Topographic Normalization in Rugged Terrain

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ABSTRACT: Remote sensing of rugged terrain is hampered by topographic effects on spectral signatures. The purpose of this study was to attempt normalization of the topographic effect encountered when analyzing Landsat Thematic Mapper (TM) imagery in the Rocky Mountains. Two normalization techniques were performed on a subscene of a TM image of Rocky Mountain National Park. The first technique was a band ratio and the second was a backwards radiance correction transformation (BRCT) employing a Minnaert constant based on a non-Lambertian assumption. The results of this study indicated that application of the BRCT approach was the more successful technique for reducing topographic effects. Further, after conducting a homogeneity of variance test, the reduction of variances in spectral values between original and non-Lambertian normalized images was found to be statistically significant.

INTRODUCTION

One of the few places in which remotely sensed data have not proven to be effective in discrimination of land-cover types is in areas of high relief. Analysis of Landsat Thematic Mapper (TM) data related to forest canopy analyses is especially limited in mountainous areas (LePrieur et al., 1988). In areas of rugged terrain, variable illumination angles and reflection geometry are caused by different slope angles and orientations. In Landsat data the effect is readily apparent as the visual impression of relief (LePrieur et al., 1988; Holben and Justice, 1979; Justice and Holben, 1979). From the late 1970s through the present, various techniques have been employed to normalize the effect as manifest on multispectral satellite images (Holben and Justice, 1979; Justice and Holben, 1979; Smith et al., 1980; Justice et al., 1980; Kowalick et al., 1983; Hugli and Frei, 1983; Stohr and West, 1985; Franklin et al., 1986; Kawata et al., 1988; LePrieur et al., 1988; Civco, 1989).

PRESENT RESEARCH

To address this problem, two normalization techniques were compared; a band ratio, and a backwards radiance correction transformation (BRCT) utilizing a Minnaert constant based on a non-Lambertian assumption. The Minnaert constant was computed for the entire scene and for a local sample area. An algorithm developed for generating slope and aspect from digital elevation data and for use in a raster based geographic information system (GIS) was used in the BRCT normalization technique. A one-way analysis of variance was conducted to quantify the relationship of brightness values (BV) from selected similar sites, consisting of Lodgepole Pine forests, before and after normalization. Another one-way analysis of variance was performed on data that consisted of BVs from two forest sample sites and values from different cover types, rock outcroppings, or barren soil. The latter analysis provided a comparison between F statistics obtained from similar cover type information with that obtained from mixed cover types.

In addition, a test for the homogeneity of variances was conducted between the original and non-Lambertian normalized bands to measure the significance of the reduction of variances due to normalization. It was hoped that integrating Landsat TM data with derived slope and aspect values per 30-m by 30-m cell, and calculating the Minnaert constant for the entire scene and for a local sample area, would result in a superior “topographic normalization” technique. The technique would not require apriori knowledge of land cover, would be practicable to implement, and would be applicable to rugged terrain and at low solar elevation angles.

RATIOS

A common method for minimizing differences in brightness values from similar surface materials due to topographic conditions, shadows, or seasonal changes in sunlight illumination factors is the ratio transformation (Lyon, 1975; Holben and Justice, 1980; Jensen, 1986). In the visible and reflected infrared (IR) regions, major spectral variations of materials are expressed in differences in the slopes of their spectral reflectance curves, with these slope differences emphasized in the ratio image. Unfortunately, variations in the albedo of materials may be obscured in ratio images if the materials have the same spectral reflectance curve (Sabins, 1987).

MINNAERT CONSTANT

Smith et al. (1980) employed an empirical photometric function, the Minnaert constant, to test the Lambertian assumption for various surfaces. The Minnaert function was developed in 1941, and has been used for photometric analysis of lunar surfaces (Justice and Holben, 1979). A constant, k, in this function describes the bidirectional reflection distribution function (BRDF) of the surfaces (Estes et al., 1983), the type of scattering dependence, and is related to surface roughness (Smith et al., 1980). Smith and others found that techniques based on a Lambertian assumption and not utilizing the Minnaert constant were ineffective in normalizing the “topographic effect” except in a limited range of slope and incidence angles (Justice and Holben, 1979; Smith et al., 1980).

