BEHAVIOR POSTULATES AND COROLLARIES—1949

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The matter of isolating and formulating a set of quantitative postulates or mathematical primary principles upon which may be based a true natural-science theory, one designed to mediate the deduction of a system as complex as that concerning mammalian behavior, is a formidable undertaking. To write out the blank form of an equation such as $x = f(y)$ is quite simple, but careful planning and a certain amount of skill are often required in the determination of what the function actually is and the approximate values of the numerical constants contained in it. Moreover, suitable units of quantification must be devised. There must ordinarily be added a preliminary checking of the consistency of the formulation with known empirical facts. All this takes much time and effort. The detailed history of such a process would itself require a fair-sized volume.

Very briefly, the present writer's procedure may be described somewhat as follows. He begins with selecting two or three principles, often isolated by earlier workers, from the complex of data involving a certain class of experiments. These are generalized and quantified as tentative equations. The attempt is then made to apply the equations to a wider range of phenomena. So long as the application of these equations agrees with empirical fact they are retained. But when it seems that a given combination of formulated principles ought to mediate the deduction of an empirically known phenomenon but does not do so, the principles in question are reëxamined to the end that with the possible revision of one or more of them they will yield the deduction sought and still mediate the true deductions already to the credit of the critical postulates.

Sometimes the deduction failure may appear to be the result of ignoring a law not yet formulated. An example of this in the present postulate set concerns the matter of stimulus-intensity dynamism (7), which appears as Postulate VI. In such a case an empirical situation is sought in the literature, where all the factors are held constant except the significant ones in question; the data which are found are plotted and an equation is fitted to these values. Usually such data are defective in one way or another, though if facilities are not available to set up a specific experiment they will be used as a first approximation. The provisional equation so secured is then tried out deductively in more general situations with other principles which presumably are also operating with it. Thus it may be seen that a large element of trial and error is involved in the process. It is to be expected that failures among such trials would be relatively more frequent in a young and fast-moving science than in an older and more stable one.

A detailed record of the theory trials made by the present writer in the behavior field is scattered through twenty-six volumes of handwritten notebooks. This series began in 1915 and extends to the present time. The system grew very slowly, especially during the early years. The first published material in the series appeared in 1929 (1). As the system took on more definite form the author began to present to his seminar groups from time to time somewhat formalized statements regarding special problems. These statements were made in the form of mimeographed memoranda which were later assembled and substantially bound. Four separate volumes of such bound memoranda are on deposit in the Yale University library and in the libraries of several other universities where a certain amount of interest has been manifested in such matters. The bound memoranda extend over the academic years 1936–1938, 1939–1940, 1940–1944, and 1945–1946 (**8**, **3**, **4**, **6**).

From time to time during the compilation of the first three of these volumes the principles which seemed most promising at the moment were gathered together in a numbered series, sometimes accompanied by a few deductions based on the postulates in question. The dates of the chief of these series follow, with the pages of the volume where they may be found:

December 2, 1937, pp. 115–116.
May 3, 1939, pp. 24–29.
January 27, 1940, pp. 40–41; 45; 47–48.
February 10, 1940, pp. 49–54.
February 12, 1940, pp. 55–59.
April 30, 1940, pp. 111–117.
July 8, 1941, pp. 44–52.
December 28, 1942, pp. 163–166.
July 13, 1943, pp. 167–170.
December 20, 1943, pp. 176–177.

On September 4, 1936, the writer used a much abbreviated version of an earlier set of postulates which was substantially like that listed above under date of December 2, 1937, as the basis of an address given by him as retiring president of the American Psychological Association. This was published in routine form (**2**) in January, 1937. The next published version of the system appeared in the joint volume, *Mathematico-Deductive Theory of Rote Learning* (**9**). The most recently preceding published form is presented in the bold-faced type sections scattered through the volume, *Principles of Behavior* (**5**).

During the years since the publication of *Principles of Behavior* numerous reasons for changes and modifications in the system as there presented have been revealed. These arose through a careful study of experimental results from Yale laboratories and from laboratories in other institutions, numerous thoughtful theoretical criticisms, and the writer's use of the postulates of the system in making concrete deductions of the systematic details of individual (non-social) behavior in connection with a book which he is now writing on this subject. As a consequence, the mathematical aspects of many of the postulates have been formulated, or reformulated, and the verbal formulation of nearly all has been modified to a certain extent. One postulate (**5**, p. 319) has been dropped in part as empirically erroneous, some postulates have been divided, and others have been combined; several new postulates have been added, and a number of the original postulates have been deduced from others of the present set and now appear as corollaries. The net result is an increase of from sixteen to eighteen postulates, with twelve corollaries.

