Blood Alcohol Concentration: modeling with mathematics

NCCTM Greensboro
October 7, 2004
Hutch Sprunt
Appalachian State University
blood alcohol concentrations and math models

- why this topic?
- what is “blood alcohol concentration”?
- Developing some mathematical models
  - elimination
    - linear
    - rational
  - absorption
    - piecewise defined
  - internet models
Why choose blood alcohol concentration?

• Societal significance

• Teenage relevance
Societal issues:

- Economic costs of alcohol abuse in the U.S. is estimated to be about $200 billion a year
  - Health care expenditures
  - Premature death
  - Impaired productivity
  - Motor vehicle crashes
  - Crime
  - Social welfare
Societal issues:

- Alcohol related motor vehicle crashes kill every 30 minutes (2003)
- 40% of U.S. traffic deaths involve alcohol; 36% of N.C. traffic deaths (2003)
- Estimated costs of alcohol related crashes in 2000 were over $114 b ($51 billion in monetary costs, $63 b in quality of life losses)
Teenage issues:

- Each day roughly 11,000 American youth (age 12 to 20) try alcohol for the first time
- Average age: boys 11  girls 13
- Three leading causes of death for 15- to 24-yr olds are motor vehicle crashes, homicides and suicides--alcohol is a factor in all three
Teenage issues:

- 24% of drivers 15 to 20 years old who died in motor vehicle crashes had been drinking (2002)
- 90% of violent crime on college campuses is alcohol related
- Total costs of underage drinking are about $58 billion per year (1998)
## Costs of Alcohol Use by Youth

<table>
<thead>
<tr>
<th>Category</th>
<th>Cost (1998 dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic Crashes</td>
<td>$18,242,000,000</td>
</tr>
<tr>
<td>Violent Crime</td>
<td>$35,937,000,000</td>
</tr>
<tr>
<td>Burns</td>
<td>$315,000,000</td>
</tr>
<tr>
<td>Drowning</td>
<td>$532,000,000</td>
</tr>
<tr>
<td>Suicide Attempts</td>
<td>$1,512,000,000</td>
</tr>
<tr>
<td>Fetal Alcohol Syndrome</td>
<td>$493,000,000</td>
</tr>
<tr>
<td>Alcohol Poisoning</td>
<td>$340,000,000</td>
</tr>
<tr>
<td>Treatment</td>
<td>$1,008,000,000</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>$58,379,000,000</strong></td>
</tr>
</tbody>
</table>
Why choose modeling?

- NCTM Standards
- North Carolina Standards
  - Advanced Functions and Modeling
### Excerpts from NCTM Standards for 9-12

<table>
<thead>
<tr>
<th>Expectations</th>
<th>Description</th>
</tr>
</thead>
</table>
| **Mathematical Models** | All students should be able to use mathematical models to represent and understand quantitative relationships:  
  - Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationship.  
  - Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts.  
  - Draw reasonable conclusions about a situation being modeled. |
| **Analysis and Change** | All students should be able to:  
  - Analyze change in variable contexts.  
  - Approximate and interpret rates of change from graphical and numerical data. |
| **Problem Solving**   | All students should be able to solve real world problems:  
  - Build new mathematical knowledge through problem solving.  
  - Solve problems that arise in mathematics and in other contexts.  
  - Apply and adapt a variety of appropriate strategies to solve problems.  
  - Monitor and reflect on the process of mathematical problem solving. |
Why choose mathematical modeling?

MATHEMATICS STANDARD COURSE STUDY AND GRADE LEVEL COMPETENCIES K-12

Advanced Functions and Modeling

Advanced Functions and Modeling provides students an in-depth study of modeling and applying functions. Home, work, recreation, consumer issues, public policy, and scientific investigations are just a few of the areas from which applications should originate. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- Describe phenomena as functions graphically, algebraically, and verbally; identify independent and dependent quantities, domain, and range, and input/output.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear, quadratic, cubic, and exponential functions to model and solve problems.
- Use systems of two or more equations or inequalities to solve problems.
- Use the trigonometric ratios to model and solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.
“math sense” materials

