Chapter 1

Introduction

The agreement on reduction of international trade barriers (GATS) including trade in the areas of intellectual property, services, capital and agriculture was achieved after eight rounds of negotiation on December 15, 1993. Later WTO (World Trade Organization) took the role of supervising the GATS agreements by reviewing the trade policies of the countries - members of the WTO. Even though WTO and other international organization support policies aimed to trade liberalization and transparency, however, there is no common agreement in the literature about the effects of the public policy on the growth rates of the countries engaged in trade and the world balanced growth rate.

There is a significant body of literature on the effects of the different public policy programs on growth in the presence of trade. Empirical studies by Dollar (1992) and Sachs and Warner (1995) demonstrate that openness to trade measured by different indicators can lead to faster growth. "Index of real exchange rate distortion" and "index of real exchange rate variability" suggested by Dollar as the measures of openness were supported by his argument that lower government intervention would result in more stable and close to the free trade equilibrium exchange rate, therefore the differences in trade regimes could be reflected in the levels of exchange rates. As his results suggest trade liberalization accompanied by exchange rate devaluation can substantially increase growth rates of developing countries.

Sachs and Warner created an alternative measure of openness given by a dummy variable that takes value of zero in the presence of average tariff rates higher than 40 percent, nontariff barriers exceeding 40 percent of the trade, depreciation of the black market exchange rate by 20 percent or more, existence of socialist system and the state monopoly in the areas of major export. As they conclude, "to some extent, opening the economy has helped to promote governmental responsibility in other areas. To that extent, trade policy should be viewed as the primary instrument of reform".

Stiglitz's (1998) argument is in the same line with conclusions by Dollar and Sachs and Warner that indicators of trade openness such as trade ratios, indices of price distortion or average tariff level are related to the growth in per-capita income.

Rodriguez and Rodrik (2000) are more pessimistic in their conclusions about the effects of policy on growth. As they argue methodological problems and measurement errors weaken the findings by Dollar (1992), Ben-David (1993), Sachs and Warner (1995), Edwards (1998), and Frankel and Romer (1999) about strong positive relationship between openness promoting policies and economic growth. According to them, the relationship between trade policies and growth is still an open question and is "far from having been settled on the empirical ground", it can be contingent and depend on external characteristics and model specifications.

In general, model specifications allow for three different channels through which trade affects growth - R&D, scale effects and technology transfer. Under each specification the effects of trade policies on growth will depend on how public policy programs affect the mechanism transmitting trade effects on growth. Theoretical work by Grossman and Helpman (1990) considers the impacts of import tariffs and export subsidy in the context of endogenous growth model with trade. They show that a small import tariff or export subsidy can increase the long-run world growth rate only if the policy is implemented by the country not specialized in the R&D. The shift of the resources towards growth enhancing R&D sector will increase world growth rate.

Standard models of learning by doing, variety expansion and quality ladder (Barro and Sala-i-Martin, 2004) generate scale effect result which has its implications for the theories of trade and growth. Rivera-Batiz and Romer (1991), for example, show that under knowledge driven specification for R&D closer integration and better communication between countries will lead to the increase in the stock of knowledge available to each country and therefore fasten growth through scale effect. Despite these theoretical predictions scale effects don't have much support on empirical grounds. In search of scale effects Backus, Kehoe and Kehoe (1992) couldn't find enough evidence for the effect of scale on growth rate of GDP per capita. Jones (1995) shows that the scale effects are not supported by the time-series predictions for industrialized countries. As he argues the share of labor devoted to R&D has increased from 0.25 percent in 1950 to 0.8 percent in 1988 without leading to much increase in TFP growth rates for the United States. According to Jones, the similar result holds for France, Germany and Japan.

The lack of the evidence for the scale effect led to the evolution of the new R&D-based growth theories that eliminate scale effects either through diminishing returns to knowledge in the R&D sector (Jones 1995, Segerstrom 1998) or through product proliferation mechanism (Dinopoulos and Thompson (1998), Peretto (1998) and Howitt (1999)).

Howitt (2000) develop[s open economy version of traditional Shumpeterian model, where using technology transfer in the form of R&D spillovers he studies cross-country income differences. His results suggest that sharing technologies allows countries that are good in R&D to grow faster compared to other countries. Connolly and Valderamma (2005) study effects of trade along the transitional path in the model of quality ladder, in which trade operates through the mechanism of technological diffusion. As they argue the welfare implications based solely on the steady state results may be misleading.

Coe and Helpman (1995) provide evidence consistent with technology transfers. In particular, they show that not only foreign R&D has positive effect on domestic productivity, but also the effect becomes stronger for more open economies, again providing argument in support of trade liberalization and transparent trade policies. The latest study by Coe, Helpman and Hoffmaister (2008) extends the original work by Coe and Helpman in two directions: first, by applying modern estimation techniques for panel data and second, by focusing on the importance of the cross-country institutional differences captured by such institutional variables as legal origin and degree of patent protection. The results of this study not only support the earlier findings by Coe and Helpman about the importance of R&D spillovers but also emphasize the importance of institutions in determining the degree of those spillovers.

Without undermining the importance of the research and development, scale effect and technological diffusion, Seater (2002) argues that theoretical arguments focusing on the pure effect of trade on growth are still missing. He develops dynamic model of trade and growth where trade affects growth through the same comparative advantage mechanism that leads to the welfare effects of trade in the static models. In the absence of the research and development, scale effect and technological diffusion, model thus allows to focus on the pure effects of trade on growth. As Seater (2002) shows trade in goods that are also factors of production can serve as a substitute of technological transfer, because in the presence of trade growth rates of countries with different technologies will be equal to the growth rates in the presence of the transfer of technological knowledge. In this paper I utilize the framework developed by Seater to study the effects of fiscal policy programs on comparative advantage mechanism and growth of countries engaged in trade. Analysis of the policy effects in the context of dynamic model of trade with the emphasis on the comparative advantage mechanism will provide additional insights on the effects of the trade liberalization on comparative advantage and growth.

The novelty of the approach used to focus on the public policy programs comes from two sources.

First, fiscal policy programs such as capital income tax or consumption tax are considered in the context of closed models with endogenous growth by many researchers, In the era of globalization and integration of more countries into the world trade system, however, it becomes important to understand the effects of domestic policy programs on the world balanced growth rate and the mechanisms through which those effects may be transmitted. Therefore incorporating domestic fiscal policies in the open economy framework of the current model aims to improve our understanding of the international effects of these policy programs.

Second, it is common to consider effects of export subsidy without considering the sources of financing ezport subsidy. Different public policy programs considered here allow to focus on fiscal policy experiments in which introduction of export subsidy is accompanied by introduction of some other tax, for example capital income tax, capturing the notion of the opportunity costs associated with introduction of export subsidy.

1.1 Model Under Autarky

There are two countries in the world economy initially closed to trade. Each country produces two different goods Y and J. Y can be used for both consumption and investment into physical capital K, whereas J is another type of capital that augments unskilled labor. Unskilled labor is normalized to 1. Each good is produced in different sector of economy. Note that interpretation of J-type capital does not necessarily limited to "human capital". Mulligan and Sala-I-Martin (1993) refer to several interpretations of the capital accumulated through the investment process in the different specifications of the two-sector endogenous growth model, but different from the physical capital: "human capital, embodied and disembodied knowledge, public capital, quality of products, number of varieties and financial capital". Seater (2002) argues that J-type capital is "anything that augments labor and it not be embodied in labor".

Production process of both goods is described by Cobb-Douglas production functions assuming constant returns to scale.

$$Y_i = A(v_i K_i)^{\alpha_i}(u_i J_i)^{1-\alpha_i} \tag{1.1}$$

$$\dot{J}_i + \delta J_i = B[(1-v_i)K_i]^{\eta_i}[(1-u_i)J_i]^{1-\eta_i} \tag{1.2}$$

where v and u are fractions of K and J types of capital respectively used in the production. Parameters A and B represent differences in total factor productivities, and parameters α and η capture differences in techniques of production. Subscript i is used to identify country under consideration.

By assumption good Y can be used for both consumption and investment into K-type capital which leads to the following accumulation condition for K:

$$\dot{K}_i + \delta K_i = A(v_i K_i)^{\alpha_i}(u_i J_i)^{1-\alpha_i} - C_i \tag{1.3}$$

Price of good Y is normalized to 1. Price level p will measure therefore relative price of good J in terms of good Y. Both countries have identical preferences described by the following utility function:

$$U = \int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \tag{1.4}$$

So each country maximizes utility function (1.4) subject to two accumulation conditions given by (1.2) and (1.3).

From solution to Hamiltonian the autarkic growth rate of country i and its relative price will be:

$$\gamma_i = \frac{1}{\theta}\left[\Delta_i - \delta - \rho\right] \tag{1.5}$$

where

$$\Delta_i = A_i^{\frac{\eta_i}{1-\alpha_i+\eta_i}} B_i^{\frac{1-\alpha_i}{1-\alpha_i+\eta_i}} \alpha_i^{\frac{\alpha_i\eta_i}{1-\alpha_i+\eta_i}} (1-\alpha_i)^{\frac{(1-\alpha_i)\eta_i}{1-\alpha_i+\eta_i}} \eta_i^{\frac{(1-\alpha_i)\eta_i}{1-\alpha_i+\eta_i}} (1-\eta_i)^{\frac{(1-\alpha_i)(1-\eta_i)}{1-\alpha_i+\eta_i}}$$

$$p_i = \frac{A_i}{B_i}\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{1-\eta_i} \times$$

$$\times \left[\left(\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\frac{\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{\frac{1-\eta_i}{1-\alpha_i+\eta_i}}\right]^{\alpha_i-\eta_i} \tag{1.6}$$

It can be shown (see Appendix) that (1.6) simplifies as follows:

$$p_i = \left[\frac{A_i \alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}}{B_i \eta_i^{\eta_i}(1-\eta_i)^{1-\eta_i}}\right]^{\frac{1}{1-\alpha_i+\eta_i}} \tag{1.7}$$

There are two things to note here. First, under assumption of identical techniques of production, implying $\alpha = \eta$, the difference between relative price levels in each country under autarky will be captured by the ratios of total factor productivities

A_i/B_i. In addition, expression in (1.7) can be utilized to rewrite (1.5) as a function of relative price level in which case the growth rate of each country under autarky will be given by:

$$\gamma_1 = \frac{1}{\theta}\left[\alpha_1^{\alpha_1} A_1 (1-\alpha_1)^{1-\alpha_1} p_1^{\alpha_1 - 1} - \delta - \rho\right] \tag{1.8}$$

$$\gamma_2 = \frac{1}{\theta}\left[\eta_2^{\eta_2} (1-\eta_2)^{1-\eta_2} B_2 p_2^{\eta_2} - \delta - \rho\right] \tag{1.9}$$

It follows from (1.8) and (1.9) that autarkic growth rate of country 1 decreases in autarkic price level in country 1 and autarkic growth rate of country 2 is increasing in autarkic price level in country 2. This result is essential in driving growth effects of trade through comparative advantage mechanism, which will be considered in the next section.

1.2 Fundamentals of Trade and Growth

As countries open to trade they have to face decisions about export and import of their production. Denote export of good Y by X_i and import of good J by $\frac{X_i}{P}$, then $X_i < 0$ will define import of good Y and $\frac{X_i}{p}$¡0 will define export of good J. In the presence of trade the modified accumulation constraints for country i will be given by:

$$\dot{K}_i + \delta K_i = A(v_i K_i)^{\alpha_i} (u_i J_i)^{1-\alpha_i} - C_i - X_i \tag{1.10}$$

$$\dot{J}_i + \delta J_i = B[(1-v_i)K_i]^{\eta_i} [(1-u_i)J_i]^{1-\eta_i} + \frac{X_i}{p} \tag{1.11}$$

With trade the present value Hamiltonian and the necessary conditions to the optimal control problem will be written as:

$$V_i = \frac{C_i^{1-\theta}}{1-\theta} e^{-\rho t} + \phi_i \left[A_i (v_i K_i)^{\alpha_i} (u_i J_i)^{1-\alpha_i} - C_i - \delta K_i - X_i\right] +$$

$$\psi_i \left[B_i((1-v_i)K_i)^{\eta_i} ((1-u_i)J_i)^{1-\eta_i} - \delta J_i + \frac{X_i}{p}\right] \tag{1.12}$$

$$\dot{\phi}_i = -\phi_i \left[\alpha_i v_i A_i \left(\frac{v_i K_i}{u_i J_i}\right)^{\alpha - 1} - \delta\right] - \psi_i B_i \eta_i (1-v_i) \left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i - 1} \tag{1.13}$$

$$\dot{\psi}_i = -\phi_i(1-\alpha_i)u_iA_i\left(\frac{v_iK_i}{u_iJ_i}\right)^{\alpha_i} - \psi_i\left[B_i(1-\eta_i)(1-u_i)\left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i} - \delta\right] \quad (1.14)$$

$$\frac{\partial V_i}{\partial C_i} = C_i^{-\theta}e^{-\rho t} - \phi_i = 0 \quad (1.15)$$

$$\frac{\partial V_i}{\partial v_i} = \phi_iA_i\alpha_iK_i\left(\frac{v_iK_i}{u_iJ_i}\right)^{\alpha_i-1} - \psi_iB_i\eta_iK_i\left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i-1} = 0 \quad (1.16)$$

$$\frac{\partial V_i}{\partial u_i} = \phi_i(1-\alpha_i)A_iJ_i\left(\frac{v_iK_i}{u_iJ_i}\right) - \psi_iB_i(1-\eta_i)J_i\left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i-1} = 0 \quad (1.17)$$

$$\frac{\partial V_i}{\partial X_i} = -\phi_i + \frac{\psi_i}{p} = 0 \quad (1.18)$$

As it follows from (1.18) there is a bang-bang control in X, meaning that country i will set the export of good Y as high as possible if the world price is lower than its autarkic price level given by the ratio of costate variables $\frac{\psi_i}{\phi_i}$. In contrary, if the world relative price is higher than its autarkic price level then export of good Y will be as small as possible. Note that the bang-bang control in X arises due to the linearity of the Hamiltonian in trade parameters.

The bang-bang nature of the control in X with the requirement of balanced trade impose the restriction on the world relative price. For countries to specialize and trade the world price should fall inside the closed interval whose boundaries are given by the autarkic price levels of each country. Without loss of generality assume that once open to trade country 1 will specialize in production of good Y and country 2 will specialize in production of good J, in which case the closed interval can be written as

$$\left[\frac{\psi_2}{\phi_2}, \frac{\psi_1}{\phi_1}\right] \quad (1.19)$$

Under assumption that techniques of production are identical across countries, meaning that $\alpha = \eta$, the balanced growth path solution for the world relative price level is determined by the ratio of total factor productivities of both countries as $\frac{A_1}{B_2}$. This result combined with the boundary condition imposed on the world relative price level (1.19) indicates the important role of the absolute advantage for the existence of world balanced growth path. When world relative price falls inside the boundary condition and equals its balanced growth path value $\frac{A_1}{B_2}$ then countries completely specialize in the production of the good for which they have not only comparative but also absolute advantage. In this context, complete specialization result will imply that each country will give up production of the good for which it has inferior technology. This guarantees that growth rates with trade for each country will be higher than their growth rates under autarky.

Seater (2002) also discusses the possibility of corner solution for the world relative price level such that it equals to the autarkic price level of either country. As he argues results arising from the corner solution of the world relative price have important interpretation for the existence of the world balanced growth and the stability of world income distribution.

Assuming that technological differences in both countries are captured by both total factor productivity parameters and the parameters measuring the techniques of production it can be shown (See Appendix) that the growth rate of each country in the presence of trade will be given by:

$$\gamma_{1,T} = \frac{1}{\theta}\left[A_1\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}p^{\alpha_1-1} - \delta - \rho\right] \tag{1.20}$$

$$\gamma_{2,T} = \frac{i}{\theta}\left[B_2\eta_2^{\eta_2}(1-\eta_2)^{1-\eta_2}p^{\eta_2} - \delta - \rho\right] \tag{1.21}$$

On the BGP both countries should grow at the same rate, however growth rates given by (1.20) and (1.21) are not equal for any arbitrary value of the world price p. Focusing on the interior solution for the price level it follows that the price level equalizing growth rates of both countries in the presence of trade will be:

$$p* = \left[\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}A_1}{\eta_2^{\eta_2}(1-\eta_2)^{1-\eta_2}B_2}\right]^{\frac{1}{1-\alpha_1+\eta_2}} \tag{1.22}$$

The price level given by (1.22) satisfy balanced trade requirement because it falls inside the closed interval given by autarkic price levels of both countries as well as balanced growth path requirement because it guarantees that countries will grow at the same rate. Substituting (1.22) into either (1.20) or (1.21) we can get the world balanced growth rate given by:

$$\gamma * = \frac{1}{\theta}\left[\Lambda_i - \delta - \rho\right] \tag{1.23}$$

where

$$\Lambda_i = \alpha_1^{\frac{\alpha_1 \eta_2}{1-\alpha_1+\eta_2}} (1-\alpha_1)^{\frac{(1-\alpha_1)\eta_2}{1-\alpha_1+\eta_2}} \eta_2^{\frac{(1-\alpha_1)\eta_2}{1-\alpha_1+\eta_2}} (1-\eta_2)^{\frac{(1-\alpha_1)(1-\eta_2)}{1-\alpha_1+\eta_2}} A_1^{\frac{\eta_2}{1-\alpha_1+\eta_2}} B_2^{\frac{1-\alpha_1}{1-\alpha_1+\eta_2}}$$

As it is noted in Arabshahi (2006) the expression for the world balanced growth in (1.23) depends only on the parameters from production of good Y in country 1 and from production of good J in country 2. This result implies that when total factor productivity parameters and techniques of production are different both between sectors and across countries complete specialization still can be achieved on the balanced growth path with country 1 specializing in production of good Y and country 2 specializing in production of good J, Note also that using (1.18) we can derive the world relative supply curve for good J, shown in Figure 1.1. If the world relative price is below autarkic price level p_2 of country 2 both countries will specialize in production of good Y. If the world relative price is equal to the autarkic price level of country 2 then country 2 will produce both goods while country 1 will continue specializing in good Y. If the world relative price falls inside the interval given by the autarkic price levels in both countries then country 2 will completely specialize in production of good J and country 1 will completely specialize in production of good Y. Finally, if the world price is above p_1 and p_2 then both countries will specialize in production of good J.

As the above analysis sugests the bang-bang control in X leads to the same "stair-step" shape relative supply curve as in Ricardian model of trade. As Feenstra (2004) argues the "stair-step" shape of the relative supply curve "reflects the linearity of the production possibility frontiers". However, as it is shown by Mulligan and Sala-i-Martin (1993) the point in time production possibility frontier (PPF) for two sector models with endogenous growth is concave under the similar setup with constant point-in-time returns and different technologies. "The stair-step" shape of the relative

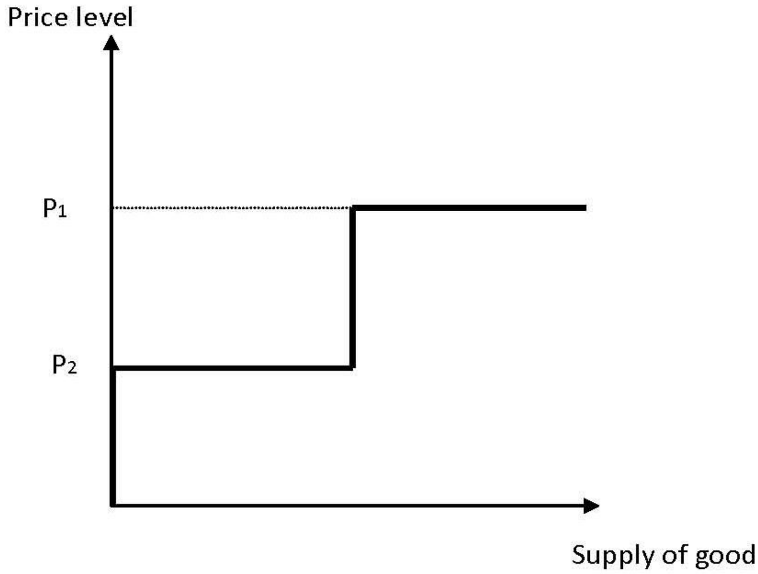

Figure 1.1: World Relative Supply of Good J

supply curve can be linked to trajectory along which economy is expanding over time.