The same sort of revision as that just described is certain to be necessary in the case of the present set of behavior postulates and corollaries. This is true of all natural-science theories; they must be continually checked against the growing body of empirical fact. In order to facilitate this winnowing and expanding process, the postulates and corollaries as of November, 1949, are here presented in an unbroken sequence.

POSTULATE I

Unlearned Stimulus-Response Connections (${}_SU_R$)

Organisms at birth possess receptor-effector connections (${}_SU_R$) which under combined stimulation (S) and drive (D) have the potentiality of evoking a hierarchy of responses that either individually or in combination are more likely to terminate the need than would be a random selection from the reactions resulting from other stimulus and drive combinations.

POSTULATE II

Molar Stimulus Traces (s') *and Their Stimulus Equivalents* (S')

A. When a brief stimulus (S) impinges on a suitable receptor there is initiated the recruitment phase of a self-propagating molar afferent trace impulse ($\dot{s}'$), the molar stimulus equivalent ($\dot{S}'$) of which rises as a power function of time ($\dot{t}$) since the termination of the stimulus, *i.e.*,

$$\dot{S}' = A\dot{t}^a + 1.0,$$

$\dot{S}'$ reaching its maximum (and termination) when $\dot{t}$ equals about .450″.

B. Following the maximum of the recruitment phase of the molar stimulus trace, there supervenes a more lengthy subsident phase ($\underset{\cdot}{s}'$), the stimulus equivalent of which descends as a power function of time ($\underset{\cdot}{t}'$), *i.e.*,

$$\underset{\cdot}{S}' = B(\underset{\cdot}{t}' + c)^{-b},$$

where $\underset{\cdot}{t}' = \underset{\cdot}{t} - .450''$.

C. The intensity of the molar stimulus trace (s') is a logarithmic function of the molar stimulus equivalent of the trace, *i.e.*,

$$s' = \log S'.$$

POSTULATE III

Primary Reinforcement

Whenever an effector activity (R) is closely associated with a stimulus afferent impulse or trace (s') and the conjunction is closely associated with the diminution in the receptor discharge characteristic of a need, there will result an increment to a tendency for that stimulus to evoke that response.

Corollary i

Secondary Motivation

When neutral stimuli are repeatedly and consistently associated with the evocation of a primary or secondary drive and this drive undergoes an abrupt diminution, the hitherto neutral stimuli acquire the capacity to bring about the drive stimuli (S_D) which thereby become the condition (C_D) of a secondary drive or motivation.

Corollary ii

Secondary Reinforcement

A neutral receptor impulse which occurs repeatedly and consistently in close conjunction with a reinforcing state of affairs, whether primary or secondary, will itself acquire the power of acting as a reinforcing agent.

POSTULATE IV

The Law of Habit Formation (${}_SH_R$)

If reinforcements follow each other at evenly distributed intervals, everything else constant, the resulting habit will increase in strength as a positive growth function of the number of trials according to the equation,

$${}_SH_R = 1 - 10^{-a\dot{N}}.$$

POSTULATE V

Primary Motivation or Drive (D).[1]

A. Primary motivation (D), at least that resulting from food privation, consists of two multiplicative components, (1) the drive proper (D') which is an increasing monotonic sig-

[1] This postulate, especially parts A, B, and C, is largely based on the doctoral dissertation of H. G. Yamaguchi (11). It is used here by permission of Dr. Yamaguchi.

moid function of h, and (2) a negative or inanition component (ϵ) which is a positively accelerated monotonic function of h decreasing from 1.0 to zero, *i.e.*,

$$D = D' \times \epsilon.$$

B. The functional relationship of drive (D) to one drive condition (food privation) is: from $h = 0$ to about 3 hours drive rises in an approximately linear manner until the function abruptly shifts to a near horizontal, then to a concave-upward course, gradually changing to a convex-upward curve reaching a maximum of 12.3 σ at about $h = 59$, after which it gradually falls to the reaction threshold ($_SL_R$) at around $h = 100$.

C. Each drive condition (C_D) generates a characteristic drive stimulus (S_D) which is a monotonic increasing function of this state.