• Statistics abound from many sources
  – Variety of forms
  – Opportunity for discussion, exploration
  – Real world data

• Tables/charts/conversions
BLOOD-ALCOHOL CONCENTRATION

* .020 - light to moderate drinkers begin to feel some effects
* .040 - most people begin to feel relaxed
* .060 - judgment is somewhat impaired, people are less able to make rational decisions about their capabilities (for example, driving)
* .080 - there is a definite impairment of muscle coordination and driving skills; this is legal level for intoxication in some states
* .10 - there is a clear deterioration of reaction time and control; this is legally drunk in most states
* .120 - vomiting usually occurs. Unless this level is reached slowly or a person has developed a tolerance to alcohol
* .150 - balance and movement are impaired. This blood-alcohol level means the equivalent of 1/2 pint of whiskey is circulating in the blood stream
* .300 - many people lose consciousness
* .400 - most people lose consciousness; some die
* .450 - breathing stops; this is a fatal dose for most people
## Impaired Driving

<table>
<thead>
<tr>
<th>BAC%</th>
<th>DRINKS CONSUMED</th>
<th>EFFECTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>0.02%</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.05%</td>
<td>2 to 3</td>
<td>1 to 2</td>
</tr>
<tr>
<td>0.08%</td>
<td>3 to 4</td>
<td>2 to 4</td>
</tr>
<tr>
<td>0.10%</td>
<td>3 to 5</td>
<td>2 to 5</td>
</tr>
<tr>
<td>0.15%</td>
<td>4 to 7</td>
<td>3 to 7</td>
</tr>
</tbody>
</table>

One drink = 1 oz. 80 proof spirits = 3 oz. glass of 12% wine = 12 oz. glass of 5% beer

Figures contained in the charts are averages, BAC and effects of alcohol on individuals vary widely.
# BAC Chart

## Male Blood Alcohol Concentration Guide

<table>
<thead>
<tr>
<th>Number of Drinks Per Hour</th>
<th>110</th>
<th>120</th>
<th>140</th>
<th>160</th>
<th>180</th>
<th>200</th>
<th>220</th>
<th>240</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
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<tr>
<td></td>
<td><strong>ONLY SAFE DRIVING LIMIT</strong></td>
<td></td>
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<td></td>
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<tr>
<td>1</td>
<td>.04</td>
<td>.03</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
</tr>
<tr>
<td>2</td>
<td>.08</td>
<td>.06</td>
<td>.05</td>
<td>.05</td>
<td>.04</td>
<td>.04</td>
<td>.03</td>
<td>.03</td>
</tr>
<tr>
<td>3</td>
<td>.11</td>
<td>.09</td>
<td>.08</td>
<td>.07</td>
<td>.06</td>
<td>.06</td>
<td>.05</td>
<td>.05</td>
</tr>
<tr>
<td>4</td>
<td>.15</td>
<td>.12</td>
<td>.11</td>
<td>.09</td>
<td>.08</td>
<td>.08</td>
<td>.07</td>
<td>.06</td>
</tr>
<tr>
<td>5</td>
<td>.19</td>
<td>.16</td>
<td>.13</td>
<td>.12</td>
<td>.11</td>
<td>.09</td>
<td>.09</td>
<td>.08</td>
</tr>
<tr>
<td>6</td>
<td>.23</td>
<td>.19</td>
<td>.16</td>
<td>.14</td>
<td>.13</td>
<td>.11</td>
<td>.10</td>
<td>.09</td>
</tr>
<tr>
<td>7</td>
<td>.26</td>
<td>.22</td>
<td>.19</td>
<td>.16</td>
<td>.15</td>
<td>.13</td>
<td>.12</td>
<td>.11</td>
</tr>
<tr>
<td>9</td>
<td>.34</td>
<td>.28</td>
<td>.24</td>
<td>.21</td>
<td>.19</td>
<td>.17</td>
<td>.15</td>
<td>.14</td>
</tr>
</tbody>
</table>

Driving Skills Impaired:

Legally Drunk in Iowa and most states:
what is blood alcohol concentration?