As Feenstra (2004) analyzes the effects of the change in the factor endowment in the context of two-good, two-factor static model of trade he shows that for a given level of capital endowment the change in labor endowment along the equilibrium of the PPF will result in the expansion of the economy along the straight line known as the Rybczynski line for change in labor. If labor endowment continues increasing then under assumption of the constant relative price economy will eventually achieve complete specialization in the production of good that uses labor more intensively. To apply Feenstra's analysis to current setting, first we need to relax Feenstra's assumption of the constant relative price level, and second, we need to recognize the fact that under dynamic approach of the current model there is a continuing accumulation of both factors of production - two different types of capital used in the production of the final outputs in both sectors.

As it was discussed above the bang-bang nature of the control in X combined with the requirement of the balanced trade leads to the complete specialization result in the current model once countries open to trade in the case of interior solution of the world relative price. The fact that countries completely specialize once they open to trade in the interior case implies that before countries open to trade there may be a movement along a path similar to Rybszynski line such that accumulation of one of the factors is happening more rapidly than of the other. Note that in the presence of the unbalanced growth with one of the factors accumulating faster than the other there will be continuous nonparallel shifts in the PPF, which are going to generate a trajectory along which an economy will move to complete specialization and trade.

1.3 Effects of Public Policy

In this section I will focus on the steady-state effects of public policy programs such as import and export taxation or export subsidy, consumption and capital income taxation. The objective is to analyze the effect of these policy programs on the comparative advantage mechanism through which trade affects growth in the dynamic setup of the model.

The effects of the capital income taxation in the context of the two sector endogenous growth model are considered by many researchers in the current literature. In particular, King and Rebelo (1990), Lucas (1990) and Jones, Manuelli and Rossi (1993) considered the steady-state effects of capital income taxation using the closed economy version of the two-sector endogenous growth model. Mino (1996) focuses on the effects of capital income taxation both in and out of the steady-state equilibrium. As he argues taxation always decreases the balanced growth rate of the economy, however initial effects of taxation on the growth rate of consumption and "human" capital accumulation will depend on the differences in factor intensities across two sectors of the economy. Bond, Wang and Yip (1996) considered three different changes in public policy regime such as capital and labor taxation and education subsidy using the framework of a general two-sector model of endogenous growth. As they show if the taxes are too distortionary then public policy can lead to either indeterminacy or instability of the balanced growth path. However, assuming that saddle-path stability of the balanced growth path is achieved they argue that both capital and labor taxation reduce the balanced growth rate, whereas education subsidy increases it.

While all these studies provide very important insights on the effects of capital income taxation on the factor accumulation process and the growth rate of economy, focusing on the open economy approach can allow us to address the issues related to the impact of capital income taxation on the channels through which trade affects growth rates of the countries engaged in trade and the world balanced growth rate. Understanding these impacts becomes especially important in the presence of globalization and integration of more countries into the world trade system. Another argument for the relevance of consumption or capital income taxation program in the context of the current model is closely related to the trade policies considered here. In the trade models the effects of import tariffs and export subsidies are often considered from the point of view of Lerner symmetry theorem. The idea behind Lerner symmetry theorem is that uniform tariff applied to all imports can be neutralized if the export subsidy of the same uniform rate is applied to all the products. In particular, this result implies that free trade equilibrium can be restored in the presence of the same rate import tariff and export subsidy. However as Panagariya (1999) argues, restricting assumptions of the theorem limit applicability of Lerner symmetry in reality. One of the important underlying assumptions of the theorem is that administrations of tariff and subsidy doesn't impose any additional costs on the society. Incorporating consumption and capital income taxation into the model with trade policies allows to introduce source of financing of export subsidy in particular and show additional costs that trade promoting policy can impose on the society.

1.3.1 Model with Public Policy Programs

To introduce trade policies I will assume that import duty is implemented at the rate τ_I and export tax is implemented at the rate τ_E. Note that for consideration of export subsidy we can assume that τ_E is negative. In addition I will assume that income from K and J types of capital is taxed at the rates τ_K and τ_J respectively, and consumption tax is implemented at rate τ_C. Let T_{Ei} denote government revenue from export taxation in country i then $T_{Ei} = \tau_{Ei} X_i$. Similarly, T_{Ii}, defined as government revenue from import taxation, will be equal to $\tau_{Ii} \frac{X_i}{p}$.

Let R_{Ki} and R_{Ji} denote returns on K and J type of capital respectively. Then goverment revenue from income earned on both types of capital will be equal to $\tau_{Ki}R_{Ki}K_i$ and $\tau_{Ji}R_{Ji}J_i$. Finally, goverment revenue from consumption tax will be equal to $\tau_{Ci}C_i$.

For now I will assume that government revenue is distributed back to the households in the form of the lump-sum transfers T_0.

Combining sources of government revenue with the expenditures in the form of lump-sum transfers the government budget constraint for country i will be given as:

$$T_{0i} = \tau_{Ki}R_{Ki}K_i + \tau_{Ji}R_{Ji}J_i + T_{Ei} + T_{Ii} + \tau_{Ci}C_i \tag{1.24}$$

Note that in the case of export subsidy T_{Ei} becomes negative and represents expenditure part of the government budget. Implemented public policy programs will modify capital accumulation conditions as follows:

$$\dot{K}_i + \delta K_i = A(v_i K_i)^{\alpha_i}(u_i J_i)^{1-\alpha_i} - (1 + \tau_{Ci})C_i - X_i - T_{Ei} + T_{0i} \tag{1.25}$$

Substituting for T_{Ei} last equation will become:

$$\dot{K}_i + \delta K_i = A_i(v_i K_i)^{\alpha_i}(u_i H_i)^{1-\alpha_i} - (1 + \tau_{Ci})C_i - (1 + \tau_{Ei})X_i + T_{0i} \tag{1.26}$$

Similarly accumulation condition for J type capital will be written as:

$$\dot{J} + \delta J = B_i \left[(1 - v_i)K_i\right]^{\eta_i} \left[(1 - u_i)J_i\right]^{1-\eta_i} + \frac{(1 - \tau_{Ii})X_i}{p} \tag{1.27}$$

Note that all the necessary conditions for present value Hamiltonian will still be given by the set of equations (1.13) - (1.17), except for the equation for marginal value of export which now will be modified as:

$$\frac{\partial V}{\partial X_i} = -\phi_i(1 + \tau_{Ei}) + \frac{(1 - \tau_{Ii})\psi_i}{p} = 0 \tag{1.28}$$

The introduction of trade policies doesn't eliminate the bang-bang control in X, which implies that marginal value of X will be positive only if the world relative price is less than $\frac{\psi_i(1-\tau_{Ii})}{\phi_i(1+\tau_{Ei})}$. If this condition is satisfied then country will specialize in production of good Y and exchange it for good J. On the other hand for country i to specialize in production of J, the world relative price should be higher than $\frac{\psi_i(1+\tau_{Ei})}{\phi_i(1-\tau_{Ii})}$. The requirement of the balanced trade leads to the new condition on the world relative price. For countries to specialize and trade the world price should fall inside the closed interval given by:

$$\left[\frac{\psi_j(1+\tau_{Ej})}{\phi_j(1-\tau_{Ii})}, \frac{\psi_i(1-\tau_{Ii})}{\phi_i(1+\tau_{Ei})}\right] \tag{1.29}$$

Above condition is a modified version of (1.19). Comparison of (1.19) and (1.29) reveals that introduction of trade policies makes the interval smaller and therefore imposes tighter restrictions on the world relative price level to guarantee the existence of the balanced trade.

The costate variables ϕ_i and ψ_i represent country i's marginal values of good Y and good J respectively, with the ratio $\frac{\psi_i}{\phi_i}$ corresponding to country i's internal price of good J in terms of Y. Introduction of import and export duties will affect the internal price of country i as follows: if country produces good Y and imports good J then both import and export taxes will reduce relative price of good J in that country. Note if import tax is combined with export subsidy then internal price will be decreasing in import tax and increasing in export subsidy. On the contrary if country specializes in production of good J and exchanges it for good Y in the world market then both taxes will increase relative price of good J, whereas export subsidy will have negative effect on the internal price of good J in that country.

To satisfy balanced trade condition in the presence of fiscal policy the world relative price should fall in the interior of the closed interval (1.29). Without loss of generality once again assume that autarkic price level in country 1 represents upper bound of the closed interval (1.29) and autarkic price level in country 2 represents lower bound of (1.29) then country 1 will specialize in the production of good Y and country 2 will specialize in production of good J. Note that once countries choose their pattern of specialization based on the level of the world relative price p, each country will choose ψ_i and ϕ_i to satisfy (1.28) with equality.

Differentiation of the necessary condition for consumption will result in the expression for the growth rate of consumption in each country given by:

$$\gamma_{Ci} = -\frac{1}{\theta}\left(\frac{\dot{\phi}_i}{\phi_i} + \rho\right) \tag{1.30}$$

where growth rate of co-state variable ϕ in country 1 is:

$$\frac{\dot{\phi}_1}{\phi_1} = \delta - A_1\alpha_1\left[\frac{v_1 K_1}{u_1 J_1}\right]^{\alpha_1 - 1} \tag{1.31}$$

At this point it will be useful to introduce capital income taxation into the problem. Using notation introduced earlier R_K and R_J will represent rents on K and J type capital respectively. Given that production functions are homogeneous of degree one in K and J, the profit maximization will imply:

$$R_{Ki} = A_i\alpha_i\left[\frac{v_i K_i}{u_i J_i}\right]^{\alpha_i - 1} = pB_i\eta_i\left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i - 1} \tag{1.32}$$

where $A_i\alpha_i\left[\frac{v_i K_i}{u_i J_i}\right]^{\alpha_i - 1} = \frac{\partial Y_i}{\partial(v_i K_i)}$ and $pB_i\eta_i\left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i - 1} = \frac{\partial(\dot{J}_i + \delta J_i)}{\partial[(1-v_i)K_i]}$

$$R_{Ji} = A_i(1-\alpha_i)\left[\frac{v_i K_i}{u_i J_i}\right]^{\alpha_i} = pB_i(1-\eta_i)\left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i} \tag{1.33}$$

where $A_i(1-\alpha_i)\left[\frac{v_i K_i}{u_i J_i}\right]^{\alpha_i} = \frac{\partial Y_i}{\partial(u_i J_i)}$ and $pB_i(1-\eta_i)\left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i} = \frac{\partial(\dot{J}_i + \delta J_i)}{\partial[(1-u_i)J_i]}$.

Combining (1.30), (1.31) and (1.32) the growth rate of consumption can be written as:

$$\gamma_{Ci} = \frac{1}{\theta}(R_{Ki} - \delta - \rho) \tag{1.34}$$

However, if capital income tax is implemented then (1.34) will become:

$$\gamma_{Ci} = \frac{1}{\theta}\left[(1-\tau_{Ki})R_{Ki} - \delta - \rho\right] \tag{1.35}$$

As Arabshahi (2007) shows on the balanced growth path Y, C, K and J will grow at the same rate denoted γ. It can be shown (see Appendix) that introduction of public policy programs considered here will not affect this result, meaning that the common growth rate for country i will be written as:

$$\gamma_{i,T} = \gamma_{Ci} = \frac{1}{\theta}\left[(1-\tau_{Ki})R_{Ki} - \delta - \rho\right] \tag{1.36}$$

On the balanced growth path all variables should grow at a constant rate. On the other hand, to satisfy the requirement of the balanced trade world relative price should fall inside the closed interval shown above. To satisfy both those conditions growth rate of price should be zero on the balanced growth path, which means that the growth rates of both costate variables should be equal. This result allows to solve for the ratios of factor shares in both sectors as a fraction of the world price;

$$\frac{v_i K_i}{u_i J_i} = \frac{(1-\tau_{Ki})}{(1-\tau_{Ji})}\frac{(1+\tau_{Ei})}{(1-\tau_{Ii})}\frac{\alpha_i}{(1-\alpha_i)}p \tag{1.37}$$

$$\frac{(1-v_i)K_i}{(1-u_i)J_i} = \frac{(1-\tau_{Ki})}{(1-\tau_{Ji})}\frac{(1-\tau_{Ii})}{(1+\tau_{Ei})}\frac{\eta_i}{(1-\eta_i)}p \tag{1.38}$$

Note that capital income taxation does not affect (1.28) and therefore does not have any impact on specialization pattern determined by the autarkic price levels in each country. However, capital income taxation affects the growth rates of both costate variables ϕ_i and ψ_i and therefore enters the solution for ratios of capital shares.

Higher capital income tax reduces return on taxed capital making its accumulation less attractive. As a result the ratios of capital shares will decrease if there is an increase in tax on return from K-type capital, and will increase if there is an increase in tax on return from J-type capital.

Increase in the tax imposed on export of either good will reduce world relative supply of that good whereas higher import tax on either good will reduce import of that good, leading to the corresponding changes in the ratio of capital shares. In particular, it follows from (1.37) and (1.38) that export and import taxes will increase the ratio of factor shares in the sector producing good Y and will decrease the ratio of factor shares in the sector producing good J. Substituting the solutions for factor shares given by (1.37) and (1.38) into the expressions for R_{Ki} given by (1.32) and using that in (1.36) will lead to the solutions for the growth rate of each country in the presence of trade and policy.

$$\gamma_{1,T}^{\tau} = \frac{1}{\theta}\left[\Omega_1 - \delta - \rho\right] \tag{1.39}$$

where

$$\Omega_1 = A_1\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}(1-\tau_{K1})^{\alpha_1}(1-\tau_{J1})^{1-\alpha_1}\left[\frac{(1-\tau_{I1})}{(1+\tau_{E1})}\right]^{1-\alpha_1} p^{\alpha_1-1}$$

$$\gamma_{2,T}^{\tau} = \frac{1}{\theta}\left[B_2\eta_2^{\eta_2}(1-\eta_2)^{1-\eta_2}(1-\tau_{K2})^{\eta_2}(1-\tau_{J2})^{1-\eta_2}\left[\frac{(1-\tau_{I2})}{(1+\tau_{E2})}\right]^{\eta_2} p^{\eta_2} - \delta - \rho\right] \tag{1.40}$$

Above growth rates are not equal for any arbitrary price level inside the closed interval $\left[\frac{p_2(1+\tau_{E2})}{(1-\tau_{I2})}, \frac{p_1(1-\tau_{I1})}{(1+\tau_{E1})}\right]$ However, I will show next that on the balanced growth path both countries should grow at the same rate. The absence of international lending or borrowing in the current model implies that trade between countries is balanced at any point in time. Therefore the accumulation condition for K type capital in each country can be written as:

$$\frac{\dot{K}_1}{K} = A_1 v_1\left[\frac{v_1 K_1}{u_1 J_1}\right]^{\alpha_1-1} - (1+\tau_{C1})\frac{C_1}{K_1} - \delta - (1+\tau_{E1})\frac{X_1}{K_1} \tag{1.41}$$

$$\frac{\dot{K}_2}{K_2} = A_2 v_2\left[\frac{v_2 K_2}{u_2 J_2}\right]^{\alpha_2-1} - (1+\tau_{C2})\frac{C_2}{K_2} - \delta - (1-\tau_{I2})\frac{X_1}{K_2} \tag{1.42}$$

Given that τ_{E1} and τ_{I2} are constant then it follows from (1.41) and (1.42) that on the balanced growth path $\gamma_{K1} = \gamma_{X1} = \gamma_{K2} = \gamma^*$. Combining this outcome with the earlier result that on the balanced growth path growth rate of Y, C, K and J are equal, then γ^* will denote common world balanced growth rate. The world balanced growth rate will be achieved at the world price level, denoted p^*, that equalizes (1.39) and (1.40), and is given as:

$$p^* = \left[\frac{A_1\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}(1-\tau_{K1})^{\alpha_1}(1-\tau_{J1})^{1-\alpha_1}}{B_2\eta_2^{\eta_2}(1-\eta_2)^{1-\eta_2}(1-\tau_{K2})^{\eta_2}(1-\tau_{J2})^{1-\eta_2}}\right]^{\frac{1}{1-\alpha_1+\eta_2}} \times$$

$$\times \left[\frac{\left(\frac{(1-\tau_{I1})}{(1+\tau_{E1})}\right)^{1-\alpha_1}}{\left(\frac{(1-\tau_{I2})}{(1+\tau_{E2})}\right)^{\eta_2}} \right]^{\frac{1}{1-\alpha_1+\eta_2}} \tag{1.43}$$

Several things are worth mentioning here. First, the world relative price is decreasing in τ_{I1} and τ_{E1} and is increasing in τ_{I2} and τ_{E2}. This result has intuitive interpretation. If country 1 specializes in production of good Y then tax on export will reduce world supply of good Y making it more expensive and forcing world relative price of good J to decrease. If country 1 imposes import tax then lower demand for good J produced in country 2 will decrease world relative price of good J. Similarly, trade policies introduced by country 2 will tend to increase relative price of good J. Note also that world relative price will be increasing in export subsidy imposed by country 1 and decreasing in export subsidy imposed by country 2.

Second, world relative price level is decreasing in capital income taxes imposed by country 1 and is increasing in capital income taxes introduced by country 2.The intuition for this result is as follows: if country 1 specializes in production of good Y then higher capital income taxes will reduce production of good Y, decreasing its world relative supply and therefore reducing the relative price of good J. Similarly, capital income taxes imposed in country 2 will reduce world relative supply of good J pushing its world relative price up. Next, under assumption that capital income taxes are different only across countries but are the same for both types of capital the world relative price elasticity with respect to capital income tax will be given by:

$$\frac{\partial \ln p}{\partial \ln \tau_{K1}}|_{\tau_{K1}=\tau_{J1}} = -\frac{1}{1-\alpha_1+\eta_2}$$

$$\frac{\partial \ln p}{\partial \ln tau_{K2}}|_{\tau_{K2}=\tau_{J2}} = \frac{1}{1-\alpha_1+\eta_2}$$

Under assumption that production of good Y is more intensive in K type capital than production of good J, meaning $\alpha_1 > \eta_2$ the world relative price elasticity with respect to capital income tax will be greater than one in absolute value. This result is similar to the result obtained by Rebelo and Stockey (1995)

Finally, consumption tax doesn't have any effect on the world relative price level. The intuition for this result is that consumption tax doesn't affect marginal value of any variable and doesn't cause distortions to the economic model considered in this paper.