D. At least some drive conditions tend partially to motivate into action habits which have been set up on the basis of different drive conditions.

POSTULATE VI

Stimulus-Intensity Dynamism (V)

Other things constant, the magnitude of the stimulus-intensity component (V) of reaction potential ($_SE_R$) is a monotonic increasing logarithmic function of S, *i.e.*,

$$V = 1 - 10^{-a \log S}.$$

POSTULATE VII

Incentive Motivation (K)

The incentive function (K) is a negatively accelerated increasing monotonic function of the weight (w) of food given as reinforcement, *i.e.*,

$$K = 1 - 10^{-a\sqrt{w}}.$$

POSTULATE VIII

Delay in Reinforcement (J)[2]

The greater the delay in reinforcement, the weaker will be the resulting reaction potential, the quantitative law being,

$$J = 10^{-jt}.$$

POSTULATE IX

The Constitution of Reaction Potential ($_SE_R$)

The reaction potential ($_SE_R$) of a bit of learned behavior at any given stage of learning is determined (1) by the drive (D) operating during the learning process multiplied (2) by the dynamism of the signaling stimulus at response evocation (V_2), (3) by the incentive reinforcement (K), (4) by the gradient of delay in reinforcement (J), and (5) by the habit strength ($_S\underset{\cdot}{H}_R$), *i.e.*,

$$_SE_R = D \times V \times K \times J \times {_S\underset{\cdot}{H}_R}.$$

where

$$_S\underset{\cdot}{H}_R = {_SH_R} \times V_1$$

and V_1 represents the stimulus intensity during the learning process.

Corollary iii

The Behavioral Summation ($\dot{+}$) *of Two Reaction Potentials,* $_SE_R$ *and* $_S\underline{E}_R$

If two stimuli, S' and S, are conditioned separately to a response (R) by $\dot{N}'$ and $\dot{N}$ reinforcements respectively, and the $_SE_R$ generalizes from S' to S in the amount of $_S\underline{E}_R$, the summation ($\dot{+}$) of the two reaction potentials at S will be the same as would result for the equiva-

[2] It is probable that this postulate ultimately will be deduced from other postulates, including II B and VI; thus it will become a corollary. In that case the phenomena represented by J would be taken over in IX by $_SH_R$, just as $_S\bar{H}_R$ now is.

lent number of reinforcements at S, *i.e.*,

$$_SE_R \dot{+} {_S\underline{E}_R}$$
$$= {_SE_R} + {_S\underline{E}_R} - \frac{_SE_R \times {_S\underline{E}_R}}{M},$$

where M is the asymtote of $_SE_R$.

Corollary iv

The Withdrawal of a Smaller Reaction Potential ($_SE_R$) *from a Larger One* (C)

If $C = {_SE_R} \dot{+} {_SE'_R}$, then

$$_SE_R = \frac{M(C - {_SE'_R})}{M - {_SE'_R}}.$$

Corollary v

The Behavioral Summation ($\dot{+}$) *of Two Habit Strengths,* $_SH_R$ *and* $_S\bar{H}_R$

Since the asymptote of $_SH_R$ is 1.0,

$$_SH_R \dot{+} {_S\bar{H}_R}$$
$$= {_SH_R} + {_S\bar{H}_R} - {_SH_R} \times {_S\bar{H}_R}.$$

Corollary vi

The Withdrawal of a Smaller Habit Strength ($_SH_R$) *from a Larger One* (C)

If

$$C = {_SH_R} \dot{+} {_SH'_R},$$

then

$$_SH_R = \frac{C - {_SH'_R}}{1 - {_SH'_R}}.$$

Corollary vii

The Problem of the Behavioral Summation ($\dot{+}$) *of Incentive Substances* (K)

If two incentive substances, f and a, have as the exponential components of their respective functional equations $A\sqrt{w}$ and $B\sqrt{m}$, the second substance will combine ($\dot{+}$) with the first in the production of the total K by taking as the exponent of the new formula: the simple addition of the units of the first substance to the product of the units of the second substance multiplied by the quotient obtained by dividing the square of the numerical portion of the second exponent by the square of the numerical portion of the first exponent, *i.e.*, in regard to exponents,

$$w \dot{+} m = w + m \times \frac{B^2}{A^2}.$$

Corollary viii

The Problem of the Behavioral Summation ($\dot{+}$) *of Stimulus-Intensity Dynamism* (V)

If two stimulus aggregates (S and S') are each scaled in terms of the absolute threshold of the stimulus in question for the subject involved, the stimulus-intensity dynamism (V) of the compound will be the simple summation of the scaled intensity values as substituted in the equation, *i.e.*,

$$V_S \dot{+} V_{S'} = 1 - 10^{-v \log(S+S')}.$$

POSTULATE X

Inhibitory Potential

A. Whenever a reaction (R) is evoked from an organism there is left an increment of primary negative drive (I_R) which inhibits to a degree according to its magnitude the reaction potential ($_SE_R$) to that response.