• Definition
• Primary factors involved
  – Absorption into the blood stream
  – Peak concentration
  – Elimination from the blood stream
What is blood alcohol concentration?

• Ratio of amount of alcohol in the blood to the amount of blood.

• 1 gram of alcohol per kilogram of blood is a ratio of 1/1000 or 0.001 alcohol/blood.

• 0.001 alcohol/blood concentration is 0.10%
Legal maximum BAC in North Carolina is .08% or 0.8 grams/kilogram

- 1 beer = 0.6 oz alcohol = 14 grams
- 150 lb adult = 68 kg
- 63% body weight from blood = 42.8 kg
- $\frac{14}{42800} = 0.000327$ or 0.327 grams/kg
  or .0327%
- 2.5 beers produces BAC of about 0.083%
BAC measured by breath

• Breath alcohol to blood alcohol ratio is 2100:1

• By volume, 2,100 ml of deep lung air will contain the same alcohol as 1ml of blood

• BAC 0.08 means there are 0.0008 grams of alcohol per ml of blood or per 2100 ml breath
Primary factors affecting peak BAC

• Consumption
  – Amount and it’s concentration
  – Time period
  – Personal habits

• Body factors
  – Gender
  – Height/weight
  – Stomach contents
  – Age (males)
• Alcohol is a central nervous system depressant

• Degree of impairment is directly proportional to concentration of alcohol in blood
Absorption of Alcohol

• Begins immediately in mouth, esophagus

• Stomach

• Small intestine
If stomach is empty:

- 20-25% of alcohol absorbed through the stomach
- 75-80% absorbed through the small intestine
Peak alcohol concentration:

- 30-45 minutes with empty stomach
- 60-90 minutes with food in stomach
Elimination of alcohol

- 95% metabolized by liver
- 5% expelled through breath and other bodily functions
Elimination of alcohol

- Healthy liver metabolizes about 0.5 oz = 15 ml of alcohol per hour

- Once saturated, BAC is actually a measure of how intake rate has exceeded outgo rate
Elimination models

BAC(t)
BAC as a function of time (hours) since peak alcohol concentration
Liver metabolizes alcohol at a constant rate (as opposed to the way kidneys eliminate chemicals)

- 0.5 oz $\approx$ 15 ml $\approx$ 14 grams of alcohol per hour metabolized
- Reduction of BAC by about 0.017 per hour
- BAC as a function of time, $t$, hours since peak concentration, $P$:

$$\text{BAC}(t) = P - 0.017t$$
150 lb adult with 2.5 beers in an hour:

- Peak 0.5 hours after drinking with
- 34 grams alcohol yielding 0.08 BAC
- Eliminates at rate of 0.017 BAC per hour:

\[
BAC(t) = 0.08 - 0.017t
\]
blood alcohol concentration over time
2.5 beers in an hour

BAC

0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01

0 1 2 3 4 5 6

time since peak (hrs)
Exploration:

How long until all alcohol is removed from the bloodstream?

solve $B(t) = 0$ to find $t = 4.7$ hours
You “feel” alcohol’s effects at a BAC of 0.01. How long will he “feel” the effects?

solve $B(t) = 0.01$ to find $t = 4.1$ hours
Suppose he had 3 beers in the hour?
Peak BAC is now about 0.104

$$BAC(t) = 0.104 - 0.017t$$

![Blood alcohol concentration over time graph](image)
How long until all alcohol is removed from the blood stream?

solve $B(t) = 0$ to find $t = 6.1$ hours

• You “feel” alcohol’s effects at a BAC of 0.01. How long will he “feel” the effects?
  
solve $B(t) = .01$ to find $t = 5.5$ hours
How long until all he can safely, no, legally drive?
Solve $BAC(t) = 0.08$ to find $t = 1.4$ hours
150 lb man drinking for one hour:

<table>
<thead>
<tr>
<th># beers</th>
<th>Peak BAC</th>
<th>Time until legally drive</th>
<th>Time until alc effects are gone</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.03</td>
<td>0</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>0.07</td>
<td>0</td>
<td>2.9</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>1.2</td>
<td>4.7</td>
</tr>
<tr>
<td>4</td>
<td>0.13</td>
<td>2.9</td>
<td>7.1</td>
</tr>
<tr>
<td>5</td>
<td>0.16</td>
<td>4.7</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Peak BAC is 30-45 minutes after the hour of drinking
Waiting for reduced BAC

