draft.

**Case 3.** $q^*(1-m) < \bar{q} < \frac{1}{2}$.

Now LHS$_{14} > 0$ and RHS$_{14} < 0$: a HQVM attracts more individuals who are productive in the military than would a random draft, and does so at a lower opportunity cost than with a random draft. Thus, for any $k \geq 0$, a HQVM yields higher welfare than with a random draft.

**Case 4.** $\frac{1-m}{2} < \bar{q} \leq q^*$.

Intuitively, the larger is $\bar{q}$, the more likely a HQVM yields higher welfare than a random draft would. Thus, with a HQVM unambiguously involving higher welfare than a random draft when $q^*(1-m) < \bar{q} < \frac{1-m}{2}$, one might think it is obvious a HQVM has higher welfare when $\bar{q} > \frac{1-m}{2}$. However, in this case, both LHS$_{14}$ and RHS$_{14}$ are positive: compared to a random draft, a HQVM attracts more individuals who are able, but does so at a higher opportunity cost. A HQVM has higher welfare than a random draft if:
where both the numerator and the denominator of the RHS of ineq. (16) are positive. The RHS of ineq. (16) is increasing in $\bar{q}$. If we let $\bar{q} = q^*$, we have a HQVM with perfect and costless testing, which was shown in Section 4 to have higher welfare than a random draft. Thus, from Cases 3 and 4, a HQVM yields higher welfare than a random draft if $q^*(1-m) < \bar{q}$.

Now we have two possibilities for a HQVM with imprecise testing to have higher welfare than a random draft. First, use the result a HQVM is preferred if $q^*(1-m) < \bar{q}$. This term can be rearranged to get:

$$m > \frac{q^* - \bar{q}}{q^*} \equiv i.$$  \hspace{1cm} (17)

Now $i$ is an index of the inaccuracy of the test. If the test is perfectly accurate, $\bar{q} = q^*$ and $i = 0$. If the test is perfectly inaccurate, $\bar{q} = 0$, and $i = 1$. The results for this section suggest the following.

**Proposition One.** A high quality volunteer military with imprecise testing results in higher welfare than a random draft if
\begin{enumerate}  
\item the inaccuracy rate of the test is less than the share of the population desired in the military, $m > i$; or  
\item if $m < i$, the value of a productive individual in the military, $k$, is low enough.  
\end{enumerate}

**Proof of Proposition One.** If $m < i$, then $\bar{q} < q^*(1-m)$, which is Case 2 from above when a HQVM attracts fewer individuals who are productive, but at a lower opportunity cost, than does a random draft. Then only if productive individuals have a value in the military that is low
enough (ineq.(15)) would a HQVM have higher welfare than a random draft. For \( m > i \), a HQVM attracts more individuals who are productive than does a draft. In that case, despite the possibility of a higher opportunity cost with a HQVM than with a random draft—-which occurs if \( \frac{1-m}{2} < \tilde{q} \)—welfare is larger with a HQVM than with a random draft. As with the case with a LQVM, with imprecise testing, the larger the military, the more likely the volunteer military is preferable to a random draft. □

7. Deferments and deadweight loss from taxation

Until now I have ignored aspects of the draft versus a volunteer military that have previously been considered. Johnson (1990), Lee and McKenzie (1992), and Ross (1994) have argued the deadweight loss (DWL) from taxation may result in a lower social cost for the draft than with a volunteer military. In response, Warner and Asch (1996) noted the reduced productivity of draftees relative to volunteers, Warner and Negrusa (2005) compared the costs of draft evasion and DWL from taxation, and Perri (2010) considered the tradeoff between the DWL of taxation and the social cost of deferments when individuals can incur costs to attain a deferred status. Although these issues are independent of the problem of the quality of military personnel and have been considered elsewhere, I now briefly consider DWL and deferments.

Suppose the test is costless and precise (and no one can fail the test who would be productive in the military, \( f = 0 \)), but all can obtain a deferment at a cost (social and private) of \( D \).\(^{14}\) The DWL from taxation = \( t \)\-[military wage payments], where \( t \) is assumed to be constant and is the DWL rate per dollar of military payroll. I also assume: 1) \( k = 1 \), so all are more valuable in

\(^{14}\) Examples of deferments are given in Perri (2010).
the military, provided the military enlists $n$ individuals and $n \leq m$; 2) $m < 1-q^*$, so the military can attract $m$ productive individuals; and 3) testing is costless.