Substituting solution for price level into either (1.39) or (1.40) will lead to the solution for the world balanced growth rate given as:

$$\gamma^* = \frac{1}{\theta}\left[\Gamma - \delta - \rho\right] \tag{1.44}$$

where

$$\Gamma = \alpha_1^{\frac{\alpha_1\eta_2}{1-\alpha_1+\eta_2}}(1-\alpha_1)^{\frac{\eta_2(1-\alpha_1)}{1-\alpha_1+\eta_2}}\eta_2^{\frac{(1-\alpha_1)\eta_2}{1-\alpha_1+\eta_2}}(1-\eta_2)^{\frac{(1-\alpha_1)(1-\eta_2)}{1-\alpha_1+\eta_2}}A_1^{\frac{\eta_2}{1-\alpha_1+\eta_2}}B_2^{\frac{1-\alpha_1}{1-\alpha_1+\eta_2}}\times$$

$$\times\left[\frac{(1-\tau_{I1})}{(1+\tau_{E1})}\right]^{\frac{(1-\alpha_1)\eta_2}{1-\alpha_1+\eta_2}}\left[\frac{(1-\tau_{I2})}{(1+\tau_{E2})}\right]^{\frac{(1-\alpha_1)\eta_2}{1-\alpha_1+\eta_2}}\times$$

$$\times(1-\tau_{K1})^{\frac{\alpha_1\eta_2}{1-\alpha_1+\eta_2}}(1-\tau_{J1})^{\frac{(1-\alpha_1)\eta_2}{1-\alpha_1+\eta_2}}(1-\tau_{K2})^{\frac{(1-\alpha_1)\eta_2}{1-\alpha_1+\eta_2}}(1-\tau_{J2})^{\frac{(1-\alpha_1)(1-\eta_2)}{1-\alpha_1+\eta_2}}$$

World balanced growth rate is decreasing in all policy parameters except export subsidy. However, under assumption that export subsidy is accompanied by capital income tax then the overall effect on growth can be negative. Note that:

$$\frac{\partial^2\gamma^*}{\partial\tau_{Ei}\partial\tau_{Ki}} < 0$$

$$\frac{\partial^2\gamma^*}{\partial\tau_{Ei}\partial\tau_{Ji}} < 0$$

where $\tau_{Ei} < 0$ in case of export subsidy. Note, however that as consumption tax doesn't enter in the (1.44) in theory it can be used to finance the export subsidy.

Overall we can conclude that when countries are large relative to each other then the effects of the policy implemented by one of the trading partners will transpire to the second country through the world price mechanism. Introduction of trade policies affects conditions for comparative advantage, while capital income taxes came into the picture through their effect on the ratios of factor shares.

We can also compare growth rate in (1.44) with the autarkic growth rates of both countries described by (1.8) and (1.9). First, as I have already mentioned imposing trade policy in the form of the import and export taxes makes the interval given by $\left[\frac{\psi_j(1+\tau_{Ej})}{\phi_j(1-\tau_{Ii})}, \frac{\psi_i(1-\tau_{Ii})}{\phi_i(1+\tau_{Ei})}\right]$ smaller compared to (1.20). Second, to guarantee the existence of the BGP in the presence of trade and public policy the world price given by (1.43) should fall inside that smaller interval. This implies that world relative price in the presence of public policy programs will lie in between autarkic price levels for both countries. Given that autarkic growth rate in country 1 is decreasing in autarkic price level and autarkic growth rate of country 2 is increasing in its autarkic price level then for the world balanced growth rate in the presence of trade and public policy to be higher than autarkic growth rate of country 1 the following condition should hold:

$$(1-\tau_{K1})^{\alpha_1}(1-\tau_{J1})^{1-\alpha_1}\left[\frac{(1-\tau_{I1})}{(1+\tau_{E1})}\right]^{1-\alpha_1} \approx 1$$

Similarly, for the world balanced growth rate in the presence of trade and public policy to be higher than autarkic growth rate of country 2 the following condition should hold:

$$(1-\tau_{K2})^{\eta_2}(1-\tau_{J2})^{1-\eta_2}\left[\frac{(1-\tau_{I2})}{(1+\tau_{E2})}\right]^{\eta_2} \approx 1$$

Note, however, that if instead of implementing export tax countries introduce export subsidy then we can no longer use above conditions to make comparisons between the growth rates with trade and policy and autarkik growth rates of both countries.

1.3.2 Existence and Uniqueness of the Balanced Growth Path

In this section I will consider the existence and uniqness of the balanced growth path for the model specified in previous section. I will use approach developed by Bond and Trask (1997) and start by showing that there will exist three different sets of equilibrium prices p, R_K and R_J satisfying zero profit conditions and no arbitrage condition. Each set of equilibrium prices will be consistent with unique pattern of specialization on the balanced growth path. Following Bond and Trask (1997) I will define unit cost functions in each sector for country i as $\Phi_{Yi}(R_{Ki}, R_{Ji})$ and $\Phi_{Ji}(R_{Ki}, R_{Ji})$. Also, it can be shown that the no arbitrage condition for the above problem can be written as:

$$\frac{(1-\tau_{Ii})}{(1+\tau_{Ei})}(1-\tau_{Ji})\frac{R_{Ji}}{p} = (1-\tau_{Ki})R_{Ki} \tag{1.45}$$

where R_{Ki} and R_{Ji} are defined as in (1.32)and (1.33) respectively. Under assumption of perfect competition output prices in both sectors should be equal to the unit costs to satisfy zero profit requirement. If the unit costs exceed the price per unit of output then sector will shut down resulting in zero output produced in that sector. So, the zero profit conditions for both sectors in country i can be written as:

$$1 = \Phi_{Yi}(R_{Ki}, R_{Ji})$$

$$p = \Phi_{Ji}(R_{Ki}, R_{Ji})$$

Totally differentiating both zero profit conditions will result in:

$$\frac{dR_{Ji}}{R_{Ji}} = -\frac{(1-\tau_{Ki})}{(1-\tau_{Ji})}\frac{\alpha_i}{1-\alpha_i}\frac{dR_{Ki}}{R_{Ki}} \tag{1.46}$$

$$\frac{dp}{p} = (1-\tau_{Ki})\left[\frac{\eta_i-\alpha_i}{1-\alpha_i}\right]\frac{dR_{Ki}}{R_{Ki}} \tag{1.47}$$

Assuming that production of Y is more intensive in K type capital than production of J, meaning $\alpha_i > \eta_i$, it follows that both price and R_{Ji} are decreasing functions of R_{Ki} Combining this result with no arbitrage condition it follows that there should exist unique values of $R^{'}_{Ki}$, $R^{'}_{Ji}$ and $p^{'}$ such that both zero profit conditions are satisfied and no arbitrage condition holds. Therefore, for the set of unique prices $R^{'}_{Ki}$, $R^{'}_{Ji}$ and $p^{'}$ both sectors will be producing.

Now consider the case when world relative price level is higher than previously determined level $p^{'}$ consistent with the operation of both sectors. Total differentiation of the zero profit condition for sector producing good J yields the following result:

$$\frac{dR_{Ji}}{R_{Ji}} = \frac{1}{(1-\tau_{Ki})(1-\eta_i)}\frac{dp}{p} - \frac{(1-\tau_{Ki})}{(1-\tau_{Ji})}\frac{\eta_i}{(1-\eta_i)}\frac{dR_{Ki}}{R_{Ki}}$$

which implies that for given R_{Ki} R_{Ji} is increasing in p. Also, it can be shown (see Appendix) that $\frac{R_{Ji}}{p}$ is a decreasing function of R_{Ki}. Given that $p > p^{'}$ derived relationship suggest that there should be unique values of $R_{Ki} > R^{'}_{Ki}$ and $R_{Ji} > R^{'}_{Ji}$ such that zero profit condition for sector producing good J is satisfied and no arbitrage condition holds. However, sector producing good Y will no longer earn zero profit, because new values for R_{Ki} and R_{Ji} satisfying no arbitrage condition are higher than $R^{'}_{Ki}$ and $R^{'}_{Ji}$ at which this sector was earning zero profit. Therefore, with new unique set of prices $p > p^{'}$, $R_{Ki} > R^{'}_{Ki}$ and $R_{Ji} > R^{'}_{Ji}$ sector producing good Y will not be profitable and will shut down. As a result country i will specialize in production of good J only. Finally, consider the case when $p < p^{'}$. Total differentiation of the zero profit condition for sector producing good Y yields:

$$\frac{dR_{Ji}}{R_{Ji}} = -\frac{(1-\tau_{Ki})}{(1-\tau_{Ji})}\frac{\alpha_i}{(1-\alpha_i)}\frac{dR_{Ki}}{R_{Ki}}$$

For given p, $R_{Ki} - \frac{R_{Ji}}{p}$ is an increasing function of R_{Ki}, implying that there will be unique values of R_{Ki}, R_{Ji} and p satisfying no arbitrage condition. However, to show that sector J will not be profitable I need to show that $\frac{d\Phi_{Ji}}{\Phi_{Ji}} - \frac{dp}{p} < 0$, meaning that adjustment in unit cost of production in J sector responds slowly to the reduction in price. It can be shown (see Appendix) that under assumption that $\alpha_i > \eta_i$ the reduction in unit cost will lag behind the price leading to non-profitability of the sector producing good J and country's complete specialization in good Y.

The above discussion can be summarized as follows:

1. If the world price equals autarkic price level in country i then both sectors operate in country i and there exists unique price vector $(p^{'}, R^{'}_{Ki}, R^{'}_{Ji})$ satisfying both zero profit conditions and no arbitrage condition.

2. If the world price is higher than autarkic price level in country i then country

i will specialize in production of good J and there will exist unique price vector (p, R_{Ki}, R_{Ji}), such that $p > p^{'}$, $R_{Ki} > R^{'}_{Ki}$ and $R_{Ji} > R^{'}_{Ji}$, satisfying no arbitrage condition and zero profit condition in sector producing good J.

3. If the world price is lower than autarkic price level in country i then country i will specialize in production of good Y and there will exist unique price vector (p, R_{Ki}, R_{Ji}), such that $p < p^{'}$, $R_{Ki} < R^{'}_{Ki}$ and $R_{Ji} < R^{'}_{Ji}$, satisfying no arbitrage condition and zero profit condition in sector producing good Y.

So, far I have shown that there exist three price vectors, with each price vector resulting in different pattern of specialization.

Next, I will show that there exists positive non-degenerate growth satisfying both transversality conditions of the problem. From (1.35) it follows:

$$C_{ti} = C_{0i} \exp\left[\frac{1}{\theta}((1-\tau_{Ki})R_{Ki} - \delta - \rho)t\right]$$

Substituting into utility function will yield:

$$U = \frac{1}{1-\theta}\int_0^\infty \exp(-\rho t)\left[C_{0i}^{1-\theta}\exp\left[\frac{1-\theta}{\theta}((1-\tau_{Ki})R_{Ki} - \delta - \rho)\right]t - 1\right]dt$$

Above integral will converge to infinity if:

$$\rho < \frac{1-\theta}{\theta}\left[(1-\tau_{Ki})R_{Ki} - \delta - \rho\right]$$

After some simplifications above condition can be written as:

$$\rho + \delta > (1-\theta)(1-\tau_{Ki})R_{Ki} + \theta\delta \tag{1.48}$$

which will be defined as condition that guarantees the existence of the positive, non-degenerate growth. Using the fact on the balanced growth path $\gamma_K = \gamma_J = \gamma_C = \gamma^*$ above condition can be further simplified to get:

$$\rho > (1-\theta)\gamma^* \tag{1.49}$$

This last specification then can be used to prove that both transversality conditions for the problem will be satisfied. The transversality conditions are:

$$\lim_{t\to\infty} K_t\phi_t = 0$$

$$\lim_{t\to\infty} J_t\psi_t = 0$$

To satisfy tranversality condition growth rate of $K_t\phi_t$ and $J_t\psi_t$ must be negative.

$$\gamma_K + \delta - (1-\tau_{Ki})R_{Ki} < 0$$

$$\gamma_J + \delta - (1-\tau_{JI})R_{Ji} < 0$$

On the balanced growth path above conditions will be written as:

$$\gamma^* - [(1-\tau_{Ki})R_{Ki} - \delta] < 0$$

$$\gamma^* - [(1-\tau_{Ji})R_{Ji} - \delta] < 0$$

Using (1.36) we can simplify both conditions to get:

$$\gamma^* - [\theta\gamma^* + \rho] \Rightarrow (1-\theta)\gamma^* - \rho < 0$$

Note that (1.49) guarantees above condition to hold and therefore transversality conditions to be satisfied.

Finally, I need to show that on the balanced growth path there will exist constant values of $k_i = \frac{K_i}{J_i} > 0$ and $c_i = \frac{C_i}{J_i} > 0$. To simplify notation I will define ratios of the factor shares of each sector in country i as follows:

$$k_y = \frac{vK}{uJ}$$

$$k_j = \frac{(1-v)K}{(1-u)J}$$

The balanced growth values of k_y and k_j are given by (1.37) and (1.38) and the value of the relative price level is defined as in (1.43). Note that as we showed earlier here are three unique price vectors corresponding to three different specialization patterns on the balanced growth path. Based on this we will identify three values of k with each one being identified with the specific price vector.

Case1: If the world relative price vector equals to the autarkic price level in country i so that equilibrium price vector is described by $(p^{'}, R^{'}_{Ki}, R^{'}_{Ji})$ then full employment condition will be written as:

$$uk_y + (1-u)k_j = k$$

Given balanced growth path solutions for k_y and k_j the value of k will be constant and positive for any $0 < u < 1$.

Case 2: If the world relative price level is above autarkic price level in country i, implying that equilibrium prices are given by $p > p^{'}$, $R_{Ki} > R^{'}_{Ki}$, $R_{Ji} > R^{'}_{Ji}$, then only sector producing good J will be operating, so that $k = k_j$ which is constant and positive for any $0 < \alpha < 1$, $0 < |\tau| < 1$, $0 < \eta < 1$ and $A > 0, B > 0$.

Case 3: If the world relative price level is below autarkic price level in country i then the equilibrium price level will be given by unique price vector (p, R_{Ki}, R_{Ji}) such that $p < p^{'}$, $R_{Ki} < R^{'}_{Ki}$ and $R_{Ji} < R^{'}_{Ji}$ and $k = k_y$ which is constant and positive for any $0 < \alpha < 1$, $0 < |\tau| < 1$, $0 < \eta < 1$ and $A > 0, B > 0$.

It remains to show that there will be constant and positive value of $c = \frac{C}{J}$ on the balanced growth path. The budget constraint for country i can be written as:

$$C_i + (\dot{K}_i + \delta K_i) + p(\dot{J}_i + \delta J_i) = R_{Ki}K_i + R_{Ji}J_i$$

or alternatively,

$$C_i + (\frac{\dot{K}_i}{K_i} + \delta)K_i + p(\frac{\dot{J}_i}{J_i} + \delta)J = R_{Ki}K_i + R_{Ji}J_i$$

Given that on the balanced growth path $\gamma_K = \gamma_J$ and after dividing above expression by J we can rewrite it as:

$$c_i + \gamma^*(k_i + p) = R_{Ji} + R_{Ki}k_i - \delta(k_i + p)$$

Assuming that $\frac{R_{Ji}}{p} = R_{Ki}$ allows to further simplify above expression:

$$c_i + \gamma^*(k_i + p) = (k_i + p)(R_{Ki} - \delta)$$

Finally, using (1.36) and slightly simplifying the above expression it will yield:

$$\frac{c_i}{k_i + p} = \frac{(\theta - 1)\gamma^* + \tau_{Ki}(\gamma^* + \delta) + \rho}{(1 - \tau_{Ki})}$$

which is positive under assumption that $\theta > 1$ Note also that the ratio on the left hand side of the above equation is increasing in the policy parameter τ_{Ki} which is the rate of taxation of return on K type capital in country i. As goverment increases tax on return from K type capital, it becomes less attractive to accumulate K-type capital, instead output of the sector producing good Y will be used to increase consumption. As investment into K-type capital slows down and consumption increase, both effects lead to increase in the ratio $\frac{c_i}{k_i+p}$

1.4 Conclusion

In this chapter I considered steady state effects of the public policy programs in the context of the open economy model with endogenous growth. There are several aspects that make current approach different from the literature in this area.

First, model allows to focus on the effects of pure trade on growth without introducing any channels through which trade can affect growth, such as scale effect, R&D or technology transfer. In fact growth effects of trade operate through the same comparative advantage mechanism that leads to welfare effects in the static models of trade.

Second, the model focuses on the trade of goods that are also factors of production.

This is different from similar approach developed by Bond and Trask (1997), and Bond, Trask and Wang (2003) that focuses on the trade of consumption good and factor of production in the form of the physical capital in the context of endogenous growth model with sectoral decomposition of production process.

Third, in the open economy models it is common to consider the case of a small country that takes the world price as given, Current framework incorporates trade between countries that are large relative to each other, such that decisions of each country will have affect on the world relative price. So, country doesn't face exogenous world price level, but rather world relative price is endogenously determined within the process of trade between countries.

This last argument is important from the point of view of the effects of fiscal policy programs considered in the model. More specifically, the assumption that countries are large relative to each other leads to the result that the policy implemented in one of the countries will affect the world relative price and balanced growth rate of both countries.

The effect of the trade policies considered here operates through the comparative advantage mechanism by affecting boundary condition imposed on the world relative price and determined by the autarkic price levels of both countries. In particular, the presence of import or export taxes puts tighter restrictions on the level of the world price consistent with balanced trade requirement. Domestic policy such as capital income taxation affects world balanced growth rates by changing the ratio of both types of capital.

The novelty of the approach used to focus on the public policy programs comes from two sources.

First, fiscal policy programs such as capital income tax or consumption tax are considered in the context of closed models with endogenous growth by many researchers, In the era of globalization and integration of more countries into the world trade system, however, it becomes important to understand the effects of domestic policy programs on the world balanced growth rate and the mechanisms through which those effects may be transmitted. Therefore incorporating domestic fiscal policies in the open economy framework of the current model aims to improve our understanding of the international effects of these policy programs.

Second, it is common to consider effects of export subsidy without considering the sources of financing ezport subsidy. Different public policy programs considered here allow to focus on fiscal policy experiments in which introduction of export subsidy is accompanied by introduction of some other tax, for example capital income tax, capturing the notion of the opportunity costs associated with introduction of export subsidy.

Overall, analysis of the growth effects of the policy suggests that both import and export taxation and capital income taxation introduced by either country lead to the reduction of the world balanced growth rate, while export subsidy have positive effect on the balanced growth rate. If we introduce source of financing of export subsidy in the form of the capital income taxes, however, then negative growth effects of capital income taxes will dominate leading to overall reduction in the world balanced growth rate. Absence of the effects of the consumption tax on growth can be justified at least in theory to finance export subsidy and generate positive growth effects.

Chapter 2

Transitional Dynamics

2.1 Introduction

In this chapter I will consider transitional dynamics of the open economy model with endogenous growth discussed in chapter 1, where growth effects of trade operate through the same comparative advantage mechanism that generates welfare effects in the static models of trade. In the first chapter I incorporated different public policy programs such as import and export taxation, export subsidy, capital income taxation and consumption tax into the model structure that allowed analyzing steady state effects of fiscal policy. As one of the main results suggests under assumption that trading countries are large relative to each other and capable of influencing the world relative price, the effects of public policy introduced by one of the trading partners will transpire through the world price mechanism affecting world balanced growth rate.

The novelty of the approach is related to the abandoning the common assumption that once open to trade country will take world price as given and will make its decisions conditional on the given price (Bond and Trask (1997)). Endogenous nature of the world relative price will also become important factor influencing structure of the transitional dynamics of the model. In particular, transitional dynamics generated by the current model will be completely different from the dynamics of the two sector closed economy endogenous growth model, despite the fact that current model utilizes main structure of dynamic closed two sector model.

Even though the ultimate objective of my current research is to evaluate both in and out of steady state effects of different public policy programs in the context of the current model, however completely new pattern of transitional dynamics generated by the model is worth discussing on its own without complicating it yet with policy experiments. So, in this chapter I will proceed with discussion of transitional dynamics of open economy model with endogenous growth considered in chapter 1 without introducing public policy programs. Before turning to the description of the dynamics, however, I want to focus on the brief literature review related to transitional dynamics of the two sector endogenous growth model.