B. With the passage of time since its formation, I_R spontaneously dissipates approximately as a simple decay function of the time (t) elapsed, *i.e.*,

$$I'_R = I_R \times 10^{-at}.$$

C. If responses (R) occur in close succession without further reinforcement, the successive increments of inhibition (ΔI_R) to these responses summate to attain appreciable amounts of I_R. These also summate with $_SI_R$ to make up an inhibitory aggregate ($\dot{I}_R$), *i.e.*,

$$\dot{I}_R = I_R \dot{+} {_SI_R}.$$

D. When experimental extinction occurs by massed practice, the $\dot{I}_R$

present at once after the successive reaction evocations is a positive growth function of the order of those responses ($\dot{n}$), *i.e.*,

$$\dot{I}_R = a(1 - 10^{-b\dot{n}}).$$

E. For constant values of super-threshold reaction potential ($_S\dot{E}_R$) set up by massed practice, the number of unreinforced responses (n) producible by massed extinction procedure is a linear decreasing function of the magnitude of the work (W) involved in operating the manipulanda, *i.e.*,

$$n = A(a - bW).$$

Corollary ix

Stimuli and stimulus traces closely associated with the cessation of a given activity, and in the presence of appreciable I_R from that response, become conditioned to this particular non-activity, yielding conditioned inhibition ($_SI_R$) which will oppose $_SE_R$'s involving this response, the amount of $\Delta {_SI_R}$ generated being an increasing function of the I_R present.

Corollary x

For a constant value of n, the inhibitory potential ($\dot{I}_R$) generated by the total massed extinction of reaction potentials set up by massed practice begins as a positively accelerated increasing function of the work (W) involved in operating the manipulandum, which gradually changes to a negative acceleration at around 80 grams, finally becoming asymptotic at around 110 grams.

Corollary xi

For a constant value of the work (W) involved in operating the manipulandum, the inhibitory potential ($\dot{I}_R$) generated by the massed total extinction of reaction potentials set up by massed practice is a negatively accelerated increasing function of the total number of reactions (n) required.

POSTULATE XI

Stimulus Generalization ($_S\bar{H}_R$, $_S\underline{E}_R$, *and* $_S\underline{I}_R$)

A. In the case of qualitative stimuli, S_1 and S_2, the effective habit strength ($_S\bar{H}_R$) generates a stimulus generalization gradient on the qualitative continuum from the simple learned attachment of S_1 to R:

$$_{S_2}\bar{H}_R = {_{S_1}H_R} \times 10^{-ad},$$

where d represents the difference between S_1 and S_2 in j.n.d.'s, and

$$_{S_2}\underline{E}_R = D \times V \times K \times J \times {_{S_1}\bar{H}_R},$$

where $D \times V \times K \times J$ are constant.

B. A stimulus intensity (S_1) generalizes to a second stimulus intensity (S_2) according to the equation,

$$_{S_2}\bar{H}_R = {_{S_1}H_R} \times 10^{-bd},$$

where d represents the difference between log S_1 and log S_2 and

$$_{S_2}\underline{E}_R = ({_{S_1}\bar{H}_R} \times V_2)(D \times K \times J),$$

where ($D \times K \times J$) are constant and V_2 is the stimulus-intensity dynamism of S_2.

C. In the case of qualitative stimulus differences, ordinary conditioning and extinction spontaneously generate a gradient of effective inhibitory potential ($_S\underline{I}_R$) which is a negative growth function of $_SI_R$ and d, *i.e.*,

$$_{S_2}\underline{I}_R = {_{S_1}I_R} \times 10^{-ad},$$

and in the case of stimulus-intensity differences,

$$_{S_2}\underline{I}_R = {_{S_1}I_R} \times 10^{-bd} \times V_2.$$

Corollary xii

When a habit is set up in association with a given drive intensity and its strength is tested under a different drive intensity, there will result a falling gradient of $_S\bar{H}_R$ and $_S\underline{E}_R$.