In the study by Smith et al. (1980), the Minnaert constant, k, was derived by first linearizing the following equation:

\[ L(\lambda, \theta) = L_{0}(\lambda) \cos^{6}i \cos^{4}e \]

where \( L \) = radiance, 
\( \lambda \) = wavelength, 
\( e \) = exitance angle = slope, 
\( L_{0} = \) radiance when \( i = e = 0 \), 
\( i \) = incidence angle, and 
\( k \) = Minnaert constant.

Then obtaining the regression value for \( k \), using

\[ L \cos e = L_{0} \cos^{6}i \cos^{4}e \]

and

\[ \log (L \cos e) = \log L_{0} + k \log (\cos i \cos e) \]

where \( y = \log (L \cos e) \), the response variable; 
\( x = \log (\cos i \cos e) \), the independent variable; and
\[ b = \log(L_n), \]

the linear form is obtained from \( y = kx + b \). The equation is then solved for \( k \), the slope of the regression line, and the Minnaert constant (Smith et al., 1980).

**Backwards Radiance Correction Transformation**

A backwards radiance correction transformation model can be developed using the non-Lambertian assumption and the Minnaert relationship (Justice et al., 1980; Smith et al., 1980).

Employing the Minnaert constant, \( k \),

\[ L_n = L(\cos \theta) / (\cos^2i \cos^2d). \]

Cosine \( i \) can be derived using the following equation:

\[ \cos i = \cos \theta_n \cos \theta_s + \sin \theta_n \sin \theta_s \cos (\phi_s - \phi_n) \]

where \( \theta_n = \) surface normal zenith angle or slope of the terrain surface,

\( \theta_s = \) solar zenith angle,

\( \phi_s = \) solar azimuth angle, and

\( \phi_n = \) surface aspect of the slope angle (Smith et al., 1980).

**Diffuse Light Component**

A diffuse light component could influence the spectral radiance values obtained from the TM imagery. Depending on the spectral band and slope and aspect, the diffuse light component could be high (A.F.H. Goetz, U. of Colorado, written communication, 1988). A diffuse light component is not considered here because, as in a similar study under non-hazy, clear sky conditions, the diffuse light component was found to be a small portion of the total irradiance (e.g., 12 percent of the total (Smith et al., 1980)). In the study by Smith et al. (1980), the elevation of the study area ranged from 1370m (4500 ft) to 2255m (7400 ft). The elevation values in this study ranged from 2560m (8399 ft) to 3198m (10,492 ft). The diffuse light component of the total irradiance should be less than in the study by Smith et al. (1980) due to the decrease in the diffuse light component with higher elevations. Diffuse irradiance affects in topographic studies of this nature have been ignored by most researchers as insignificant (Estes et al., 1983).

On 21 November 1988, photographs of the study area were taken at approximately the time of the satellite’s overpass. No terrain induced shadows appeared in the sample sites; therefore, it is doubtful that shadows were present in the sample sites when the imagery was acquired, two weeks earlier.

**METHODOLOGY**

**STUDY AREA**

The study area is located in Rocky Mountain National Park (RMNP) in Colorado, due west of Estes Park, and approximately 8 km west of the Park Headquarters (Figure 1.). The area is 2700m² in size. The usefulness of this study area for testing the effectiveness of the normalization techniques is based on its relatively homogeneous tree cover and its topographic character. Stands of pure and partly mixed Lodgepole Pine (Pinus contorta) can be found on sites of various slopes and aspects. The key areas focused on an eastern oriented moderate slope west of Beaver Meadows (mean slope value of 9.1 degrees), consisting of predominately Lodgepole Pine (sample site 1); an area of moderately steep terrain on the northeast slope of Beaver Mountain above sample site 1 (mean slope value 17.7 degrees), with close to 100 percent Lodgepole Pine (sample site 2); and an area of moderately steep terrain on the southeast slope of the ridge due east of the Hidden Valley Ski Area (mean slope value 23.1 degrees), again primarily of Lodgepole Pine (sample site 3) (Figure 2).