2.1.1 Review of Existing Literature

The transitional dynamics of the two sector endogenous growth model are discussed among others by Mulligan and Sala-i-Martin (1993), Bond, Wang and Yip (1995), Faig (1995), Mino (1996). Mulligan and Sala-i-Martin considered a general setup where model exhibits constant returns to scale at the private level but also incorporates increasing or decreasing returns at the social level. The analysis of the transitional dynamics of the model is based on the numerical simulations One of the main findings is that for the values of intertemporal elasticity of substitution greater than 1 both policy functions defined in terms of the allocation of human capital to the production of final output and consumption to capital ratio are downward sloping during the transition, implying that due to the wealth effect people choose higher consumption levels and allocate higher fraction of labor to the production of the final output relative to physical capital when the level of the physical capital is low. The important empirical implication of the transitional dynamics is that growth rate of economy doesn't respond symmetrically to the reduction of both types of capital. Loss in physical capital results in faster growth, whereas losses in human capital lead to longer period of low growth.

Bond, Wang and Yip presented transitional dynamics of general two sector endogenous growth model. The dynamic properties of the model are analyzed using the system consisting of three differential equations in price (p), consumption per human capital (c) and ratio of physical capital to human capital (k). As they show the transitional dynamics depend on factor intensity parameters. In particular under assumption that production of the output used for accumulation of physical capital is more intensive in human capital the price adjustment process is unstable, meaning that price level jumps to its steady state value and dynamic properties of the system are analyzed using two-dimensional system of differential equations in consumption per human capital and the ratio of physical capital to human capital. In this setup if the ratio of physical to human capital is below its steady state value then increase in output producing physical capital will serve as a stabilizing force leading economy to the balanced growth path.

Under more realistic assumption that production of the output used in accumulation of physical capital is more intensive in physical capital, price adjustment process is stable. In this case, however, the dimensionality of the system can be reduced by projecting the system of three differential equations in price, consumption per human capital and the ratio of physical per human capital into two-dimensional space of c and k because evolution of price is independent of the ratios of consumption to human capital and physical to human capital.

Faig develops graphical tools to analyze dynamic properties of the model with physical and human capital and then uses derived framework to analyze effects of fiscal policies and stochastic shocks. As results of the analysis suggest stochastic shocks to technology will have stronger effect on output and employment, with substantially weaker effect on wages. On the other hand the effects of transitory shocks to public expenditures are much more important than permanent changes in policy regime.

Mino focuses on the analytical framework to analyze the dynamics of the two-sector model with physical and human capital in the presence of capital income taxation. His arguments are in the same line with Bond, Wang and Yip that dynamics of the economy and effects of capital income taxation depend on assumptions regarding the relative factor intensities in both sectors of production. As he argues taxation always decreases the balanced growth rate of the economy, however initial effects of taxation on the growth rate of consumption and "human" capital accumulation will depend on the differences in factor intensities across two sectors of the economy.

Even though the contribution of these studies to the understanding of the transitional dynamics of two-sector endogenous growth model is significant, however they all focus on the closed economy setup and ignore any structural changes that can arise in open economy context. For example, Ventura (1997) argues that East Asian growth miracle can be explained by "structural transformation" when faster accumulation of capital leads to the expansion of capital-intensive sector and contraction of labor-intensive sector and not just continuing production of both goods with more capital-intensive techniques. As Mino concludes, "Since the literature on sectoral shifts has usually ignored the possibility of endogenous growth, the open-economy version may provide interesting contribution to the field".

One of the works in this area focusing on the open economy model with sectoral decomposition of economy and endogenous growth is the paper by Bond and Trask (1997).

Model developed by Bond and Trask consists of three sectors: capital goods sector, education sector and consumption goods sector, where only output produced in capital goods and consumption goods sectors is assumed to be tradable. Under assumption of small open economy that takes world relative price level as given, Bond and Trask show that when world relative price of capital goods is at the level consistent with production of both tradable goods and nontradable good then transitional dynamics will not exist for any initial level of factor endowments, because open economy can adjust the level of the ratio of physical to human capital by increasing import of the good that uses either physical or human capital more intensively. In the case when world relative price of capital goods leads to the production of only one of the tradable goods and the nontradable good then the transitional dynamics are similar to the dynamics of the closed economy case.

Even though in my paper I also focus on the open economy model with sectoral decomposition of economy and endogenous growth, however my work is different from the paper by Bond and Trask in several aspects. One of the differences is that in my model both tradable goods are factors of production. Given the assumption of irreversable investment countries cannot eliminate any disbalances in the ratio of both types of capital by sending the excess stock of a capital abroad. In the current model countries are trading flow of investment, which is a reasonable assumption for some types of physical capital such as buildings,factories, etc.

Another important difference is that, as I have already mentioned in the current model I abandon the assumption of small open economy facing world relative price as given and endogenize world relative price level.

These key differences lead to substantially richer pattern of transitional dynamics consistent under some assumptions with Ventura's argument of "structural transformation" and capable of generating different paths of transition in the presence of trade depending on the direction of the deviation of the ratios of factor endowments in both countries from their steady state levels.

2.2 Review of the Model with Trade and Growth

In this section I just want to briefly review the basic structure of my model.

The production side of the economy is described by the operation of two sectors: sector producing good Y that can be used for investment into K-type capital and consumption and sector producing output that can be used only for investment into J-type capital, defined as any type of capital augmenting labor but not embodied in labor.

When countries open to trade they need to choose the optimal flow of export and import of both goods which leads to the following accumulation conditions for each country:

$$\dot{K}_i + \delta K_i = A(v_i K_i)^{\alpha_i}(u_i J_i)^{1-\alpha_i} - C_i - X_i \tag{2.1}$$

$$\dot{J}_i + \delta J_i = B[(1-v_i)K_i]^{\eta_i}[(1-u_i)J_i]^{1-\eta_i} + \frac{X_i}{p} \tag{2.2}$$

where $X_i > 0$ represents export of good Y for country i and $X_i < 0$ represents import of good Y.

Similar logic implies in interpretation of X_i/p.

Given identical preferences for both countries the present value Hamiltonian for country i and necessary conditions will be written as:

$$V_i = \frac{C_i^{1-\theta}}{1-\theta}e^{-\rho t} + \phi_i \left[A_i(v_iK_i)^{\alpha_i}(u_iJ_i)^{1-\alpha_i} - C_i - \delta K_i - X_i\right] +$$

$$\psi_i \left[B_i((1-v_i)K_i)^{\eta_i}((1-u_i)J_i)^{1-\eta_i} - \delta J_i + \frac{X_i}{p}\right] \tag{2.3}$$

$$\dot{\phi}_i = -\phi_i \left[\alpha_i v_i A_i \left(\frac{v_iK_i}{u_iJ_i}\right)^{\alpha-1} - \delta\right] - \psi_i B_i \eta_i (1-v_i) \left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i - 1} \tag{2.4}$$

$$\dot{\psi}_i = -\phi_i(1-\alpha_i)u_iA_i\left(\frac{v_iK_i}{u_iJ_i}\right)^{\alpha_i} - \psi_i\left[B_i(1-\eta_i)(1-u_i)\left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i} - \delta\right] \tag{2.5}$$

$$\frac{\partial V_i}{\partial C_i} = C_i^{-\theta}e^{-\rho t} - \phi_i = 0 \tag{2.6}$$

$$\frac{\partial V_i}{\partial v_i} = \phi_i A_i \alpha_i K_i \left(\frac{v_iK_i}{u_iJ_i}\right)^{\alpha_i - 1} - \psi_i B_i \eta_i K_i \left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i - 1} = 0 \tag{2.7}$$

$$\frac{\partial V_i}{\partial u_i} = \phi_i(1-\alpha_i)A_iJ_i\left(\frac{v_iK_i}{u_iJ_i}\right) - \psi_i B_i(1-\eta_i)J_i\left[\frac{(1-v_i)K_i}{(1-u_i)J_i}\right]^{\eta_i - 1} = 0 \tag{2.8}$$

$$\frac{\partial V_i}{\partial X_i} = -\phi_i + \frac{\psi_i}{p} = 0 \tag{2.9}$$

The necessary condition for choice of X doesn't depend on its own value, implying the bang-bang nature of control in X. The interpretation of this result is that if the world relative price level is lower than the ratio of costate variables which represents autarkic price level in country i then marginal value of export of good Y will be positive and country should set the export of good Y as high as possible.

Similarly, if the world relative price level is higher than autarkic price level in country i then marginal value of export of good Y will be negative and country should set it as low as possible. The requirement of the balanced trade suggests that for countries to trade the marginal value of export of good Y should be positive for one country and negative for the other country in which case country with positive marginal value of export of good Y will specialize in production of good Y and exchange it for good J produced in the other country. So, the requirement of the balanced trade imposes restriction on the world price, such that for countries to specialize and trade the world relative price level should fall inside the closed interval given by the autarkic price levels in both countries. This result represents the idea of comparative advantage.

If without the loss of generality we will assume that the closed interval is written as $[p_2, p_1]$ then it means that country 2 will specialize in the production of good J and country 1 will specialize in production of good Y following the pattern of comparative advantage.

The bang-bang nature of control in X followed by the complete specialization result is the outcome of the linearity of the Hamiltonian in trade parameters. As I have already argued in Chapter 1 the complete specialization results will hold not only along the balanced growth path (BGP) but also along the transitional path in the neighborhood of the BGP. In particular I have shown in chapter 1 there are three possible set of prices consistent with the different patterns of specialization. If price vector is given by $[p', R'_{Ki}, R'_{Ji}]$ consistent with the autarkic price level in country i then country i will be operating both sectors, because this price vector will satisfy zero profit conditions in both sectors and no arbitrage condition.

If price vector is given by $[p > p', R_{Ki} > R'_{Ki}, R_{Ji} > R'_{Ji}]$ then sector producing good Y will be non-profitable in country i and country i will operate only sector producing good J.

Finally, if price vector is given by $[p < p', R_{Ki} < R'_{Ki}, R_{Ji} < R'_{Ji}]$ then country i will operate only sector producing good Y and shut down sector producing good J because it will be non-profitable.

The existence of the three possible set of prices with corresponding patterns of specializations suggest that if prices deviate from the level p', R'_{Ki} and R'_{Ji} then countries will specialize in the production of the good for which it has comparative advantage, implying that complete specialization results will hold along the transitional path in the neighborhood of the BGP.

2.3 Transitional Dynamics

In considering transitional dynamics of the model it is important to emphasize two things. First, as I have argued above linearity of the Hamiltonian in X leads to the complete specialization result not only on the BGP but also along the transition to the BGP in the presence of trade. Second, I will focus on the transitional dynamics around BGP with trade when country 1 specializes in the production of good Y and country 2 specializes in the production of good J.

On the BGP the solution for factor ratios in each country then will be given by:

$$k_1^* = \frac{\alpha_1}{1-\alpha_1} p^* \tag{2.10}$$

$$k_2^* = \frac{\eta_2}{1-\eta_2} p^* \tag{2.11}$$

where $k_1 = \frac{K_1}{J_1}$ is the ratio of the factor shares in country 1, $k_2 = \frac{K_2}{J_2}$ is the ratio of factor shares in country 2 and p^* is the value of the world relative price on the BGP and is equal to:

$$p* = \left[\frac{A_1 \alpha_1^{\alpha_1} (1-\alpha_1)^{1-\alpha_1}}{B_2 \eta_2^{\eta_2} (1-\eta_2)^{1-\eta_2}} \right]^{\frac{1}{1-\alpha_1+\eta_2}}$$

There are four possible cases of transitional dynamics depending on the deviations of the factor shares from their steady state values in each country.

2.3.1 Transitional Dynamics: Case 1

In the first case I am assuming that $k_1 < k_1^*$ and $k_2 > k_2^*$. So, country 1 specializes in production of good Y but it deviated from the BGP value of the ratio of capital shares by having more of the J-type capital and less of the K-type capital. On contrary, country 2 has more of the K-type capital and less of the J-type capital compared to the ratio of the capital ratios on the BGP.

As country 1 has more of the J-type capital than its BGP value, it sets investment into J-type capital equal to 0. Note that I am considering transitional dynamics in the neighborhood of the BGP where country 1 specializes in production of good Y and imports good J from country 2. As country 1 sets investment into J-type capital equal to 0, it means that good J will no longer be imported from country 2 along the transition to the BGP.

This conclusion follows from the following equation:

$$\dot{J}_1 + \delta J_1 = \frac{X_1}{p} = 0$$

$$\Rightarrow X_1 = 0$$

$$\Rightarrow \frac{\dot{J}}{J} = -\delta$$

Intuitevely, this result implies that as Country 1 has too much of good J its price in country 1 will be very low and country 1 will no longer has comparative advantage in production of good Y and therefore will not have incentive to trade good Y in exchange of good J.

In particular, the relative price of good J in country 1 will be determined as the ratio of marginal products of both types of capital $\frac{MP_J}{MP_K}$. From the production function of good Y I can calculate the relative price of good J in country 1 as follows:

$$Y = A_1 K_1^{\alpha_1} J_1^{1-\alpha_1}$$

$$MP_J = \frac{\partial Y}{\partial J} = A_1(1-\alpha_1)k_1^{\alpha_1}$$

$$MP_K = \frac{\partial Y}{\partial K} = A_1\alpha_1 k_1^{\alpha_1 - 1}$$

$$p_1 = \frac{MP_J}{MP_K} = \left[\frac{1-\alpha_1}{\alpha_1}\right] k_1 \tag{2.12}$$

It follows from (2.12) that the dynamics of the price along the transitional path will be determined by accumulation conditions for K and J capital. As country 1 sets investment into J type capital equal to 0, it means that J-type capital will be left to depreciate along the transition to the BGP. The accumulation condition for K-type capital will then be written as:

$$\dot{K}_1 + \delta K_1 = A_1 K_1^{\alpha_1} J_1^{1-\alpha_1} - C_1$$

Country 1 will be maximizing utiliy given by (1.4) subject to the above accumulation condition for K type capital. Therefore the present value Hamiltonian and the necessary conditions for the problem can be written as:

$$H = \frac{C_1^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \mu \left(A_1 K_1^{\alpha_1} J_1^{1-\alpha_1} - C_1 - \delta K_1 \right) \tag{2.13}$$

$$\frac{\partial H}{\partial C_1} = C_1^{-\theta} e^{-\rho t} - \mu = 0 \tag{2.14}$$

$$\dot{\mu} = -\mu \left(A_1 \alpha_1 k_1^{\alpha_1 - 1} - \delta \right) \tag{2.15}$$

$$lim_{t \to \infty} K_{1t} \mu_t = 0 \tag{2.16}$$

Combining (2.14) and (2.15) I can write growth rate of consumption in country 1 as:

$$\frac{\dot{C}_1}{C_1} = \frac{1}{\theta} \left(A_1 \alpha_1 k_1^{\alpha_1 - 1} - \delta - \rho \right) \tag{2.17}$$

Dynamics in this case can be written in terms of the variables $c_1 = \frac{C_1}{K_1}$ and $k_1 = \frac{K_1}{J_1}$ and expressed in terms of the following two dimensional system:

$$\begin{bmatrix} \frac{\dot{c_1}}{c_1} \\ \frac{\dot{k_1}}{k_1} \end{bmatrix} = \begin{bmatrix} 1 & \frac{(\alpha_1 - 1)(\theta - \alpha_1) A_1 k_1^{\alpha_1 - 2}}{\theta} \\ -1 & A_1 (\alpha_1 - 1) k_1^{\alpha_1 - 2} \end{bmatrix} \begin{bmatrix} c_1 - c_1^* \\ k_1 - k_1^* \end{bmatrix}$$

Note that the above dynamic system looks exactly like dynamic system of the one-sector model (see Barro and Sala-ia-Martin, Chapter 5, 2004). Transitional dynamics for country 1 are shown on Figure 2.1.

Now, consider Country 2. On the BGP country 2 was specializing in production of good J and was importing good Y from country 1. The import of good Y was used for consumption and investment into K-type capital. Along the transitional path corresponding to the case 1 considered here country 1 no longer has incentive to trade, therefore country 2 has to open sector producing good Y to provide itself with

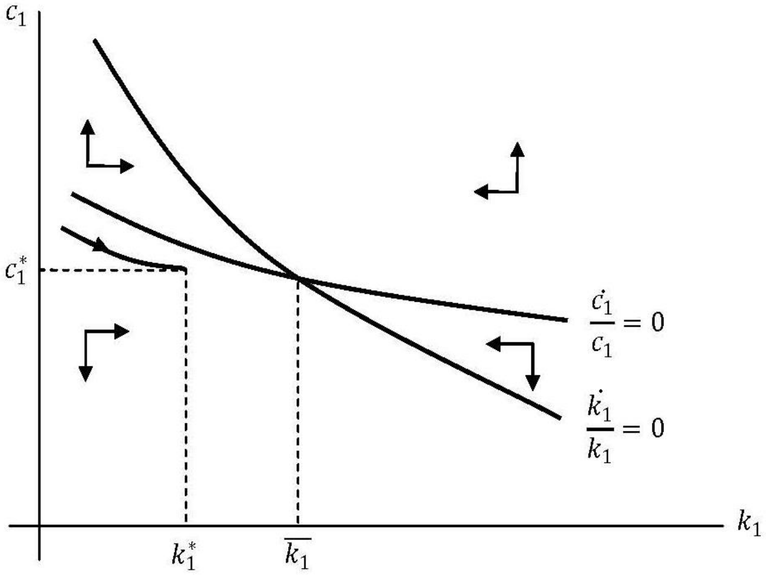

Figure 2.1: Case 1: Transitonal Dynamics for Country 1

consumption. Contry will act as closed economy two sector model until price level in country 1 will reach the level consistent with the BGP in the presence of trade. At that price level both countries will have incentive to trade, so country 2 will set $v = 0$ and $u = 0$ and specialize in production of good J to exchange it for good Y from country 1. The price dynamics are shown on Figure 2.2. It followed from the necessary condition for X that for countries to trade the world relative price should fall inside the closed interval given by the autarkic price levels in both countries $[p_2, p_1]$. It follows from (2.12) that under the conditions of case 1 relative price of good J in country 1 will be lower than the world relative price level in the presence of trade. However, as country 1 will be accumulating K-type capital and depreciating J-type capital along the transitional path the relative price level of good J will be increasing until it reaches the world relative price level consistent with the presence of trade along the BGP. As soon as price in country 1 reaches level p^* both countries will open to trade.

2.3.2 Transitional Dynamics: Case 2

In the second case I am assuming that $k_1 < k_1^*$ and $k_2 < k_2^*$.

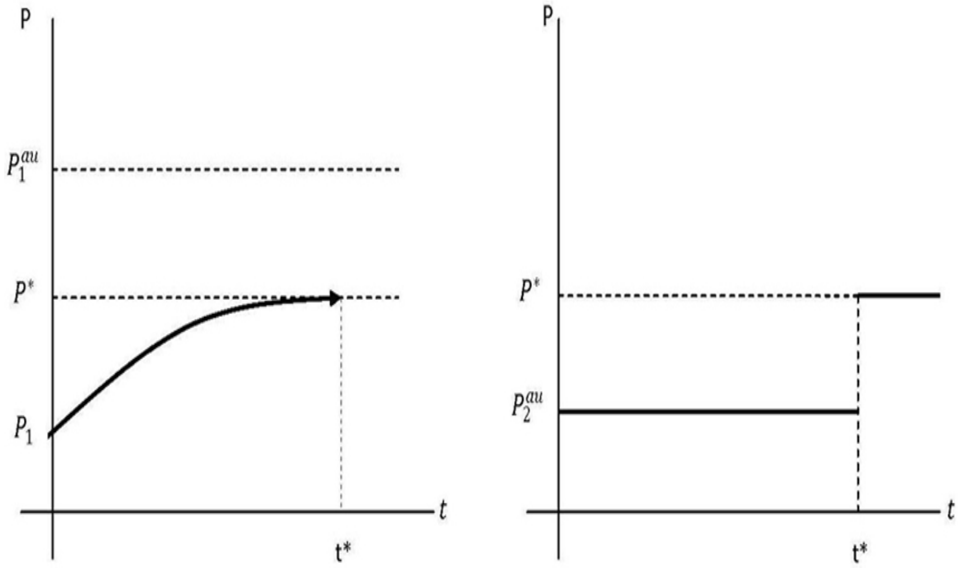

Figure 2.2: Case 1: Price Adjustment Process

Note that for country 1 condition has not changed: country 1 still is assumed to have more of J-type capital than K-type capital. Therefore solution for country 1 will be exactly the same as in case 1. However, for country 2 condition is different than in case 1. Now, country 2 has more J-type capital than K. So, it no longer has incentive to accumulate J. Therefore, country 2 sets investment into J-type capital equal to 0 and accumulates only K-type capital. The present value Hamiltonian and the necessary conditions for country 2 will be given by the following set of the equations:

$$H = \frac{C_2^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \mu \left(A_2 K_2^{\alpha_2} J_2^{1-\alpha_2} - C_2 - \delta K_2 \right) \tag{2.18}$$

$$\frac{\partial H}{\partial C_2} = C_2^{-\theta} e^{-\rho t} - \mu = 0 \tag{2.19}$$

$$\dot{\mu} = -\mu \left(A_2 \alpha_2 k_1^{\alpha_2 - 1} - \delta \right) \tag{2.20}$$

$$lim_{t\to\infty} K_{2t} \mu_t = 0 \tag{2.21}$$

Using (2.19) and (2.20) I can solve for the growth rate of consumption in country 2, which will be given as:

$$\frac{\dot{C}_2}{C_2} = \frac{1}{\theta}\left(A_2\alpha_2 k_1^{\alpha_2-1} - \delta - \rho\right) \tag{2.22}$$

As in the previous case for country 1, dynamic system for country 2 can be written in terms of the variables $c_2 = \frac{C_2}{K_2}$ and $k_2 = \frac{K_2}{J_2}$.