- Time until legally drive
- Time until alc effects are gone

# beers in an hour vs. hours since peak
In contrast to liver and alcohol, caffeine elimination is a good study:

- Kidneys eliminate a proportion of a chemical over a time period
- After peak, kidneys eliminate 13% of the caffeine in the body each hour

Percent of peak amount, \( P \), of caffeine in body as function of time, \( t \), in hours since peak:

\[
C(t) = P(0.87)^t
\]
In an 8-ounce cup of COFFEE:
- Drip: 165 mg
- Brewed: 130 mg
- Instant: 95 mg
- Decaffeinated: 4 mg

In an 8-ounce cup of TEA:
- Brewed: 45 mg
- Instant: 35 mg
- Green tea: 30 mg
In a 12-ounce can of SODA:
- Coca Cola: 45.6 mg
- Diet Coke: 45.6 mg
- Surge: 51.0 mg
- Dr. Pepper: 39.6 mg
- Pepsi: 37.2 mg
- Diet Pepsi: 35.4 mg
- Mountain Dew: 55 mg

In a 1.5-ounce CHOCOLATE BAR:
- Hershey’s Special Dark: 31 mg
- Hershey Bar (milk choc): 10 mg
Caffeine Elimination by Kidneys
3 cups of drip coffee

Caffeine in body (mg) vs. hours since peak amount.
A different alcohol elimination model:

• % of alcohol metabolized by the liver depends on the amount of alcohol in the body…not a constant proportion as with kidneys

• As the amount of alcohol in the body increases, the proportion eliminated in an hour decreases
• Let $p$ be the proportion of alcohol broken down in a given hour
• Let $a$ be the number of grams of alcohol in the body at the beginning of the hour

\[ p = \frac{10}{4.2 + a} \]

(constants vary with individual)

• This is called capacity-limited metabolism
<table>
<thead>
<tr>
<th>hrs since peak</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (start conc grams)</td>
<td>42.00</td>
<td>32.91</td>
<td>24.04</td>
<td>15.53</td>
<td>7.66</td>
</tr>
<tr>
<td>p (prop removed)</td>
<td>0.22</td>
<td>0.27</td>
<td>0.35</td>
<td>0.51</td>
<td>0.84</td>
</tr>
</tbody>
</table>

**proportion of alc removed in an hour**

![Graph showing the proportion of alcohol removed in an hour against the initial alcohol amount.](image_url)
<table>
<thead>
<tr>
<th>hrs since peak</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>0.35</td>
<td>0.51</td>
<td>0.84</td>
</tr>
</tbody>
</table>

**capacity limited metabolism**

- **Starting alcohol amount (grams)**
- **Proportion of alcohol removed**

**Graph:**
- Blue line: a (start conc grams)
- Pink line: p (prop removed)
<table>
<thead>
<tr>
<th>hrs since peak</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
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<td>0.22</td>
<td>0.27</td>
<td>0.35</td>
<td>0.51</td>
<td>0.84</td>
</tr>
<tr>
<td>amount alc removed (gr)</td>
<td>9.09</td>
<td>8.87</td>
<td>8.51</td>
<td>7.87</td>
<td>6.46</td>
</tr>
</tbody>
</table>

**Amount alc removed (gr) over time**

- **Amount alc removed (gr)**: The graph shows the amount of alcohol removed over time, with the x-axis representing hours since peak concentration and the y-axis representing grams removed that hour. The data points indicate a decrease in the amount of alcohol removed over time, suggesting a reduction in alcohol concentration.

- **Graph Analysis**:
  - The initial peak concentration is evident at the start (hrs since peak = 1).
  - As time progresses (hrs since peak = 5), the amount of alcohol removed (gr) decreases steadily, indicating effective removal processes.
  - The trend suggests a logarithmic decrease in alcohol concentration, which is common in biological systems where the rate of removal reduces over time.