**In Situ Collection of Ground Truth Data**

The ground truth data for this project consisted of quantitative identification of the vegetative cover, in RMNP, using transects. Three sites were chosen because they had different slopes and aspects and because of their predominant populations of Lodgepole Pine with similar canopy densities. The point quarter sampling method (Smith, 1980) was employed in a random stratified manner to determine percentage species composition in each area.

Each transect covered 300 m by 180 m. From a control point on the west side of sample site 1, a 300 m main transect was plotted following a compass bearing of 100 degrees from magnetic North. To plot the main transect in sample site 2, a compass bearing of 140 degrees from magnetic North was followed from a control point on the North side of the site. To plot the main...
transact for sample site 3, the direction followed from the South side of the site was magnetic North (Figure 2). The sub-transsects were then plotted and the point quarter sampling method used to record species composition. A data base was created in dBASE III PLUS, and utilized in quantifying species composition. The quantitative measure used in this study was relative density: i.e.,

\[
\text{relative density} = \frac{\text{individuals of species A}}{\text{total individuals of all species}} \times 100.
\]

The relative density was calculated for each species within each sample site area and for each sub-transsect. The results of the \textit{in situ} analysis indicated a predominance of Lodgepole Pine throughout the sample sites, except for the eastern section of sample site 1, where the percentage of Lodgepole Pine was found to be less than 60 percent, due in part to Beaver Creek crossing the site through that section. The pixel values from that section were excluded from further use in the study, due to the sufficient number of values that were available from the western and center sections of the sample site (Figure 2).

**Slope and Aspect Derivation**

The slope and aspect maps were developed by first digitizing 305 \(x\) (east), \(y\) (north), and \(z\) (elevation) points from the 7.5-minute series 1:24,000-scale USGS quadrangle topographic maps. Both critical and stratified points were digitized within and surrounding the boundaries of the 2700m\(^2\) study area. A grid matrix with values every 30 m by 30 m was created using a kriging interpolation method, a normal search routine, and ten nearest points in the Golden Software program GRID. The elevation file was then converted into an ERDAS GIS format using GRIDT, a program developed at the University of Colorado. Slope and aspect values were derived using vector geometry (after Ritter, 1987). The elevation values of the four immediately adjacent neighbors of each grid cell were used to calculate the normal vector to the surface.

**Digital Image Preprocessing**

Image processing for this study was conducted primarily using the ERDAS system on a personal computer. A seven-band Landsat Thematic Mapper scene acquired on 7 November 1982 was used. The false color composite image of bands 5, 4, and 2 highlighted the borders and site composition between vegetation and barren ground effectively, and was used to extract four control points for rectification and resampling purposes. The derived root-mean-square (RMS) error was 0.57 grid cells. The coordinates were then rectified to correspond with those of the Universal Transverse Mercator projection. A nearest neighbor resampling procedure was chosen instead of another procedure, i.e., a bilinear interpolation or a cubic convolution, due to its superior retention of absolute spectral radiance.

**Normalization Procedures**

Various ratios or indices have been used to detect vegetation on remotely sensed images (Tucker, 1980). A TM band 5/4 ratio was chosen to test the normalization process because it has been shown to be an indicator of vegetation health (Rock et al., 1986; Rock and Vogelmann, 1986; Vogelmann, 1988).

A program entitled \textit{NORMAL} was written to carry out the BRCT normalization technique. The solar elevation at the time of the satellite overpass, was calculated to be 29.39\(^\circ\), after determining the hour angle and the direction cosine. By soliciting the solar elevation and azimuth, and integrating the elevation file and a single band TM image, the \textit{NORMAL} program normalized the TM image using the non-Lambertian based BRCT technique, and a Minnaert constant.

To compute the Minnaert constant for a TM band, the \textit{NORMAL} program first computed the cosine \(i\) and cosine \(e\) for each pixel, recorded the original BVS of the TM band, and wrote them to a "cosine" file. Another program, written in C and entitled \textit{MINNAERT CONSTANT VARIABLES}, calculated the independent and dependent variables using the cosine file created in NORMAL. Utilizing the independent and dependent variables, a linear regression program then calculated the Minnaert constant for the particular band. The Minnaert constants for bands 1, 2, 3, 4, 5, and 7 were calculated in this manner, and the images were normalized.