The two-dimensional system below characterizes dynamics in country 2 under conditions of case 2.

$$\begin{bmatrix} \frac{\dot{c}_2}{c_2} \\ \frac{\dot{k}_2}{k_2} \end{bmatrix} = \begin{bmatrix} 1 & \frac{(\alpha_2-1)(\theta-\alpha_2)A_2 k_2^{\alpha_2-2}}{\theta} \\ -1 & A_2(\alpha_2-1)k_2^{\alpha_2-2} \end{bmatrix} \begin{bmatrix} c_2 - c_2^* \\ k_2 - k_2^* \end{bmatrix}$$

Again, we can see that dynamic system for country 2 looks exactly like dynamic system for country 1 in the previous case and is consistent with the dynamics of one-sector model. Figure 2.3 describes dynamic adjustment process in country 2 under conditions of the second case.

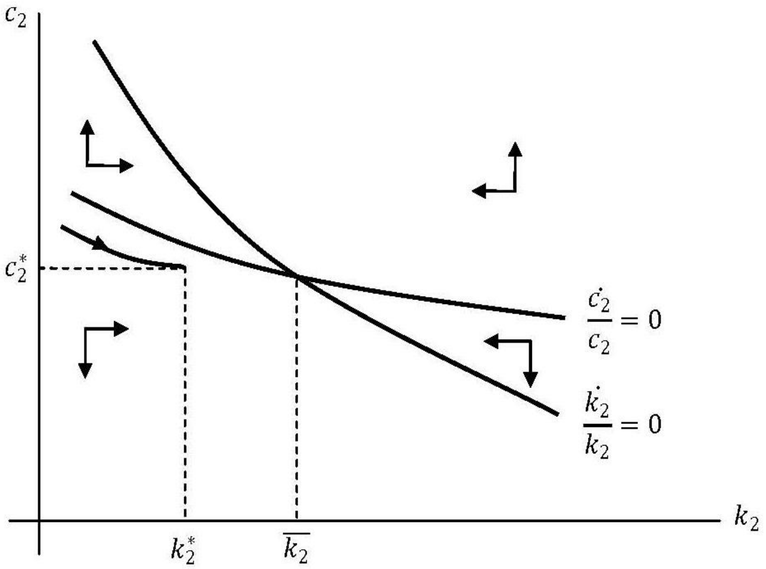

Figure 2.3: Case 2: Transitonal Dynamics for Country 2

Figure 2.4 captures price dynamics for both countries for case 2.

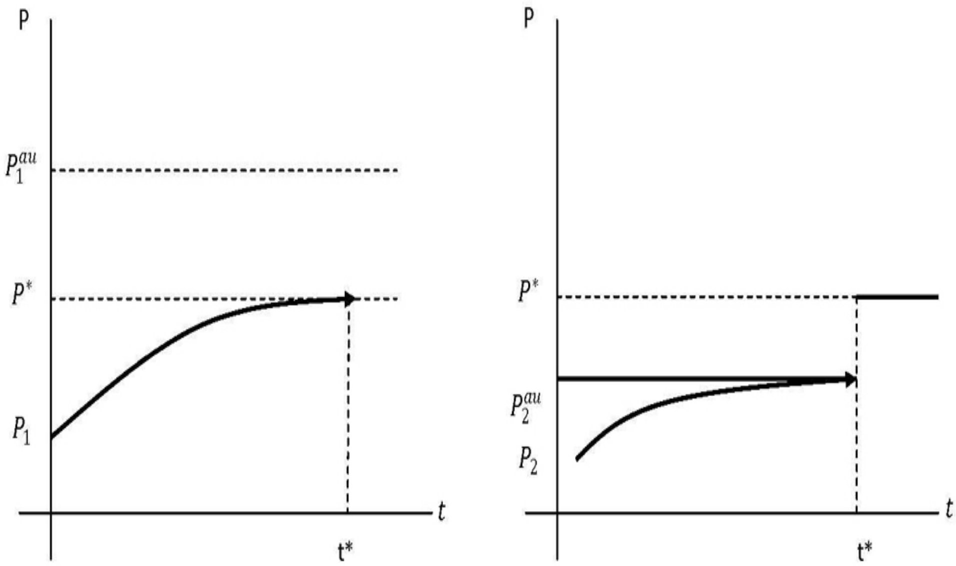

Figure 2.4: Case 2: Price Adjustment Process

Under conditions of case 2 both countries start with higher level of J-type capital than K-type capital relative to the values on the BGP. Therefore, in both countries relative price level of good J is lower than the world price level associated with the BGP in the presence of trade (p^*). It can also be shown (see Appendix) that starting relative price level in country 2 is lower than its autarkic price level associated with the operation of both sectors. As countries accumulate K-type capital and depreciate J-type capital along the transitional path, the relative price level in both countries increases. As it was shown above on the BGP the world relative price level in the presence of trade should fall inside the closed interval given by autarkic price levels in both countries, with autarkic price level in country 2 representing lower bound of the closed interval and the autarkic price level of country 1 representing upper bound of the closed interval. So, as relative price increases in both countries along the transitional path, country 2 will achieve its autarkic price level before the price level in country 1 will reach p^*. Therefore, when relative price level in country 2 is at its autarkic level country 2 will operate both sectors and stay in autarky until price level in country 1 will reach p^* then countries will open up to trade, with each country specializing in the production of good for which it has comparative advantage.

2.3.3 Transitional Dynamics: Case 3

In this case I am assuming that $k_1 > k_1^*$ and $k_2 < k_2^*$.

So, at the starting point country 1 has more K than J relative to its BGP value, whereas country 2 has more J than K-type capital relative to its BGP value. The important specific of this case is that at starting point each country has more of the good for each it has comparative advantage on the BGP.

As country 1 has more K than J, it will set investment into good K equal to 0, which leads to the following solution for X.

$$C_1 + X_1 = A_1 K_1^{\alpha_1} J_1^{1-\alpha_1}$$

$$X_1 = A_1 K_1^{\alpha_1} J_1^{1-\alpha_1} - C_1 \tag{2.23}$$

Accumulation of J-type capital in the presence of complete specialization is given as:

$$\dot{J}_1 + \delta J_1 = \frac{X_1}{p}$$

Substituting from (2.23) the modified accumulation condition for J will become:

$$\dot{J}_1 + \delta J_1 = \frac{A_1 K_1^{\alpha_1} J_1^{1-\alpha_1} - C_1}{p}$$

Then the present value Hamiltonian and the necessary conditions for this case will be:

$$H = \frac{C_1^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \mu \left(\frac{A_1 K_1^{\alpha_1} J_1^{1-\alpha_1} - C_1}{p} - \delta J_1 \right) \tag{2.24}$$

$$\frac{\partial H}{\partial C_1} = C_1^{-\theta} e^{-\rho t} - \frac{\mu}{p} = 0 \tag{2.25}$$

$$\dot{\mu} = -\mu \left(\frac{(1-\alpha_1) A_1 k_1^{\alpha_1}}{p} - \delta \right) \tag{2.26}$$

$$lim_{t \to \infty} J_{1t} \mu_t = 0 \tag{2.27}$$

Using (2.25) and (2.26) I can solve for the growth rate of consumption in country 1, which will be given as:

$$\frac{\dot{C}_1}{C_1} = \frac{1}{\theta}\left(\frac{(1-\alpha_1)A_1 k_1^{\alpha_1}}{p} + \frac{\dot{p}}{p} - \delta - \rho\right) \tag{2.28}$$

Equation (2.28) combined with the following two conditions will determine paths of C_1, J_1 and K_1.

$$\frac{\dot{J}_1}{J_1} = \frac{A_1 k_1^{\alpha_1} - c_1 k_1}{p} - \delta \tag{2.29}$$

$$K_{1t} = K_{10} e^{-\delta t} \tag{2.30}$$

To complete solution for country 1 I need to determine the growth rate of the world relative price, p. Before I do that, however, I will proceed with solution for country 2.

According to condition of case 3, country 2 has more of the J-type capital than K-type capital, therefore constraint of non-negative investment in J-type capital binds and country 2 sets investment into J-type capital equal to 0.

$$\dot{J}_2 + \delta J_2 = 0$$

$$\frac{\dot{J}_2}{J_2} = -\delta$$

As country 2 no longer invests into J-type capital, it will export the output of this sector to country 1, which implies:

$$\frac{X_2}{p} = B_2 K_2^{\eta_2} J_2^{1-\eta_2} \tag{2.31}$$

Using (2.31) the present value Hamiltonian for country 2 will be:

$$H = \frac{C_2^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \lambda\left(-C_2 - \delta K_2 + p B_2 K_2^{\eta_2} J_2^{1-\eta_2}\right) \tag{2.32}$$

$$\frac{\partial H}{\partial C_2} = C_2^{-\theta} e^{-\rho t} - \lambda = 0 \tag{2.33}$$

$$\dot{\lambda} = \lambda \left(\delta - p B_2 \eta_2 k_2^{\eta_2 - 1} \right) \tag{2.34}$$

$$lim_{t \to \infty} K_{2t} \lambda_t = 0 \tag{2.35}$$

Using (2.33) and (2.34) I can solve for the growth rate of consumption in country 2, which will be given as:

$$\frac{\dot{C}_2}{C_2} = \frac{1}{\theta} \left(p \eta_2 B_2 k_2^{\eta_2 - 1} - \delta - \rho \right) \tag{2.36}$$

Equation (2.36) combined with the following two conditions will determine paths of C_2, J_2 and K_2.

$$\frac{\dot{K}_2}{K_2} = p B_2 k_2^{\eta_2 - 1} - c_2 - \delta \tag{2.37}$$

$$J_{2t} = J_{20} e^{-\delta t} \tag{2.38}$$

To close solution for case 3 I need to determine the growth rate of the world relative price. To do that I will use the trade balance condition given by:

$$A_1 J_1 k_1^{\alpha_1} - C_1 = p B_2 J_2 k_2^{\eta_2}$$

The above condition states that export of country 1 must be equal to the import of country 2. Total differentiation of the trade balance condition combined with some algebra (see Appendix) leads to the solution for the growth rate of the world relative price, which will be given by:

$$\frac{\dot{p}}{p} = \frac{\theta}{(\theta p B_2 k_2^{\eta_2} + c_1 k_{12})} \times A_1 k_1^{(\alpha_1 - 1)} k_{12} \left[\frac{(1-\alpha_1)(A_1 k_1^{\alpha_1} - c_1 k_1)}{p} - \delta \right]$$
$$- \frac{\theta}{(\theta p B_2 k_2^{\eta_2} + c_1 k_{12})} \times p B_2 k_2^{\eta_2} \left[\eta_2 p B_2 k_2^{\eta_2 - 1} - \eta_2 c_2 - \delta \right]$$

$$-\frac{\theta}{(\theta p B_2 k_2^{\eta_2}+c_1k_{12})} \times \frac{c_1k_{12}}{\theta}\left[\frac{(1-\alpha_1)A_1k_1^{\alpha_1}}{p}-\delta-\rho\right]$$

where $k12=\frac{K_1}{J_2}$. Note that the ratio of K_1 to J_2 will be constant along the transition to the BGP under this case of transitional dynamics. As country 1 depreciates K-type capital at rate δ and country 2 depreciates J-type capital at rate δ the ratio will stay constant along the transitional path.

The dynamic system describing the transitional behavior of the world economy under scenario of case 3 can be expressed in terms of the following five variables p, $c_1=\frac{C_1}{K_1}$, $k_1=\frac{K_1}{J_1}$, $c_2=\frac{C_2}{K_2}$, $k_2=\frac{K_2}{J_2}$. The resulting five dimensional system is given below:

$$\begin{bmatrix} \frac{\dot{p}}{p} \\ \frac{\dot{c_1}}{c_1} \\ \frac{\dot{k_1}}{k_1} \\ \frac{\dot{c_2}}{c_2} \\ \frac{\dot{k_2}}{k_2} \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} & n_{13} & n_{14} & n_{15} \\ n_{21} & n_{22} & n_{23} & n_{24} & n_{25} \\ n_{31} & n_{32} & n_{33} & 0 & 0 \\ n_{41} & 0 & 0 & n_{44} & n_{45} \\ n_{51} & 0 & 0 & n_{54} & n_{55} \end{bmatrix} \begin{bmatrix} p-p^* \\ c_1-c_1^* \\ k_1-k_1^* \\ c_2-c_2^* \\ k_2-k_2^* \end{bmatrix}$$

where

$$n_{11}=\frac{\theta A_1k_1^{\alpha_1-1}k_{12}}{(\theta p B_2 k_2^{\eta_2}+c_1k_{12})}\frac{(1-\alpha_1)(c_1k_1-A_1k_1^{\alpha_1})}{p^2}-\frac{\theta^2A_1k_1^{\alpha_1-1}k_{12}B_2k_2^{\eta_2}}{(\theta p B_2 k_2^{\eta_2}+c_1k_{12})^2}$$

$$\times\left(\frac{(1-\alpha)(Ak-ck)}{p}-\delta\right)-\frac{\eta_2\theta p B_2^2k_2^{2\eta_2-1}}{(\theta p B_2 k_2^{\eta_2}+c_1k_{12})}-\frac{\theta B_2k_2^{\eta_2}c_1k_{12}}{(\theta p B_2 k_2^{\eta_2}+c_1k_{12})^2}$$

$$\times\left(\eta_2 p B_2 k_2^{\eta_2-1}-\eta_2c_2-\delta\right)+\frac{(1-\alpha_1)A_1k_1^{\alpha_1}}{p^2}\frac{c_1k_{12}}{(\theta p B_2 k_2^{\eta_2}+c_1k_{12})}$$

$$+\frac{\theta B_2k_2^{\eta_2}c_1k_{12}}{(\theta p B_2 k_2^{\eta_2}+c_1k_{12})^2}\left(\frac{(1-\alpha_1)A_1k_1}{p}-\delta-\rho\right)$$

$$n_{12}=\frac{\theta p B_2k_2^{\eta_2}k_{12}}{(\theta p B_2 k_2^{\eta_2}+c_1k_{12})^2}\left(\eta_2 p B_2 k_2^{\eta_2-1}-\eta_2c_2-\delta\right)-\frac{k_{12}^2A_1k_1^{\alpha_1-1}\theta}{(\theta p B_2 k_2^{\eta_2}+c_1k_{12})^2}$$

$$\times\left(\frac{(1-\alpha_1)(A_1k_1^{\alpha_1}-c_1k_1)}{p}-\delta\right)-\frac{(1-\alpha_1)k_1}{p}\frac{\theta A_1k_1^{\alpha_1-1}k_{12}}{(\theta pB_2k_2^{\eta_2}+c_1k_{12})}$$
$$-\frac{\theta k_{12}pB_2k_2^{\eta_2}}{(\theta pB_2k_2^{\eta_2}+c_1k_{12})^2}\left(\frac{(1-\alpha_1)A_1k_1^{\alpha_1}}{p}-\delta-\rho\right)$$

$$n_{13}=\frac{(\alpha_1-1)\theta A_1k_{12}k_1^{\alpha_1-2}}{(\theta pB_2k_2^{\eta_2}+c_1k_{12})}\left(\frac{(1-\alpha_1)(A_1k_1^{\alpha_1}-c_1k_1)}{p}-\delta\right)+\frac{\theta A_1k_1^{\alpha_1-1}k_{12}}{(\theta pB_2k_2^{\eta_2}+c_1k_{12})}$$
$$\times\frac{(1-\alpha_1)(A_1\alpha_1k_1^{\alpha_1-1}-c_1)}{p}-\frac{\alpha_1(1-\alpha_1)A_1k_1^{(\alpha_1-1)}}{p}\frac{c_1k_{12}}{(\theta pB_2k_2^{\eta_2}+c_1k_{12})}$$

$$n_{14}=\frac{\theta\eta_2pB_2k_2^{\eta_2}}{\theta pB_2k_2^{\eta_2}+c_1k_{12}}$$

$$n_{15}=\frac{\theta\eta_2pB_2k_2^{\eta_2-1}c_1k_{12}}{(\theta pB_2k_2^{\eta_2}+c_1k_{12})^2}\left(\frac{(1-\alpha_1)A_1k_1^{\alpha_1}}{p}-\delta-\rho\right)-\frac{\eta_2\theta^2pB_2k_2^{\eta_2-1}A_1k_1^{\alpha_1-1}k_{12}}{(pB_2k_2^{\eta_2}\theta+c_1k_{12})^2}$$
$$\times\left(\frac{(1-\alpha_1)(A_1k_1^{\alpha_1}-c_1k_1)}{p}-\delta\right)-\frac{\eta_2\theta pB_2k_2^{\eta_2-1}c_1k_{12}}{(\theta pB_2k_2^{\eta_2}+c_1k_{12})^2}\left(\eta_2pB_2k_2^{\eta_2-1}-\eta_2c_2-\delta\right)$$
$$-(\eta_2-1)\eta_2pB_2k_2^{\eta_2-2}\frac{\theta pB_2k_2^{\eta_2}}{(\theta pB_2k_2^{\eta_2}+c_1k_{12})}$$

$$n_{21}=-\frac{(1-\alpha_1)A_1k_1^{\alpha_1}}{\theta p^2}+\frac{n_{11}}{\theta}$$

$$n_{22}=\frac{n_{12}}{\theta}$$

$$n_{23}=\frac{\alpha_1(1-\alpha_1)A_1k_1^{\alpha_1-1}}{\theta p}+\frac{n_{13}}{\theta}$$

$$n_{24}=\frac{n_{14}}{\theta}$$

$$n_{25} = \frac{n_{15}}{\theta}$$

$$n_{31} = \frac{A_1 k_1^{\alpha_1} - c_1 k_1}{p^2}$$

$$n_{32} = \frac{k_1}{p}$$

$$n_{33} = \frac{c_1 - \alpha_1 A_1 k_1^{\alpha_1 - 1}}{p}$$

$$n_{41} = \frac{(\eta_2 - \theta) B_2 k_2^{\eta_2 - 1}}{\theta}$$

$$n_{44} = 1$$

$$n_{45} = \frac{(\eta_2 - 1)(\eta_2 - \theta) p B_2 k_2^{\eta_2 - 2}}{\theta}$$

$$n_{51} = B_2 k_2^{\eta_2 - 1}$$

$$n_{54} = -1$$

$$n_{55} = (\eta_2 - 1) p B_2 k_2^{\eta_2 - 2}$$

The above five dimensional system can be written as $\dot{z} = N z_t$, where $\dot{z}$ is the five-dimensional vector of the growth rates of the variables, p, c_1, k_1, c_2 and k_2 and z_t is a vector of deviations of the variables from their steady state values. The solution to this system can be approximated as follows:

$$z_t \cong M e^{-\kappa t} M^{-1} z_0$$

where M is the matrix of the eigenvectors, κ is the diagonal matrix of eigenvalues of matrix N and z_0 is the vector of initial deviations of the variables from their steady state.