Thus, we have the HQVM from Section 4: the volunteer wage equals $q^*+m$, and individuals with $q \in [q^*, q^*+m]$ are enlisted. With a draft and testing, assume the military sets the wage, $w_d$, to just attract the same individuals as with a HQVM (recall the assumption no one can fail the test), so there is no misallocation from the “wrong” individuals being in the military. The tradeoff here is between lower DWL with a draft and the cost of deferments. Those with $q-D > w_d$ will “purchase” a deferment.\textsuperscript{15} Thus, if the military sets $w_d = q^*+m-D$,\textsuperscript{16} those with $q > q^*+m$ will obtain deferments, those with $q < q^*$ will be discharged after testing, and those with $q \in [q^*, q^*+m]$ will be enlisted.

The reduced DWL from a draft is the DWL rate ($t$) multiplied by the lower wage ($D$) and the number enlisted ($m$). The cost of deferments is $D$ times the number who defer ($1-m-q^*$). Thus a HQVM is preferred to a draft if:

$$t < \frac{1-m-q^*}{m} \equiv \tilde{t}. \quad (18)$$

\textit{Ineq.}(18) is similar to the result in Perri (2010), who finds, in a model with homogeneous labor, a fairly large military force relative to the military labor pool would be required for the DWL from taxation to be sufficiently large so as to offset the cost of deferments. It is possible

\textsuperscript{15} Deferments act like buyouts that were available in the U.S. Civil War, except the latter involved no social costs. For analysis of the U.S. Civil War draft, see Perri (2008).

\textsuperscript{16} Paying a lower wage with a draft would result in more deferring and the military attracting fewer than $m$ productive individuals.
World War Two involved a large enough demand for military personnel so the draft might have been comparable to a volunteer military in social cost.\textsuperscript{17}

Using $\text{ineq.}(18)$, $\frac{\partial \tilde{I}}{\partial m}$ and $\frac{\partial \tilde{I}}{\partial q^*}$ are both negative. Thus, the greater the demand for military personnel ($dm > 0$), and the fewer individuals who would be productive in the military ($dq^* > 0$), the less likely is a HQVM to be preferred to a random draft. Increases in either $m$ or $q^*$ imply fewer individuals will choose to incur the cost to be deferred (since the draft wage increases in $m$ and $q^*$), and a larger $m$ also means a larger savings in the wage bill with a draft, and thus even lower DWL with a draft versus a volunteer military.

Birchenall and Koch (2012(b)) have an interesting analysis of DWL from financing the military. They note that heterogeneous civilian skills suggests, with a draft, a larger military implies fewer individuals with high earnings are left in the civilian population, requiring higher marginal tax rates and causing greater DWL. Thus, their analysis suggests the opposite conclusion from what has previously been found (and was found in this section), which is that DWL is likely to be higher with a draft compared to a volunteer military as the size of the military increases.

8. Discussion and summary

It has been argued the draft may enable the military to attract more able individuals than a volunteer military and thus increase welfare. In my theoretical model, I find this may be the case if a volunteer military simply takes the least able individuals. However, when the military tests individuals and does not take the lowest quality applicants, neither a random draft nor a draft with testing increases welfare, and both usually decrease welfare---assuming a costless and

\textsuperscript{17} As is the case herein, Perri (2010) ignores some costs of the draft relative to a volunteer military such as higher turnover and reduced effort.
precise test, and ignoring deadweight loss from taxation. When there is a sufficient number of productive individuals available to the military, and testing would be used with both a volunteer military and a draft, the same number of productive individuals are attracted with either system, but a draft involves a higher opportunity cost.

Although a random draft with no testing may result in lower opportunity cost than with a volunteer military with testing, fewer productive individuals would actually be attracted with a draft. The latter effect dominates, so a volunteer military yields higher welfare than a random draft with no testing. A random draft will attract some of those at the highest level of civilian productivity---those with the highest opportunity cost of being in the military.

When the military cannot attract the desired number of productive individuals, a draft may attract more individuals who are productive than would a volunteer military, but the higher opportunity cost of the former leads to lower welfare with a draft versus a volunteer military (see the Appendix).

With either a low quality volunteer military or a high quality volunteer military with imprecise testing, the larger the size of the military, the more likely welfare is higher with a volunteer military than with a draft. This is the opposite of what is found in models of homogeneous labor quality, when the deadweight loss from taxation is considered (Johnson, 1990, Lee and McKenzie, 1992, Ross, 1994, Warner and Asch, 1996, and Perri, 2010). When testing is costly, it is less likely a volunteer military will have higher welfare than a random draft with a larger military since the draft does not involve testing. Thus, in general, the effect of military size on the relative efficiency of the draft an a volunteer military is ambiguous.

Compare the results herein to those in Birchenall and Koch (2012 (a)). They focus on selection bias in a Roy model of self-selection. With perfect information (analogous to my model
when testing is costless and precise), they find similar welfare effects to those found herein if military and civilian productivity are positively associated. This suggests my assumption of constant military productivity does not drive our results. Birchenall and Koch do not look at welfare effects unless they are obvious, say, when military and civilian output are both higher with a volunteer military. Also, with imprecise testing, I find a draft could have either higher or lower civilian and military output than with a high quality volunteer, even with a positive association between military and civilian output.