A sample area containing sample site 2 was also extracted from bands 1, 2, 3, 4, 5, and 7. This 450- by 450-m area was used to compare Minnaert constant values obtained from the entire scene with those from a smaller area, and served to illustrate the application of a locally derived Minnaert constant. To successfully derive the Minnaert constant, a range of incidence and exitance angles are required.

**Analysis**

A quantitative analysis routine measured the effectiveness of the normalization techniques by quantifying the relationship of the sample site data before and after normalization. The analysis also measured the relationship of BVS data from different cover types. The first application of the analysis quantified the relationship of the BVS between sample sites 1, 2, and 3 for a given image before and after normalization. The second application measured the relationship of BVS from sample sites 1 and 2 with an array of data substituted for sample site 3 which consisted of pixel values representing rock outcroppings or barren and rocky soil.

It was assumed that if the normalization techniques were successful, then the spectral response values from the three sample sites would become similar because the sites have the same vegetative cover. The effect of slope and aspect, therefore, would be minimized. For each image the relationship of the spectral responses between forested sample sites was expected to be significantly different from the relationship of spectral responses between the forested-rocky ground sample sites.

To measure the relationship of the BVS between sets of data, a one-way analysis of variance was used. The research hypothesis was that the pixel values from the data sets will be significantly different in all cases. The null hypothesis in the first application was that for each band the samples could have been drawn from the same population (i.e., the mean BVS of sample site 1 = the mean BVS of sample site 2 = the mean BVS of sample site 3). Similarly, the null hypothesis for the second application was that the mean BVS of sample site 1 = sample site 2 = the rocky ground sample site.

A statistical measure of the significance of the reduction in BVS variance between the sample site data obtained from the original and the non-Lambertian normalized images, for a given band, was based on the test for the homogeneity of variances (Zar 1984). The test was defined by

\[
F = \frac{\text{mean square error between (original)}}{\text{mean square error between (normalized)}}
\]

The degrees of freedom in this case could equal 98 for both the numerator and denominator. Instead, one quarter the usual number of degrees of freedom was used to represent non-adjacent pixel value sampling.

From each sample site, 33 pixels were selected for analysis. The 33 pixels representing rocky ground were chosen from known areas of rock outcroppings and from areas of similarly high spectral response. From each single band image, 132 pixel values were extracted.

F statistics were computed from both the forested sample sites data and the forested sample sites-rocky and barren ground
data, for the following images: the original bands 2,3,4, and 5; the ratio band 5/4; and the non-Lambertian normalized bands 2,3,4, and 5.

If the normalization techniques were successful, lower F statistics should result from the analysis of the forested sample sites data sets. Analysis of the forested sample sites-rocky ground data sets should result in higher F statistics, indicating a significant difference due to the variation in cover types.

An F statistic testing for the homogeneity of variances between image sample site values was computed between the original and non-Lambertian normalized bands 2,3,4, and 5. Acceptance of the research hypothesis is an indication of a significant reduction in the variances between original and normalized images.

RESULTS

The Minnaert constants computed from both the entire image data set and the sample area are recorded in Table 1. Original and Minnaert constant normalized band 3,4, and 5 images are displayed in Figures 3a, 3b, 4a, 4b, 5a, and 5b, respectively. Inspection of the photographs reveal the removal of the impression of relief, in the non-Lambertian normalized images (Figures 3b, 4b, and 5b). Also, the areas in which the sample sites are located (marked by 1, 2, and 3 on the images) display a more consistent grey tone in the normalized images than in the originals. The grey shade of the area around sample site 3 in particular becomes more homogenous with the other sites after normalization. The difference in grey tone between sample site 3 and the other two sites in the original images is due in part to the southeastern orientation of the site and its moderately steep slope angle.

The band 4 sample area images are displayed in Figures 6a and 6b. The upper right corner of the images represent a south-western facing slope. This slope has a healthy conifer population, though the canopy coverage is not as dense as the opposing northeastern facing slope located in the lower left portion of the images. Beaver Creek separates the opposing slopes. In the original image (Figure 6a) the presence of the conifer population on the southwestern facing slope (upper right) is not detected, whereas the normalized image (Figure 6b) reveals the existence of the forest. These images display the use of a locally derived Minnaert constant.