High dimensionality of the above system doesn't allow for analytical solution, therefore I proceed with simulation exercise for this case of transitional dynamics. As it follows from the entries of matrix N, the above system is a function of the parameters α_1, η_2, A_1, B_2, ρ, θ, δ and k_{12}. To start simulation exercise I assumed the following values for these parameters: $\alpha_1 = 0.3$, $\eta_2 = 0.25$, $\theta = 2$, $\rho = 0.0025$, $\delta = 0.025$, $A_1 = 1$, $B_2 = 0.2$ and $k_{12} = 15$. As time units are assumed to be measured by quarters the value of $\rho = 0.0025$ implies that annual real interest rate is 1 percent and the value of $\delta = 0.025$ assumes annual depreciation rate of 10 percent. Values of θ and A_1 are consistent with what is commonly used in model calibration exercises (see for example Mulligan and Sala-i-Martin, 1993). The value of α_1 is consistent with the stylized fact that the share of physical capital in Cobb-Douglas production function is 1/3. The choice of the initial value for η_2 is determined based on the assumption that $\alpha_1 > \eta_2$, meaning that the share of the physical capital in the production of labor augmenting type of capital is smaller than in the production of physical capital. The choice of values for parameters B_2 and k_{12} is rather arbitrary: this particular choice of the values was determined to achieve smooth convergence pattern for this benchmark case corresponding to the above values of parameters.

After presenting the results of the simulation exercise under initial parameter values I will proceed with the results of comparative static exercise from the changes in the values of the parameters.

Figures 2.5-2.9 show the paths of the deviations of the five variables, p, c_1, k_1, c_2, k_2 from their steady state and their growth rates.

As it follows from the figures asymptotic convergence to the steady state in country 2 is substantially slower than convergence in country 1. In particular, it takes about 12 years for country 2 to asymptotically converge to its steady state whereas for country 1 it takes only about a year to asymptotically converge to the steady state. Related to the duration of the asymptotic convergence of countries to the steady state one can also see that deviation of the growth rates from the steady state are smaller

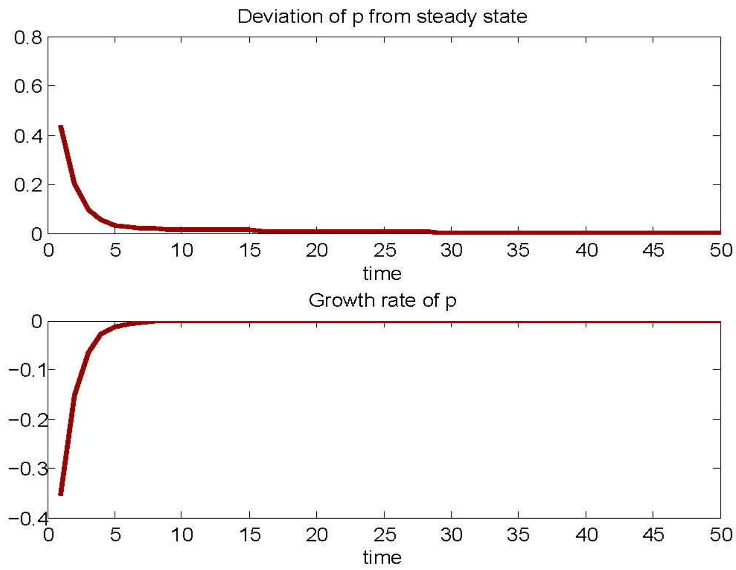

Figure 2.5: Case 3: Paths of the Deviation of Price From its Steady State and its Growth Rate

in magnitude for country 2 than in country 1 but it takes longer for country to adjust to steady state. To understand what determins the speed and pattern of asymptotic convergence of countries to the steady state I experimented with the values of all parameters and below present the changes that occur in the behavior of the countries compared to above discribed benchmark case.

Changes in θ. I change the value of θ in the range of $1.8 - 2.2$. Increase in the value of theta from 2 to 2.2 keeping all the other parameters at their initial values makes all the initial deviations of the variables and their growth rates from the steady state stronger in magnitude, whereas reduction of the value of theta from 2 to 1.8 eliminates smooth convergence pattern in the transitional behavior of all the variables and leads to the complex eigenvalues of matrix N. Resulting cyclical behavior of the growth rates of the variables of country 2 along the transitional path is presented in figures 2.10 and 2.11.

Changes in B_2. Reduction in the value of B2 in the range from 0.2 to 0.04 leads to two noticeable changes in the behavior of variables of country 2. it introduces jumps in the convergence patterns of p and c2 and increases duration of asymptotic

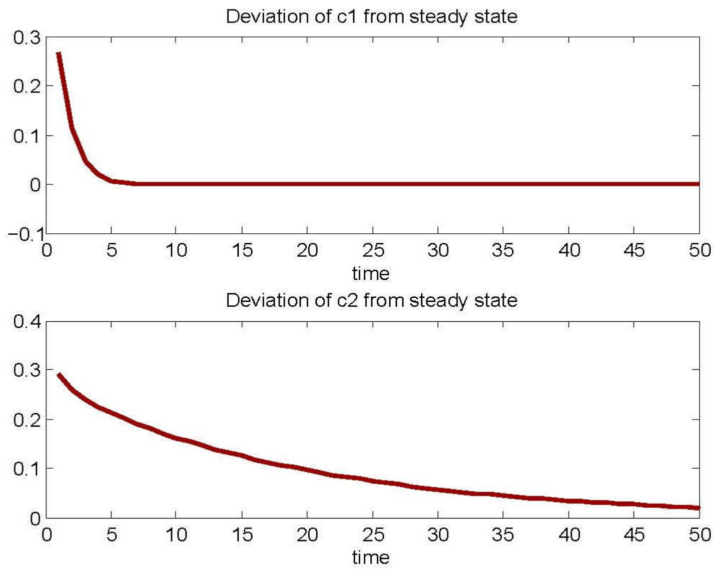

Figure 2.6: Case 3: Paths of the Deviation of c1 and c2 From their Steady States

convergence of country 2 to the steady state. On the other hand increase in the value of B2 from 0.2 to 0.6 and above leads to the complex eigenvalues of matrix N and fast asymptotic convergence to the steady state, where transitional paths of the variables are characterized by the short presence of oscillations around the steady state. Figures 2.12 − 2.14 present transitional dynamics consistent with changes in B_2 from 0.2 to 0.04.

Changes in A_1. Reduction of A_1 from its benchmark value of 1 leads to the presence of the complex eigenvalues and cycles in the growth rates of the variables of country 2 along the transition. Figures 2.15 and 2.16 show the growth rates of c_2, k_2 for the value of A_1 equal to 0.8. It is interesting to note that reduction in the total factor productivity parameters of each country on its own seems to affect more the behavior of country 2 rather than country 1.

Increase in value of A_1 doesn't introduce any substantial changes in the pattern of asymptotic convergence of the variables to their steady state compared to the benchmark case. The only noticeable difference is some increase in the magnitude of initial deviations of the variables and their growth rates from the steady state.

Changes in k_{12}. Results of this simulation exercise are robust to reduction of

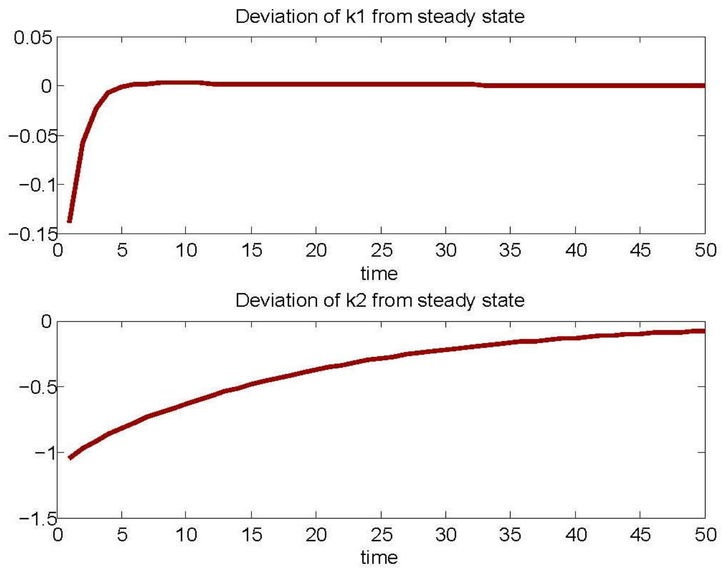

Figure 2.7: Case 3: Paths of the Deviation of k1 and k2 From their Steady States

k_{12} below its initial value of 15, however increase in k_{12} above 15 again leads to the presence of complex eigenvalues. Figures 2.17 and 2.18 show the presence of cycles in the growth rates of c2 and k2 for the value of k12=20.

Duration of the adjustment to the steady state and the convergence patterns obtained under initial values of parameters are robust to the changes in all the other parameters. Results of the above experiments with changes in parameters lead to conclusion that there are three important factors that may affect both duration and pattern of asymptotic convergence to the steady state under scenario of this case of transitional dinamics. Those factors include technological differences among countries, initial ratio of capital goods of both countries for which each has comparative advantage and constant relative risk aversion parameter determining curvature of the utility function. In particular, technological backwardness of countries may lead to slower adjustment process and presence of cycles in the pattern of convergence to the steady state. Steeper slope of the utility function and higher level of K-type capital in the country specializing in its production relative to the level of J-type capital in the other country with comparative advantage in its production may also contribute to the cyclical adjustment to the steady state.

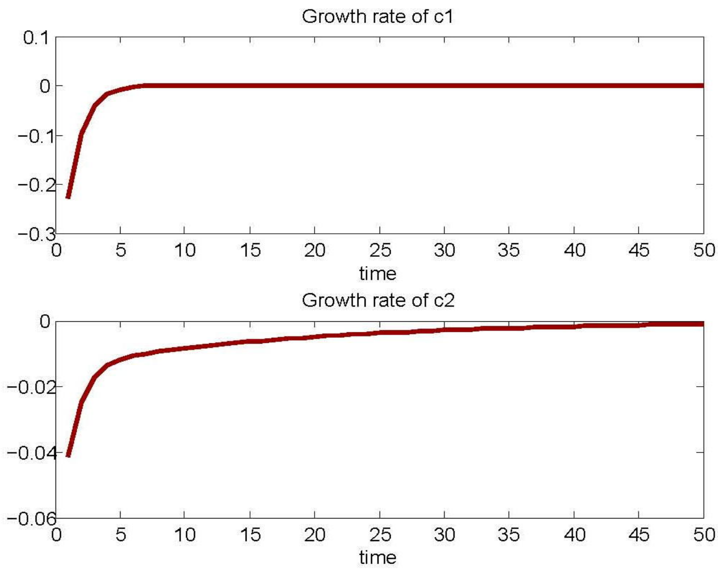

Figure 2.8: Case 3: Growth Rates of c1 and c2 Along the Transition

2.3.4 Transitional Dynamics: Case 4

In this case I am assuming that $k_1 > k_1^*$ and $k_2 > k_2^*$.

For country 1 solution will be the same as in the previous case.

Country 2 will set investment into K-type capital equal to 0. As country 2 imports good Y from country 1, now all the import will be used for consumption in country 2.

$$C_2 = X_2$$

Therefore accumulation condition of J-type capital in country 2 will be written as:

$$\dot{J}_2 + \delta J_2 = B_2 K_2^{\eta_2} J_2^{1-\eta_2} - \frac{C_2}{p}$$

The present value Hamiltonian and the necessry conditions for country 2 then will be given as:

$$V_2 = \frac{C_2^{1-\theta}}{1-\theta} e^{-\rho t} + \mu_t \left(B_2 K_2^{\eta_2} J_2^{1-\eta_2} - \frac{C_2}{p} - \delta J_2 \right)$$

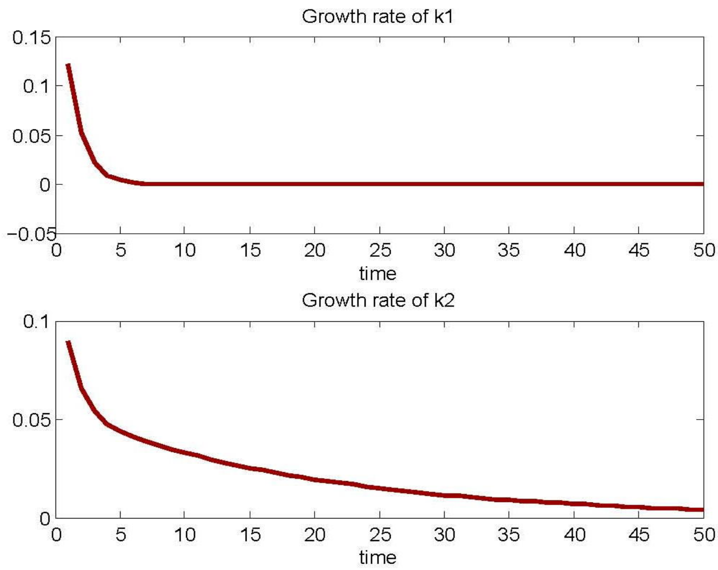

Figure 2.9: Case 3: Growth Rates of k1 and k2 Along the Transition

$$\frac{\partial V_2}{\partial C_2} = C_2^{-\theta} e^{-\rho t} - \frac{\mu_2}{p} = 0$$

$$\dot{\mu_2} = \frac{\partial V_2}{\partial J_2} = -\mu_2 \left((1-\eta_2) B_2 K_2^{\eta_2} J_2^{-\eta_2} - \delta\right)$$

$$lim_{t\to\infty} J_{2t}\mu_{2t} = 0$$

From the solution to Hamiltonian it foolows that the paths of C_2, K_2 and J_2 will be determined by the following set of the equations:

$$\frac{\dot{C_2}}{C_2} = \frac{1}{\theta}\left[(1-\eta_2) B_2 k_2^{\eta_2} + \frac{\dot{p}}{p} - \delta - \rho\right] \tag{2.39}$$

$$K_{2t} = K_{20} e^{-\delta t} \tag{2.40}$$

$$\frac{\dot{J_2}}{J_2} = B_2 k_2^{\eta_2} - \frac{c_2 k_2}{p} - \delta \tag{2.41}$$

Similarly to the previous case, I am going to use trade balance condition to solve

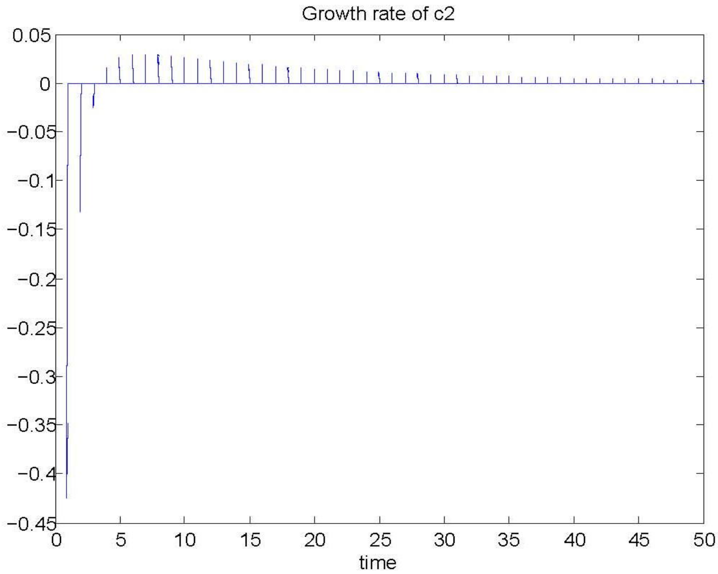

Figure 2.10: Case 3: Growth Rate of c2 for the Value of Theta Equal to 1.8

for the growth rate of the world relative price. Trade balance in this case will be written as:

$$A_1 K_1^{\alpha_1} J_1^{1-\alpha_1} - C_1 = C_2$$

Total differentiation of the trade balance condition leads to the solution for the growth rate of the world relative price as a function of the same five variables as in case 3 given by:

$$\frac{\dot{p}}{p} = \frac{\theta A_1 k_1^{\alpha_1 - 1}}{c_2 k_{21} + c_1} \left(\frac{(1-\alpha_1)(A_1 k_1^{\alpha_1} - c_1 k_1)}{p} - \delta \right) - \frac{c_1}{c_2 k_{21} + c_1} \left(\frac{(1-\alpha_1) A_1 k_1^{\alpha_1}}{p} - \delta - \rho \right)$$

$$- \frac{c_2 k_{21}}{c_2 k_{21} + c_1} \left((1-\eta_2) B_2 k_2^{\eta_2} - \delta - \rho \right)$$

where $k_{21} = \frac{K_2}{K_1}$. Note again that this ratio will remain constant along the transition to the BGP with both countries depreciating K-type capital at rate δ along the transition. As in the previous case of transitional dynamics the transitional behavior of this world economy will be described by five dimensional system which is presented

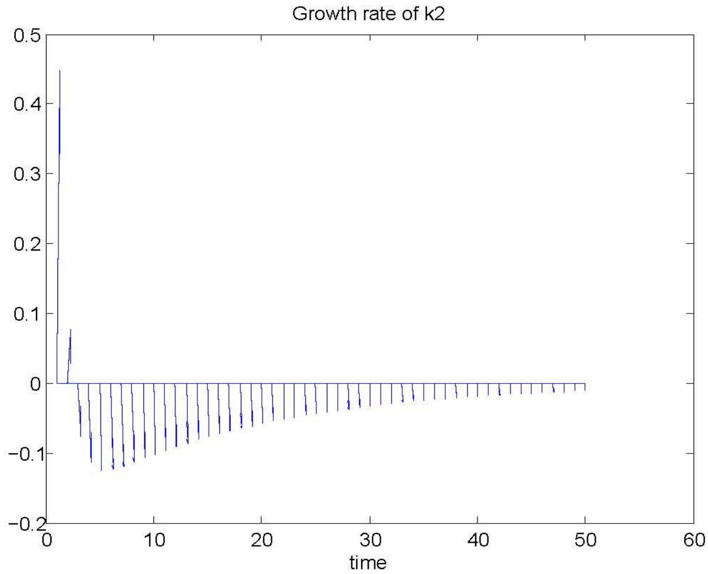

Figure 2.11: Case 3: Growth Rate of k2 for the Value of Theta Equal to 1.8

below.