In general, I find a random draft would dominate a volunteer military with testing (that is, a high quality military) only if testing is relatively costly or imprecise. By ignoring other factors such as higher turnover costs plus evasion costs with a draft, I have understated the likelihood a high-quality volunteer military dominates a draft.
Appendix

The case of a relatively large military (\( m > 1-q^* \)).

If \( m > 1-q^* \), there are not enough productive individuals to satisfy the military’s demand. As suggested in Section 3, this does not appear to be the case in the U.S. today. However, continued increases in the fraction of the U.S. population who are obese or otherwise unfit, accompanied by an unexpected increase in the desired size of the U.S. military, could result in such a situation. Also, for a county like Israel, this case may be relevant today. As in Section 4, assume a test is costless and precise except that \( f \) of those drafted who do not wish to be in the military intentionally fail the test.

With a draft and testing, the number of productive individuals obtained is again given by eq.(6). If \( w_d > q^* \), the average opportunity cost of those in the military for the \( w_d-q^* \) individuals who do not try to fail the test is \( \frac{q^*+w_d}{2} \), and the average opportunity cost of the \( (1-f)(1-w_d) \) individuals who try to fail but do not is \( \frac{1+w_d}{2} \). Welfare with a draft and testing, \( \Omega_{\text{draft/test}} \), is then:

\[
\Omega_{\text{draft/test}} = k[w_d - q^* + (1-f)(1-w_d)] - \frac{1}{2}[(w_d - q^*)(w_d + q^*) + (1-f)(1-w_d)(1+w_d)]. \tag{A1}
\]

In this case, the military must choose the wage,\(^{18}\) given a higher wage will yield more in the military, because those willing to serve increase one for one with a wage increase, and those who try to fail but do not decrease by \( 1-f \) for each unit increase in the wage. However, a higher wage will also increase the average opportunity cost of both types who are enlisted. The welfare-maximizing wage is determined by:

\[
\frac{\partial \Omega_{\text{draft/test}}}{\partial w_d} = f(k - w_d) = 0. \tag{A2}
\]

From eq.(A2), we can conclude \( w_d = \min(1,k) \). If \( k < 1 \), \( w_d = k \), and, if \( k > 1 \), \( w_d = 1 \), since that is the highest wage anyone would earn in the civilian sector (when \( q = 1 \)).\(^{19}\)

With a HQVM, and a wage equal to \( w_v \), the number of productive individuals attracted is \( w_v-q^* \), and these individuals have an average opportunity cost of \( \frac{q^*+w_v}{2} \). Thus, with a volunteer military, the welfare-maximizing wage is determined by maximizing \( \Omega_{\text{HQVM}} \):

\[
\Omega_{\text{HQVM}} = [(w_v - q^*) (k - \frac{w_v+q^*}{2})], \tag{A3}
\]

\[
\frac{\partial \Omega_{\text{HQVM}}}{\partial w_v} = k - w_v = 0. \tag{A4}
\]

---

\(^{18}\) As noted in the Section 4, we could have considered the optimal choice of the military wage when \( m < 1-q^* \). In this Appendix, it is shown the results are similar when the military wage is chosen to maximize welfare and when it is assumed to be set to enlist as many productive individuals as possible in the military, provided the number does not exceed \( m \).\(^{19}\)

\(^{19}\) I assume \( k > q^* \). If this is not true, then no productive individuals would be enlisted with a HQVM.
Thus, \( w_v = w_d = \min(1, k) \). With a draft and testing, if \( w_d \) is set above \( q^* \), so not all productive individuals will try to fail the test, a draft and a volunteer military set identical wages. Letting \( w_d = w_v = k \), with \( k > q^* \) (or else no productive individuals would be enlisted with a HQVM), substitute in eqs. (A1) and (A3) for \( w_d \) and \( w_v \). I find a HQVM is preferred to a draft with testing if:

\[
(1 - f)(1 + k^2) > 2k(1 - f) + 2q^*(q^* - k). \tag{A5}
\]

With \( k > q^* \), the second term in RHS(A5) is negative. Thus, if LHS(A5) at least equals the first term on RHS(A5), ineq. (A5) holds. This simplifies to \( k^2 - 2k + 1 \equiv z \geq 0 \). Now \( \frac{\partial z}{\partial k} = 2(k-1) \), and \( \frac{\partial^2 z}{\partial k^2} = 2 \). The minimum value of \( z \) occurs when \( k = 1 \) and \( z = 0 \). Thus, \( z \geq 0 \), with the strict inequality holding when \( k \neq 1 \). Clearly ineq. (A5) holds and the HQVM dominates the draft in this case.