F STATISTICS

The results of the one-way analyses of variance at the 95 percent confidence interval are recorded in Table 2. The critical limit for these analyses when the numerator degrees of freedom = 2 and the denominator degrees of freedom = 96 is approximately 3.07. The null hypothesis was rejected in all cases.

The F statistics testing for the homogeneity of variances between images for bands 2,3,4, and 5 are listed in Table 3. At the 95 percent confidence level the F critical limit at 24 degrees of freedom for both the numerator and denominator is 1.98. The null hypothesis was rejected in all cases.

Table 1. Minnaert Constants

<table>
<thead>
<tr>
<th>IMAGE</th>
<th>MINNAERT CONSTANT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Entire Scene</td>
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<tr>
<td>Band 1</td>
<td>0.09</td>
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<tr>
<td>Band 2</td>
<td>0.19</td>
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<td>Band 3</td>
<td>0.31</td>
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<td>Band 4</td>
<td>0.43</td>
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<tr>
<td>Band 5</td>
<td>0.96</td>
</tr>
<tr>
<td>Band 7</td>
<td>0.14</td>
</tr>
</tbody>
</table>

FIG. 3. Thematic Mapper band 3 image of study area. Sample site locations marked by 1,2, and 3: (a) original; (b) non-Lambertian normalized.
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Fig. 4. Thematic Mapper band 4 image of study area. Sample site locations marked by 1, 2, and 3: (a) original; (b) non-Lambertian normalized.

Fig. 5. Thematic Mapper band 5 image of study area. Sample site locations marked by 1, 2, and 3: (a) original; (b) non-Lambertian normalized.

effect of slope and aspect on the brightness values of the pixels in the study area has been significantly reduced.

The F statistics computed from the mixed cover type data files showed an expected increase (Table 2), signifying that, on the non-Lambertian normalized images, differences in cover type could be distinguished.

A comparison of the F statistics generated from the ratio image data and the Minnaert constant normalized data indicated superior results were obtained using the BRCT technique.

Minnaert constants derived from the sample areas varied from the constants derived from the entire scenes (Table 1). Because these Minnaert constants were derived from smaller data sets with a lesser yet sufficient range of incidence and exitance angles, they are believed to be a more accurate measure of the BRDF for the sample areas than the Minnaert constants derived from the corresponding entire scenes.

Qualitatively assessing the sample area image normalized using the non-Lambertian assumption and a Minnaert constant derived from that area reveals an apparently realistic normalization result (Figure 6b).

CONCLUSIONS

Previous studies have developed various aspects of topographic normalization processes for remotely sensed data. The contribution of this paper is based on the development and testing of procedures practicable to implement which normalize Landsat TM imagery with a BRCT based on a non-Lambertian assumption and using a Minnaert constant without a priori knowledge of scene composition. Also, a comparison of this technique made with a band ratio revealed that the BRCT technique was the superior normalizing method. Further, this paper suggests that, after a study area has been located on an image, the procedures could be utilized to process specific areas using a local Minnaert constant to enhance analysis capabilities. Due to the limited size of the study area used in this example, further testing using larger data sets which cover a larger geographical area may be useful.

ACKNOWLEDGMENTS

The author would like to thank Michael Hodgson, who suggested the research, directed a thesis on this topic, and who wrote the NORMAL program. Jon Barbour’s contribution in writing the MINNAERT CONSTANT VARIABLES and linear regression...
programmed, and assistance in the field helped make this project possible. Alex Goetz provided the Landsat data and an early review of the study. Nel Caine, Bob Alexander, and David Hastings provided very valuable ideas and comments. Thomas Wisselgol performed the graphics for Figure 1.