$$\begin{bmatrix} \frac{\dot{p}}{p} \\ \frac{\dot{c_1}}{c_1} \\ \frac{\dot{k_1}}{k_1} \\ \frac{\dot{c_2}}{c_2} \\ \frac{\dot{k_2}}{k_2} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\ d_{31} & d_{32} & d_{33} & 0 & 0 \\ d_{41} & d_{42} & d_{43} & d_{44} & d_{45} \\ d_{51} & 0 & 0 & d_{54} & d_{55} \end{bmatrix} \begin{bmatrix} p - p^* \\ c_1 - c_1^* \\ k_1 - k_1^* \\ c_2 - c_2^* \\ k_2 - k_2^* \end{bmatrix}$$

where

$$d_{11} = -\frac{\theta A_1(1-\alpha_1)k_1^{\alpha_1-1}}{c_1 + c_2 k_{21}} \frac{A_1 k_1^{\alpha_1} - c_1 k_1}{p^2} + \frac{c_1}{c_1 + c_2 k_{21}} \frac{(1-\alpha_1)A_1 k_1^{\alpha_1}}{p^2}$$

$$d_{12} = -\frac{\theta A_1(1-\alpha_1)k_1^{\alpha_1-1}}{c_1 + c_2 k_{21}} \frac{A_1 k_1^{\alpha_1} - c_1 k_1}{p} - \frac{\theta A_1(1-\alpha_1)k_1^{\alpha_1}}{p(c_1 + c_2 k_{21})}$$
$$+ \frac{\theta A_1 k_1^{\alpha_1-1}\delta}{(c_1 + c_2 k_{21})^2} + \frac{c_2 k_{21}}{(c_1 + c_2 k_{21})^2}((1-\eta_2)B_2 k_2^{\eta_2} - \delta - \rho)$$
$$- \frac{c_2 k_{21}}{(c_1 + c_2 k_{21})^2}\left(\frac{(1-\alpha_1)A_1 k_1^{\alpha_1}}{p} - \delta - \rho\right)$$

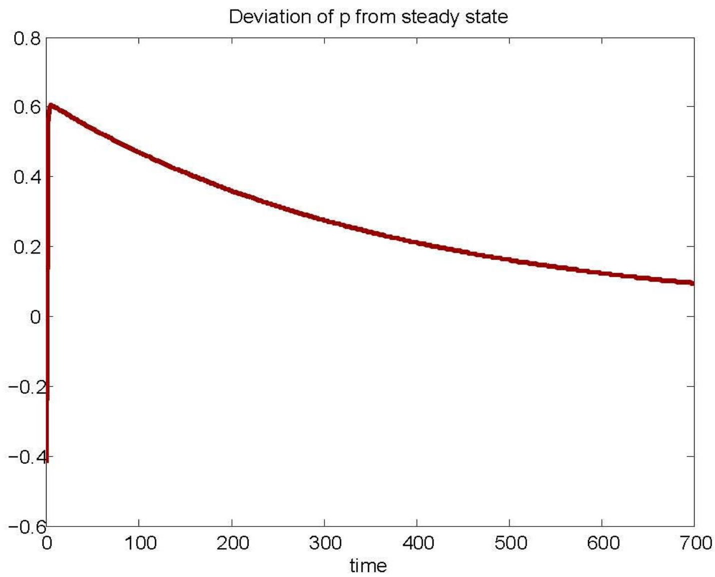

Figure 2.12: Case 3: Deviation of Price for the Value of B2 Equal to 0.04

$$d_{13} = -\frac{\theta A_1(1-\alpha_1)^2 k_1^{\alpha_1-2}}{c_1+c_2k_{21}}\frac{(A_1k_1^{\alpha_1}-c_1k_1)}{p} + \frac{(A_1\alpha_1k_1^{\alpha_1-1}-c_1)}{p}$$
$$\times\frac{\theta A_1(1-\alpha_1)k_1^{\alpha_1-1}}{(c_1+c_2k_{21})} + \frac{\theta(1-\alpha_1)A_1k_1^{\alpha_1-2}\delta}{c_1+c_2k_{21}} - \frac{\alpha_1(1-\alpha_1)A_1k_1^{\alpha_1-1}c_1}{p(c_1+c_2k_{21})}$$

$$d_{14} = -\frac{\theta A_1(1-\alpha_1)k_{21}k_1^{\alpha_1-1}}{(c_1+c_2k_{21})^2}\frac{(A_1k_1^{\alpha_1}-c_1k_1)}{p} + \frac{\theta A_1k_1^{\alpha_1-1}k_{21}\delta}{(c_1+c_2k_{21})^2} - \frac{c_1k_{21}}{(c_1+c_2k_{21})^2}$$
$$\times((1-\eta_2)B_2k_2^{\eta_2}-\delta-\rho) + \frac{c_1k_{21}}{(c_1+c_2k_{21})^2}\left(\frac{(1-\alpha_1)A_1k_1^{\alpha_1}}{p}-\delta-\rho\right)$$

$$d_{15} = -\frac{c_2k_{21}\eta_2(1-\eta_2)B_2k_2^{\eta_2-1}}{c_1+c_2k_{21}}$$

$$d_{21} = -\frac{(1-\alpha_1)A_1k_1^{\alpha_1}}{\theta p^2} + \frac{d_{11}}{\theta}$$

$$d_{22} = \frac{d_{12}}{\theta}$$

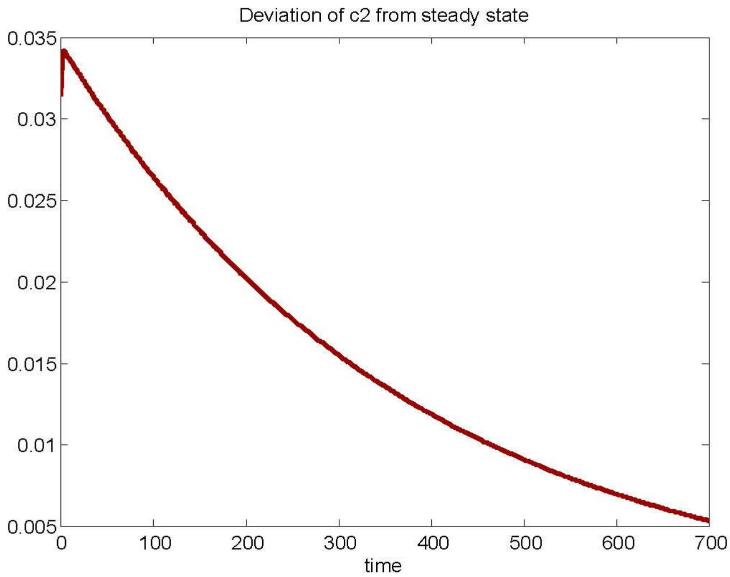

Figure 2.13: Case 3: Deviation of c2 for the Value of B2 Equal to 0.04

$$d_{23} = \frac{\alpha_1(1-\alpha_1)A_1 k_1^{\alpha_1 - 1}}{\theta p} + \frac{d_{13}}{\theta}$$

$$d_{24} = \frac{d_{14}}{\theta}$$

$$d_{25} = \frac{d_{15}}{\theta}$$

$$d_{31} = \frac{A_1 k_1^{\alpha_1} - c_1 k_1}{p^2}$$

$$d_{32} = \frac{k_1}{p}$$

$$d_{33} = \frac{c_1 - \alpha_1 A_1 k_1^{\alpha_1 - 1}}{p}$$

$$d_{41} = \frac{d_{11}}{\theta}$$

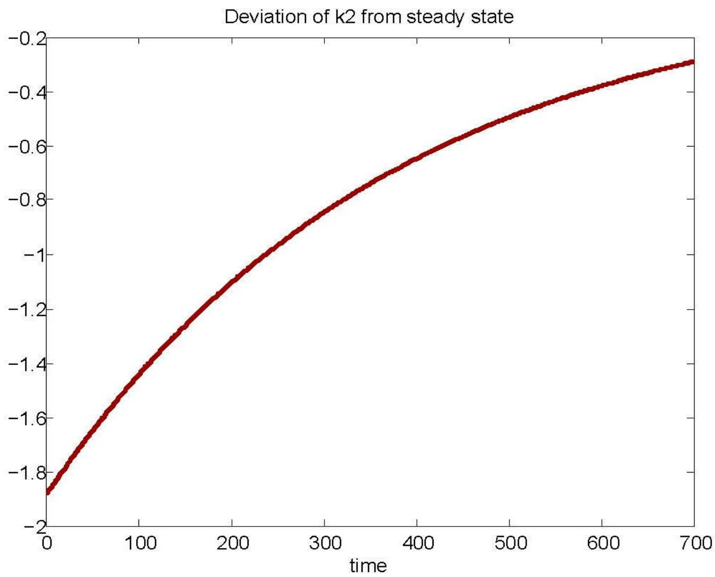

Figure 2.14: Case 3: Deviation of k2 for the Value of B2 Equal to 0.04

$$d_{42} = \frac{d_{12}}{\theta}$$

$$d_{43} = \frac{d_{13}}{\theta}$$

$$d_{44} = \frac{d_{14}}{\theta}$$

$$d_{45} = \frac{\eta_2(1-\eta_2)B_2k_2^{\eta_2-1}}{\theta} + \frac{d_{15}}{\theta}$$

$$d_{51} = \frac{c_2k_2}{p^2}$$

$$d_{54} = -\frac{k_2}{p}$$

$$d_{55} = \eta_2B_2k_2^{\eta_2-1} - \frac{c_2}{p}$$

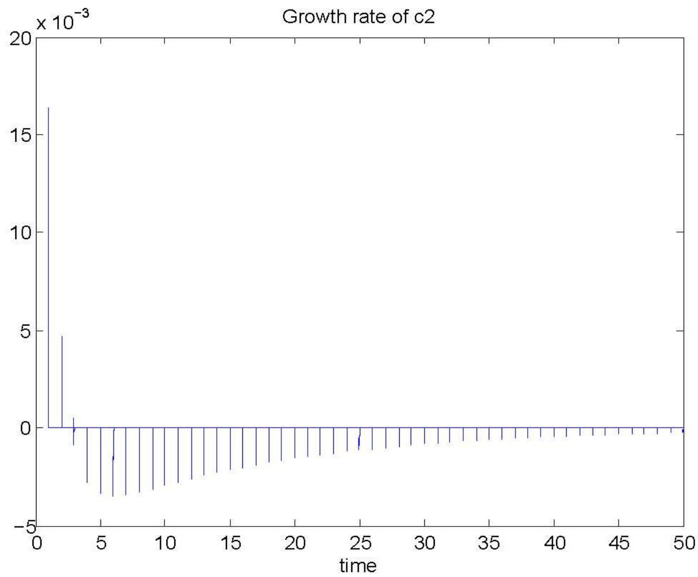

Figure 2.15: Case 3: Growth Rate of c2 for the Value of A1 Equal to 0.8

As in the previous case the above five dimensional system can be written as $\dot{z} = Dz_t$, where $\dot{z}$ is the five-dimensional vector of the growth rates of the variables, p, c_1, k_1, c_2 and k_2 and z_t is a vector of deviations of the variables from their steady state values. The solution to this system can be approximated using eigenvectors and eigenvalues of matrix D. This case of transitional dynamics also doesn't allow for analytical solution, therefore I proceed with the same type of simulation exercise as I did in the previous case.

As initial values of parameters for this case of transitional dynamics, I chose $\alpha_1 = 0.3$, $\eta_2 = 0.25$, $\theta = 2$, $\rho = 0.0025$, $\delta = 0.025$, $A_1 = 1$, $B_2 = 0.8$ and $k_{21} = 18$. As in the previous case of transitional dynamics here as well the choice of values for B_2 and k_{21} is rather arbitrary and later I will discuss the effects of the changes in those parameters on the transitional behavior of the variables. Figures 2.19 − 2.22 describe transitional behavior of the world economy for the benchmark case with the initial values of the parameters.

As we can see from the figures the behavior of the most of the variables is characterized by the presence of cycles. Opposite to the previous case of transitional dynamics when the presence of the cycles usually was associated with the variables of

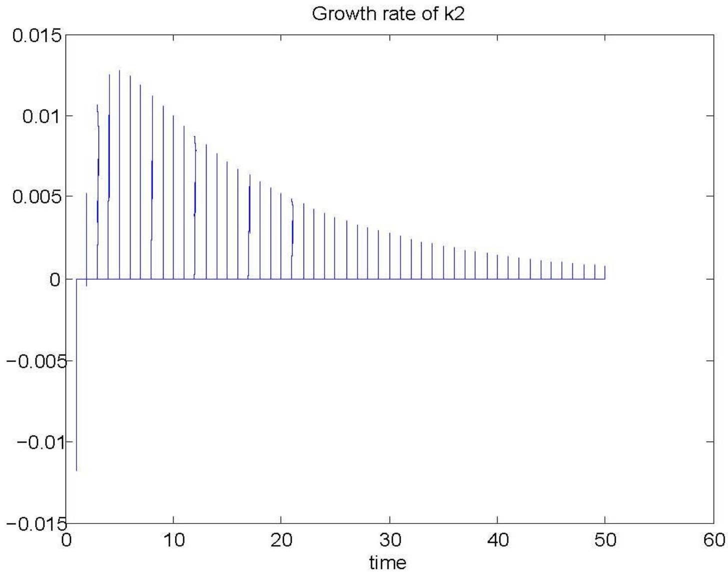

Figure 2.16: Case 3: Growth Rate of k2 for the Value of A1 Equal to 0.8

country 2, here cycles characterize transitional behavior of country 1 as well. Therefore, my next task is to consider how changes in the values of the parameters are going to affect the cyclical behavior demonstrated under the benchmark scenario of the fourth case of transitional dynamics to determine parameters responsible for the cyclical behavior of this dynamic system.

Changes in θ. To analyze the effects of constant relative risk aversion parameter on the pattern of transitional dynamics I consider the values of theta in the range 1.5-2.2. Reduction in the value of theta from its benchmark value doesn't lead to substantial changes in results, however increase in the value of theta from 2 to 2.2 causes dramatic changes in the pattern of transition shown in figures $2.23 - 2.27$. It eliminates complex eigenvalues of matrix D and any evidence of cyclical behavior. Instead, we can observe non-monotonic pattern of asymptotic convergence in all variables and their growth rates with exception of deviations of k1 and c1 from their steady state that demonstrate smooth asymptotic convergence pattern with duration of about 15 quarters. The intuitive explanation for this result is that the higher value of θ implies higher willingness of the households tp smooth their consumption. Therefore increase in the value of theta from 2 to 2.2 eliminates cycles and leads to

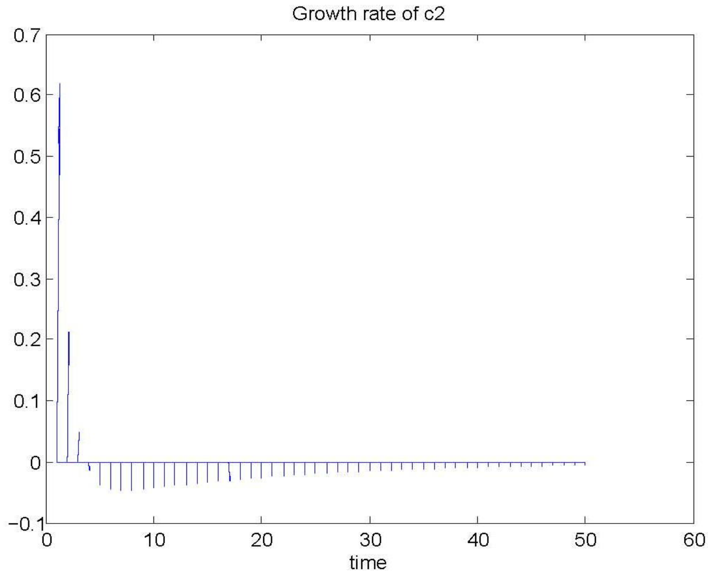

Figure 2.17: Case 3: Growth Rate of c2 for the Value of k12 Equal to 20

smoother convergence pattern.

Changes in B_2. For the range of the values from 0.5 to 0.9 for the total factor productivity parameter of country 2 there is no substantial differences in the transitional behavior of the variables. The differences in duration and pattern of convergence becomes significant for the value of B_2 starting from 0.01 and below. Figures 2.28 − 2.32 below show the transitional process of the variables for value of B_2 equal to 0.01. As we can see the cyclical behavior is replaced by slow monotonic convergence of both countries to the steady state, one more time emphasizing the conclusion drawn from the previous case of dynamics that technological backwardness slows down the transition to the steady state. Reduction in total factor productivity for country 1 combined with lower value of B_2 doesn't add any new insights in the transitional behavior of this world economy.

The results of the benchmark case with the initial values of parameters are robust to the changes in the remaining parameters.

This case of transitionl dynamics also emphasized the importance of total factor productivity parameters as well as parameter θ determining curvature of the utility function. Low levels of technology may lead to very slow monotonic convergence

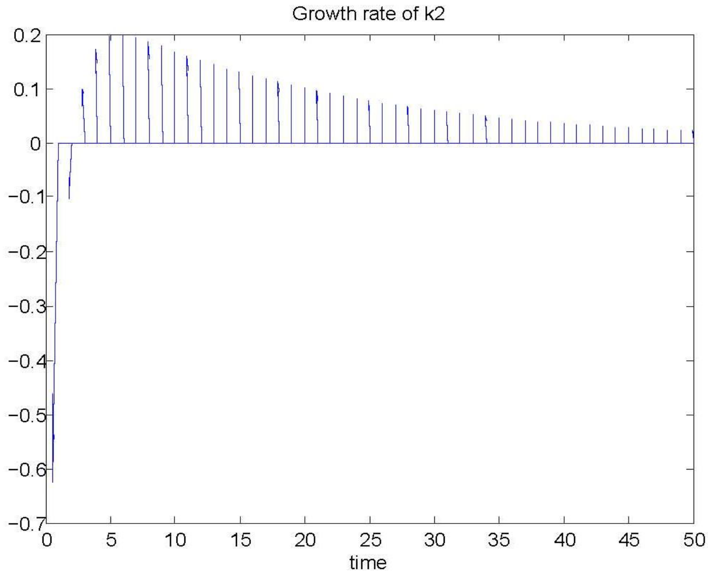

Figure 2.18: Case 3: Growth Rate of k2 for the Value of k12 Equal to 20

towards steady state, while higher values of θ may lead to increased non-monotonicity under initial values of parameters in the effort to eliminate cycles. It is important to note, however, that changes in parameter θ don't affect behavior of the system for low values of B_2.

2.4 Conclusion

In this chapter I studied transitional dynamics of the two-sector open economy model with trade and growth. Trade in goods that are also factors of production leads to a completely new pattern of transitional dynamics. In this chapter I focused on the transitional dynamics in the neighborhood of the balanced growth path consistent with the interior solution of the model in which each country completely specializes in the production of the good for which it has comparative advantage. As I have shown there are four possible cases of transitional dynamics depending on the direction of the deviations of the ratios of factors of production from their steady state values in both countries. In two of those four cases transitional path is characterized by the autarkic behavior of both countries with trade occuring only on the balanced growth path. The other two cases of transitional dynamics are characterized by the presence of the trade both on and along the transition to the balanced growth path. Endogeneity of the world relative price level, however, imposes complexity on the solution. As world relative price depends on the control and state variables of both countries the dynamic systems in the presence of trade are characterized by five differential equations in five unknowns. High dimensionality of the systems under consideration led me to exploit model simulation techniques to analyze behavior of the model along the transitional path in the presence of trade. Results of the simulation exercises lead to several conclusions. First, presence of complex eigenvalues and cycles can be a common pattern describing transitional behavior of the system Second, the technological differences may become important in determining both duration and pattern of assymptotic convergence to the balanced growth path. In particular, technologicall backwardness may lead to substantial slow down of asymptotic convergence. Finally, for some initial values of the other parameters, changes in constant relative risk aversion parameter and the ratio of factors of production in both countries for which each has comparative advantage may be responsible for changes in both pattern and duration of assymptotic convergence to the balanced growth path.