POSTULATE XII

Afferent Stimulus Interaction

All afferent impulses (s's) active at any given instant mutually interact, converting each other into $\breve{s}$'s which differ qualitatively from the original s's so that a reaction potential ($_sE_R$) set up on the basis of one afferent impulse (s) will show a generalization fall to $_{\breve{s}}E_R$ when the reaction (R) is evoked by the other afferent impulse ($\breve{s}$), the amount of the change in the afferent impulses being shown by the number of j.n.d.'s separating the $_sE_R$'s involved according to the principle,

$$d = \frac{\log \frac{_sE_R}{_{\breve{s}}E_R}}{J}.$$

POSTULATE XIII

Behavioral Oscillation

A. Reaction potential ($_SE_R$) oscillates from moment to moment, the distribution of $_SO_R$ deviating slightly from the Gaussian probability form in being leptokurtic with β_2 at about 4.0, *i.e.*, the form of the distribution is represented by the equation,[3]

$$y = y_0 \frac{1}{\left(1 + \frac{x^2}{a^2}\right)^m}.$$

B. The oscillation of $_SE_R$ begins with a dispersion of approximately zero at the absolute zero (Z) of $_SH_R$, this at first rising as a positive growth function of the number of subthreshold reinforcements (N) to an unsteady maximum, after which it remains relatively constant though with increasing variability.

C. The oscillations of competing reaction potentials at any given instant are asynchronous.

[3] This equation is taken from **10**, p. lxiii.

POSTULATE XIV

Absolute Zero of Reaction Potential (Z) and the Reaction Threshold ($_SL_R$)

A. The reaction threshold ($_SL_R$) stands at an appreciable distance (B) above the absolute zero (Z) of reaction potential ($_SE_R$), *i.e.*,

$$_SL_R = Z + B.$$

B. No reaction evocation (R) will occur unless the momentary reaction potential at the time exceeds the reaction threshold, *i.e.*, unless,

$$_S\dot{\bar{E}}_R > {_SL_R}.$$

Corollary xiii

The Competition of Incompatible Reaction Potentials ($_S\bar{E}_R$)

When the net reaction potentials ($_S\bar{E}_R$) to two or more incompatible reactions (R) occur in an organism at the same instant, each in a magnitude greater than $_SL_R$, only that reaction whose momentary reaction potential ($_S\dot{\bar{E}}_R$) is greatest will be evoked.

POSTULATE XV

Reaction Potential ($_SE_R$) as a Function of Reaction Latency ($_St_R$)

Reaction potential ($_SE_R$) is a negatively accelerated decreasing function of the median reaction latency ($_St_R$), *i.e.*,

$$_SE_R = a\, {_St_R}^{-b}.$$

POSTULATE XVI

Reaction Potential ($_SE_R$) as a Function of Reaction Amplitude (A)

Reaction potential ($_SE_R$) is an increasing linear function of the Tarchanoff galvanic skin reaction amplitude (A), *i.e.*,

$$_SE_R = cA.$$

POSTULATE XVII

Complete Experimental Extinction (n) as a Function of Reaction Potential ($_SE_R$)

A. The reaction potentials ($_SE_R$) acquired by massed reinforcements are a negatively accelerated monotonic increasing function of the median number of massed unreinforced reaction evocations (n) required to produce their experimental extinction, the work (W) involved in each operation of the manipulandum remaining constant, *i.e.*,

$$_SE_R = a(1 - 10^{-bn}) + c.$$

B. The reaction potentials ($_SE_R$) acquired by quasi-distributed reinforcements are a positively accelerated monotonic increasing function of the median number of massed unreinforced reaction evocations (n) required to produce their experimental extinction, the work (W) involved in each operation of the manipulandum remaining constant, *i.e.*,[4]

$$_SE_R = a \times 10^{bn} + c.$$

POSTULATE XVIII

Individual Differences

The "constant" numerical values appearing in equations representing primary molar behavioral laws vary from species to species, from individual to individual, and from some physiological states to others in the same individual at different times, all quite apart from the factor of behavioral oscillation ($_SO_R$).

[4] The equation of XVII B is regarded with more than usual uncertainty. Fortunately the true function can be determined by a straightforward empirical procedure.

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