REFERENCES


Table 2. Analysis of Variance Results

<table>
<thead>
<tr>
<th>IMAGE</th>
<th>F STATISTIC</th>
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<tr>
<td>Between Sample Sites</td>
<td>Between Cover Types</td>
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<tr>
<td>Original:</td>
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</tr>
<tr>
<td>Band 2</td>
<td>64.09</td>
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<tr>
<td>Band 3</td>
<td>87.42</td>
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<td>Band 4</td>
<td>651.56</td>
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<tr>
<td>Band 5</td>
<td>132.84</td>
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<td>Ratio:</td>
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<td>Band 5/4</td>
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<td>Non-Lambertian normalized:</td>
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<td>Band 2</td>
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<td>Band 3</td>
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<td>Band 4</td>
<td>31.89</td>
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<td>Band 5</td>
<td>41.35</td>
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Table 3. Results of Analysis of Homogeneity Test Between Original and Non-Lambertian Images

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<td>Band 2</td>
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<td>Band 3</td>
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<td>Band 4</td>
<td>7.04</td>
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<tr>
<td>Band 5</td>
<td>34.92</td>
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Fig. 6. Thematic Mapper band 4 image of sample area. (a) original; (b) non-Lambertian normalized.
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Forum

Topographic Normalization in Rugged Terrain

While Colby (1991) raises some interesting questions about appropriate normalization techniques for rugged terrain, the test statistics used to compare models are unclear. Equation 1 specifies a Minnaert constant model as

$$L(\lambda; \varepsilon) = \ln(\lambda) \cos^{4\lambda} \cos^{4\lambda-1} \varepsilon,$$

where $k(\lambda)$ and $\ln(\lambda)$ are parameters to be estimated by the linear regression applied to a log form of the above equation. Should the model be correctly specified, an assumption required for test statistics to be valid, including the usual additive error term, $\varepsilon$, and subscripts to denote target, $t$, and pixel, $j$, Equation 1 becomes

$$L_t(\lambda; \varepsilon) = \ln_t(\lambda) \cos^{4\lambda} \varepsilon_t \cos^{4\lambda-1} \varepsilon_j + \varepsilon.$$

Linear regression using the log form is only appropriate should the error term be multiplicative, though non-linear least squares would provide similar parameter estimates and hypothesis testing.

Though the author references a one-way analysis of variance to test the hypothesis that the mean corrected radiance variables for each site are equal:

$$H_0: \ln_1 = \ln_2$$
$$H_1: \ln_1 \neq \ln_2,$$

no reference is made as to whether the F-test statistics provided in Tables 2 and 3 test the above hypothesis, or correspond to the homogeneity of variance test given by the author as

$$F = \frac{\text{mean squared error between (original)}}{\text{mean squared error between (normalized)}}.$$

While unequal error variances (between sites) would tend to result in an overstatement of the significance of the equality of means F-tests, the appropriate F-test for equality of variance would be to ratio the mean squared errors for each site (Snedecor and Cochran, 1967). For the F-statistic to be appropriate, the numerator and denominator sum of squares must be independent and $\chi^2$ distributed under the null hypothesis (Morrison, 1976). Should the normalization procedure given by Equation 4 be used to calculate the denominator sum of squares, these conditions would not be met. In general, because a transformation of a random variable $Y$, to $Z = c Y$ results in the $\text{Var}(Z) = c^2 \text{Var}(Y)$, the F-ratio, as it appears to be stated above, would yield an expected value of $\text{Var}(Y)\text{Var}(Z) = 1/a^2$, rather than the expected value of 1. The usual test for equality of means is the F-ratio of the mean sum of squares due to regression to the mean sum of squares due to error (Draper and Smith, 1966). A test to compare the Minnaert model to the original data is possible by adding a constant term, $c_0$, to Equation 1 and possibly overfitting the model:

$$L_t(\lambda; \varepsilon) = c_0 + \ln_t(\lambda) \cos^{4\lambda} \varepsilon_t \cos^{4\lambda-1} \varepsilon_j + \varepsilon.$$

Should the confidence interval for $\ln_t$ contain zero, then evidence supporting the Minnaert model over a simple site average, $c_0$, does not exist. Should the hypothesis that $k(\lambda_1) = k(\lambda_2)$ be rejected, then the data would suggest that band ratioing for bands $\lambda_1$ and $\lambda_2$ is inappropriate.

REFERENCES


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Response

In her comment on the article, "Topographic Normalization in Rugged Terrain," the author raises two points concerning the clarity of the test statistics, and three suggestions regarding applications. The points refer to:

- the nature of the distribution of the error terms resulting from a linear regression using the log form of Equation 1, and
- clarification of the correspondence between statistical tests and the tables of results.