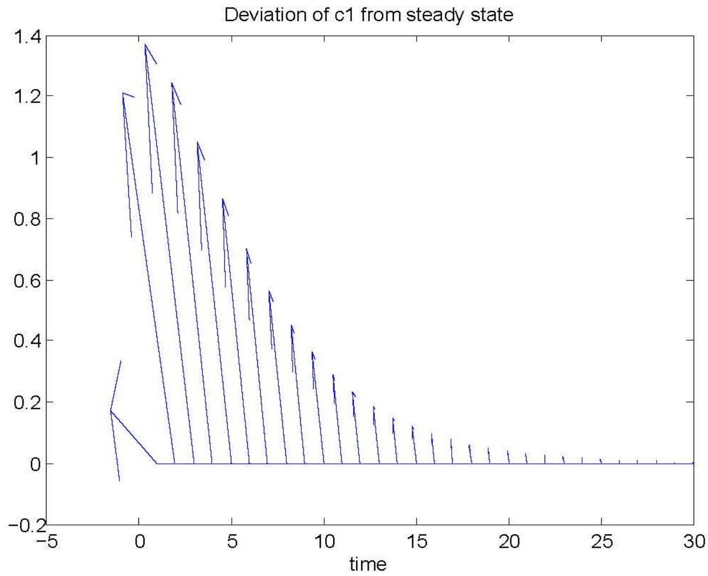

Figure 2.19: Case 4: Deviation of c1 From its Steady State

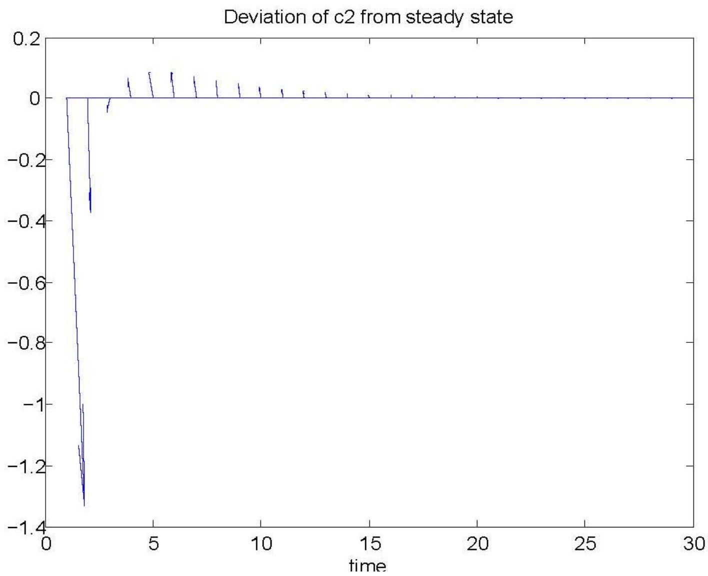

Figure 2.20: Case 4: Deviation of c2 From its Steady State

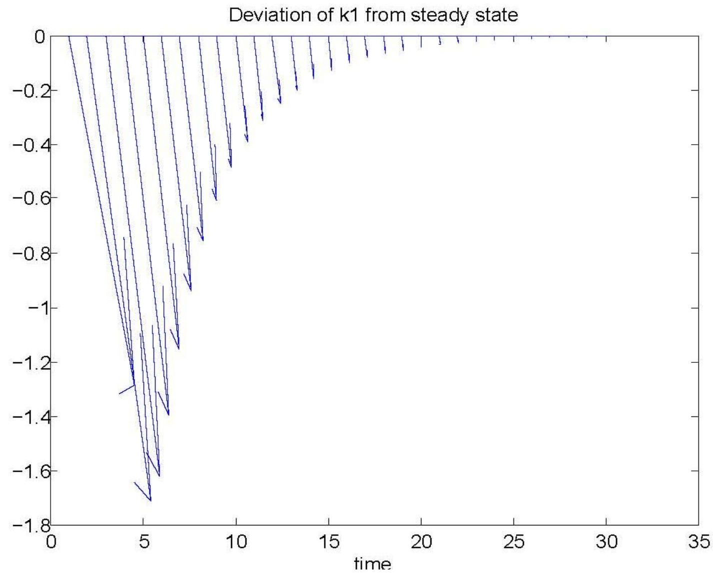

Figure 2.21: Case 4: Deviation of k1 From its Steady State

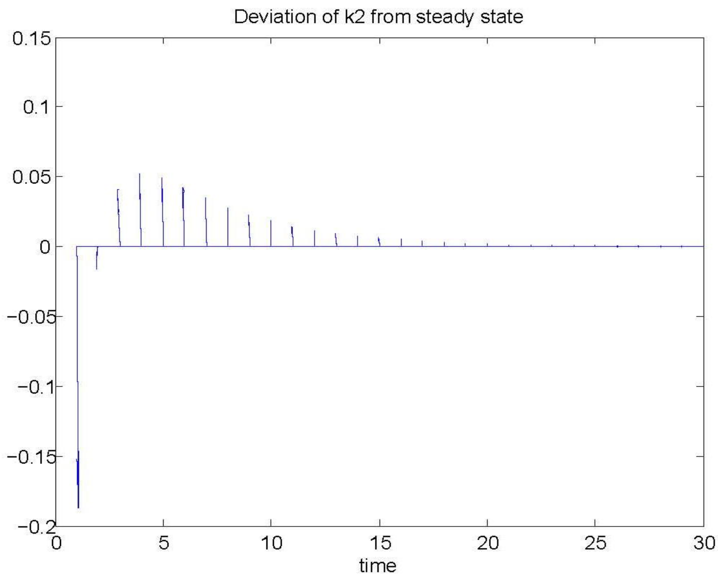

Figure 2.22: Case 4: Deviation of k2 From its Steady State

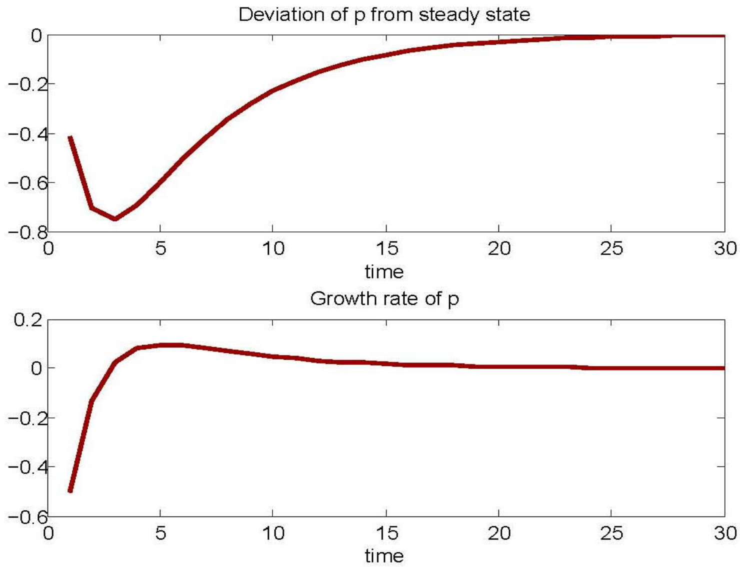

Figure 2.23: Case 4: Deviation and Growth Rate of the Price for Theta=2.2

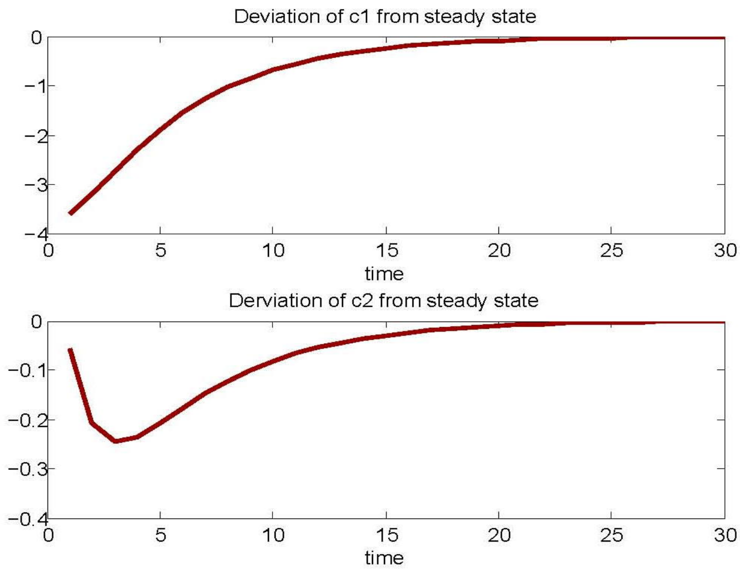

Figure 2.24: Case 4: Deviation and Growth Rate of c1 for Theta=2.2

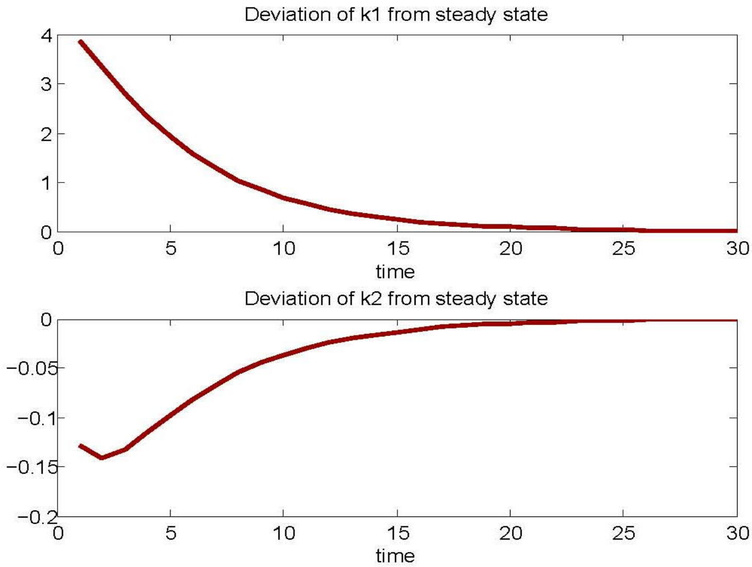

Figure 2.25: Case 4: Deviation and Growth Rate of the k1 for Theta=2.2

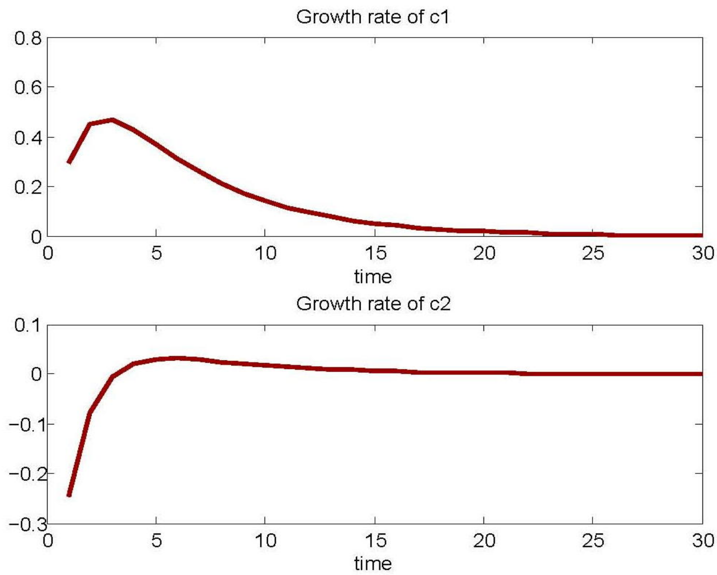

Figure 2.26: Case 4: Deviation and Growth Rate of c2 for Theta=2.2

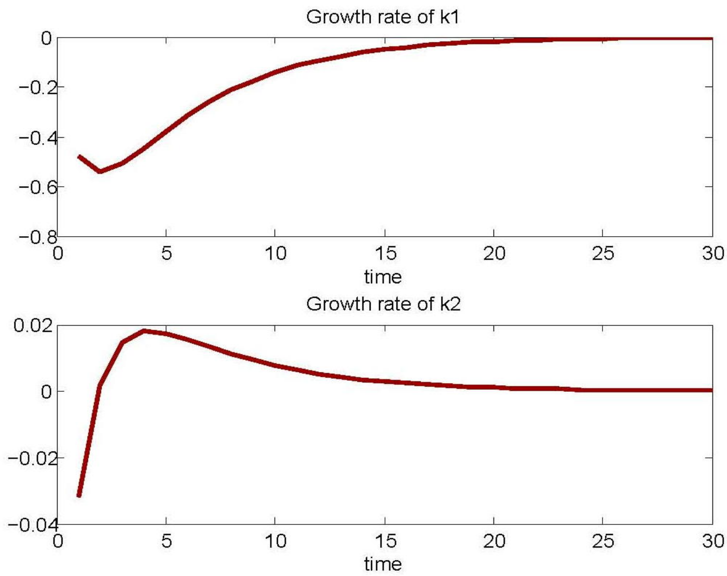

Figure 2.27: Case 4: Deviation and Growth Rate of k2 for Theta=2.2

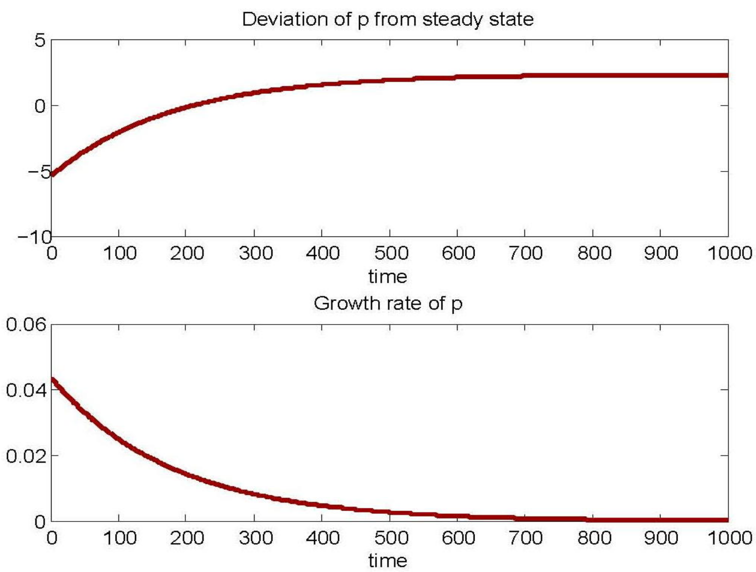

Figure 2.28: Case 4: Deviation and Growth Rate of Price for B2=0.01

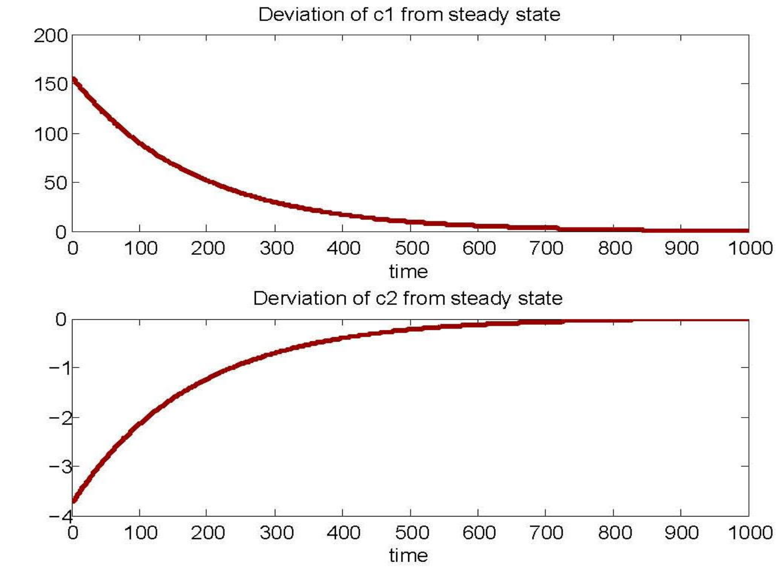

Figure 2.29: Case 4: Deviation of c1 and c2 from BGP for B2=0.01

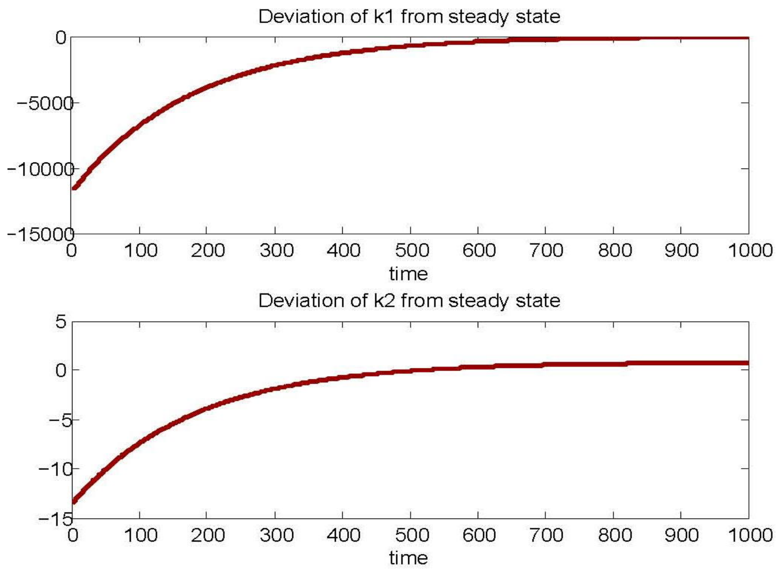

Figure 2.30: Case 4: Deviation of k1 and k2 from BGP for B2=0.01

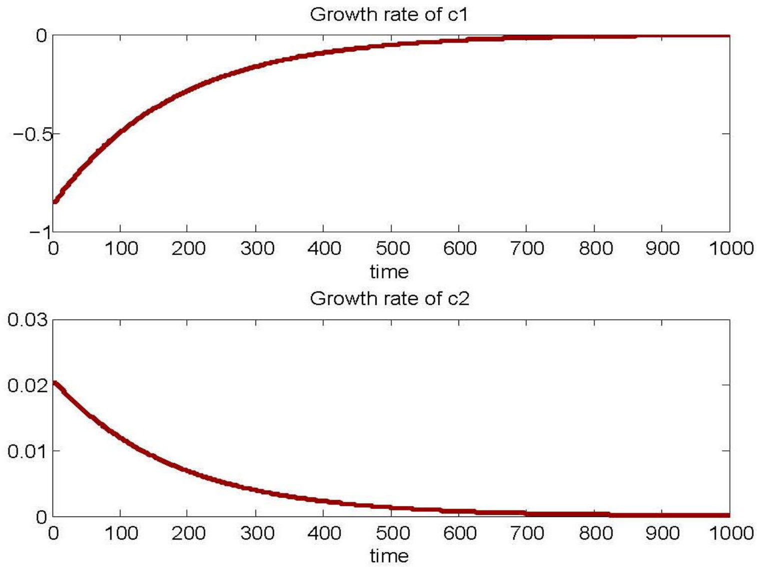

Figure 2.31: Case 4: Growth Rates of c1 and c2 Along the Transition for B2=0.01

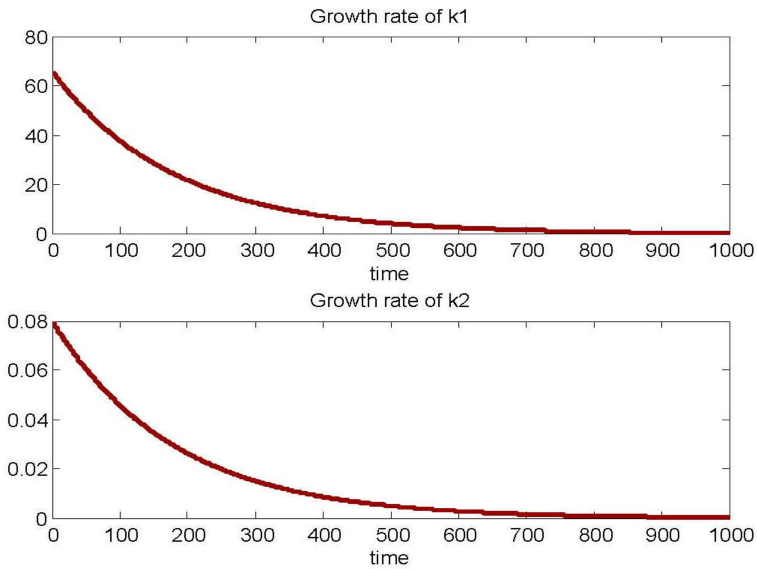

Figure 2.32: Case 4: Growth Rates for k1 and k2 Along the Transition for B2=0.01