With \( w_d = w_v = k < 1 \), a HQVM enlists \( k - q^* \) productive individuals, and (using eq. (6)) a draft with testing enlists \( k - q^* + (1 - f)(1 - k) \) individuals. However, the draft brings in higher opportunity cost individuals for whom it is not socially optimal to be enlisted. Although a military with the draft is assumed to choose the wage that maximizes welfare, this maximization occurs given that reluctant individuals who do not fail the test will be inducted. A volunteer military accounts for the opportunity cost of all individuals inducted when the wage is chosen.

If \( k \geq 1 \), \( w_d = w_v = 1 \). In this case, it is optimal to have all \( 1 - q^* \) productive individuals in the military, and a draft and a HQVM would be identical.

With a draft and testing, as discussed in Section 4, it may be optimal to set \( w_d = 0 \). Then no productive individual wishes to be in the military. Now the number of productive individuals inducted equals \( (1 - f)(1 - q^*) \), and the average opportunity cost of those attracted is \( \frac{1 + q^*}{2} \). Thus, using eq. (A3) with \( w_v = 1 \), it is easy to see the draft has the same average opportunity cost as a HQVM, but attracts fewer productive individuals, and thus is dominated by a HQVM. When \( w_v = k \), welfare with a high quality volunteer military (HQVM) is given by eq. (A3) with \( w_v = k \), and equals \( \frac{1}{2}(k - q^*)^2 \). With a draft, \( (1 - f)(1 - q^*) \) productive individuals are inducted, with an average opportunity cost of \( \frac{1 + q^*}{2} \), so welfare with a draft is \( (1 - f)(1 - q^*)(k - \frac{1 + q^*}{2}) \). A HQVM is preferred to the draft if (simplifying terms):

\[
k^2 - 2kq^* + [q^*]^2 > (1 - f)(1 - q^*)(2k - 1 - q^*). \tag{A6}
\]

If LHS(A6) \( \geq (1 - q^*)(2k - 1 - q^*) \), then LHS(A6) > RHS(A6). This occurs if \( k^2 - 2k + 1 \geq 0 \), which was shown to be true in the proof following ineq. (A5) above.

Now consider whether a HQVM is preferred to a random draft when no testing is used in the latter. Consider the case where the volunteer wage equals one \( (k \geq 1) \).\(^{20}\) Using eq. (A3), welfare\(^{20}\) When \( w_v = k \), the HQVM dominates a random draft if \( (k - q^*)^2 > m[2(1 - q^*)k - 1] \). This is least likely to hold the larger is \( m \), so let \( m = 1 \). The inequality then becomes \( k^2 + [q^*]^2 > 2k - 1 \). Since \( \frac{1}{2} < k < 1 \) in this case, let \( k = \frac{1}{2} + \varepsilon \), \( 0 < \varepsilon < \frac{1}{2} \). Then the inequality becomes \( \frac{1}{4} + [q^*]^2 > \varepsilon(1 - \varepsilon) \). The RHS of the inequality is maximized when \( \varepsilon = \frac{1}{2} \), yielding a RHS = \( \frac{1}{4} \). Thus, a
with the HQVM is then \((1 - q^*)(k \frac{1+q^*}{2})\), and welfare with a random draft is given by \(eq.(1)\). A HQVM is preferred to a random draft if:

\[
(1 - q^*)(1 - m)k > \frac{1}{2}[(1 - q^*)(1 + q^*) - m].
\]

Both sides of \(ineq.(A7)\) are linear and decreasing in \(m\). If \(m = 1\), \(LHS_{(A7)} = 0\), and \(RHS_{(A7)} = -\frac{1}{2}[q^*]^2\). Since \(m > 1 - q^*\) in this case, the smallest value of \(m\) is slightly larger than \(1-q^*\). Substituting \(1-q^*\) in \(ineq.(A6)\) for \(m\) yields \(LHS_{(A6)} = k q^*(1-q^*)\) and \(RHS_{(A6)} = \frac{1}{2} q^*(1-q^*)\). Since we must have \(k > \frac{1}{2}\), the HQVM dominates a random draft for all \(m > 1 - q^*\).

In Section 4 and in this Appendix, it is shown neither a draft with testing nor a random draft ever dominates a HQVM when testing is costless and precise. When the military is relatively small \((m < 1-q^*)\), the draft attracts the same number of productive individuals as does a HQVM, but at a higher opportunity cost. With a larger military \((m > 1-q^*)\), the draft may result in more of the productive individuals enlisted, but at an opportunity cost sufficiently high so welfare is reduced.

HQVM dominates a random draft in this case. Note, I previously assumed \(k > m\) (in order for a HQVM to dominate a LQVM), but that is irrelevant for the proof in this footnote since I simply needed to show a HQVM dominates a random draft when \(w_e = k\).
References