The suggestions refer to:

- the use of 1/a² in the homogeneity of variance test rather than assuming a 1/1 relationship,
- the use of an "overfitted" model, and
- the appropriateness of the band ratio technique if the Minnaert constants for the bands are not equal.

The first point states that linear regression using the log form of Equation 1 is only appropriate should the error term be multiplicative. Equation 1 and the succeeding linear and log transformations were obtained from Smith et al. (1980), and are so referenced. The procedure was used to derive the Minnaert constant (k) by Smith et al. (1980), Justice and Holben (1979), and Justice et al. (1980). The first two articles are referenced in the Second Edition of the Manual of Remote Sensing, in the chapter "Removal of Topographic Effects" (Estes et al., 1983). I am aware of no evidence to believe that the assumption made in previous research regarding the multiplicative nature of the error term in Equation 3 is incorrect. In fact, in Smith et al. (1980), Figure 3 displays the scatter of data resulting from the linear log transformation of Equation 2 (Smith et al., 1980), Equation 1 (Colby, 1991). Inspection of the distribution of residuals in the scatterplot for bands 5 and 7 suggests that the errors would be multiplicative rather than additive.

The second point describes the null and research hypotheses of the one-way analysis of variance as pertaining to the mean corrected radiance values for each site, whereas the F-test statistics for the one-way analyses of variance were derived from the relationship between the three separate sample sites of the digital images of bands 2, 3, 4, and 5. The research and null hypotheses for the one-way analyses are stated in the text of the paper, and the resulting F-statistics are listed in Table 2. The reduction of the F-statistics from the same band images before and after normalization is being highlighted, together with a comparison of the F-statistics derived from the band ratio technique, and between forested sample sites, and forested-rocky ground (different cover type) sample sites. The null and research hypotheses for the homogeneity of variance test are not explicitly stated in the original paper, but are similar as for the one-way analyses of variance; that is,

\[ H_0: \sigma_1^2 = \sigma_2^2 \]
\[ H_A: \sigma_1^2 \neq \sigma_2^2 \]

The F statistics testing for the homogeneity of variances between images for bands 2, 3, 4, and 5 are listed in Table 3.

Addressing the first suggestion, an analysis using 1/a² in place of 1/1 was undertaken using the mean values of cos i and cos e (excluding extremes) for the pixels of sample sites 1, 2, and 3 for band images 2, 3, 4, and 5. The results show a² = 1.12 (band 2), 1.28 (band 3), 1.46 (band 4), and 2.6 (band 5). With F statistic and a² values calculated using Minnaert constants derived from the entire image, the results of the homogeneity of variance tests found that the difference between the original and normalized images were significant at 0.13 (band 2), 0.06 (band 3), 0.12 (band 4), and 0.18 (band 5). However, if the more accurate Minnaert constants derived from the site values were used to both normalize the original image and to calculate a², then the above significance thresholds are expected to decrease, as suggested in Colby (1991). For example, if the Minnaert constant derived from the sample sites for band 3 is used just in the calculation of a², then the threshold drops to 0.03, which would be statistically significant. It should be kept in mind that the test for the homogeneity of variances measures a reduction in variability which is generally being interpreted as a measure of the reduction of the topographic effect. As with the previous comparison of F statistics from the one-way analyses of variance, statistical significance is not a requirement to consider the technique effective.

The suggestion of using the "overfitted" model is already incorporated in determining the slope of the regression line (the Minnaert constant). According to previous research, if the confidence interval for Ln contains 0, then the non-Lambertian assumption is not valid and the Lambertian model may be more appropriate (Smith et al., 1980).

Regarding the final suggestion, the slope of the regression line which determines the Minnaert constant is spectrally dependent; therefore, it is doubtful that the constants for different bands will be the same for one image. Information from the relationship between the spectral reflectance responses in different wavelengths for the same phenomena is sought through band ratios. Requiring equal Minnaert constants as a pre-condition for the use of a band ratio would severely limit the information that could be extracted, and in any case would be a pre-condition that rarely could be met. The band ratio technique may be useful in some applications, for example, visual inspection of an image with some removal of the topographic effect.

REFERENCES


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