- **Implications**:
  - Understanding this rate of removal is crucial for predicting the time it takes to achieve a safe alcohol concentration.
  - This data can inform strategies for treatment and rehabilitation programs.

- **Conclusion**:
  - The data and graph provide a clear visual representation of how alcohol concentration decreases over time, emphasizing the importance of monitoring and managing alcohol intake.
Absorption of alcohol

Peak concentration in

- 30-45 minutes if stomach empty
- 60-75 minutes if food in stomach
Linear model of absorption and elimination
150 lb man; 3 beers in an hour

BAC: 150 lb. man; 3 beers in one hour
Linear model of absorption and elimination
150 lb man; 3 beers in an hour

Piecewise defined function: BAC as a function of time, t hours, since started drinking

\[
BAC(t) = \begin{cases} 
0.07t & 0 < t \leq 1.5 \\
0.07 \times 1.5 - 0.017(t - 1.5) & t > 1.5
\end{cases}
\]
Peak BAC depends on:

- Amount and concentration of alcoholic drink
- Time span and rate of consumption
- Type and amount of food in stomach
- Drinking habits of the individual
- Medical conditions and/or other drugs
- Gender
- Physique (height, weight, body type)
- Age (males)
BAC is a factor of total body water concentration

- **Women**
  - 55% +/- 5.5% body weight is water

- **Men**
  - 68% +/- 8.5% of body weight is water
Effects of total body water (TBW) differences on BAC

- **Gender**

**BAC for male/female same size people; 3 beers in 1 hour**

![BAC graph](image)

- **BAC**
- **time since started drinking (hr)**

- **Male BAC**
- **Female BAC**
Effects of total body water (TBW) differences on BAC

• Weight
  – More weight, higher TBW, lower BAC

• Age (males)
  – Older, less TBW, higher BAC

• Height
  – More height, more TBW, lower BAC
A Total Body Water model:

TBW as a function of gender, age (males), height, weight

Male TBW(liters) =
2.447 - 0.09516Age(yrs) + 0.1074ht(cm) + 0.3362wt(kg)

Female TBW = -2.097 + 0.1069ht + 0.2466wt
Interactive models on the internet

- http://www.prevlink.org/therightstuff/community/LB166/BAC.html
- http://www.lastcall.org/topics/bac_calculator.htm
- http://www.ncsec.org/team1/applet.htm
Blood Alcohol Content Calculator

This program provides an estimate of Blood Alcohol Concentration (BAC) given information on weight, gender, number of drinks consumed and the time from when drinking was initiated until the present.

The estimate of BAC should not be used by individuals to decide whether or not to drive after drinking -- impairment can result from any amount of alcohol.

Each 'drink' entered into the formula represents .54 oz alcohol, which is based on a 12 oz beer containing 4.5% alcohol by volume. The BAC Estimator can be used with any alcoholic beverage provided you know the amount of alcohol consumed: divide the number of ounces of alcohol by .54 and enter the result instead of the number of beers. For example, if the user's beverage contained .75 oz of alcohol per drink, divide .75 by .54. Thus, each drink corresponds to 1.39 beers. Multiply this number by the number of drinks to find the total number to be entered. If the user consumed two drinks, the result would be equivalent to 2.78 beers.
### BAC Calculator

<table>
<thead>
<tr>
<th>Weight:</th>
<th>150 ‼️</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drinks:</td>
<td>4 ‼️</td>
</tr>
<tr>
<td>Hours:</td>
<td>1 ‼️</td>
</tr>
<tr>
<td>Gender:</td>
<td>Female ‼️</td>
</tr>
<tr>
<td>Calc BAC</td>
<td>0.1099</td>
</tr>
</tbody>
</table>

**Breath-Alcohol Level**  
**Blood Alcohol Level (BAC)**
If you move a slider bar you will immediately see the model run and the graph change.
The last model uses “Stella” software: http://www.ncsec.org/team1/model.htm
These slides are available in .pdf format at http://www.appstate.edu/~spruntwh

or send an email to spruntwh@appstate.edu

thanks for coming
Underlying message to students about alcohol and drugs?

Just say know....

Knowledge is the most powerful tool we have.