Snell’s Law & Critical Refraction

- Because seismic sources radiate waves in all directions
- Some ray must hit interface at exactly the critical angle, $i_c$
- This critically oriented ray will then travel along the interface between the two layers
- If more oblique than critical, all wave energy is reflected
  - The reflected energy is useful too!
    - E.g. Chapter 7

\[
\frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}
\]

\[
\frac{\sin i_c}{v_1} = \sin 90
\]

\[
\sin i_c = \frac{v_1}{v_2}
\]

\[
i_c = \arcsin \left( \frac{v_1}{v_2} \right)
\]
Critical Refraction and Wave Fronts

- When a ray meets a new layer at the critical angle...
  - The ray travels along the interface
    - What layer is it in?
- Rays, aren’t real, so consider the wave fronts...
  - Wave fronts travel in both layers
  - Wave front in top continues on the same trajectory
  - Wave front in Bottom has to be perpendicular to the ray
  - But the layers have different velocities
  - This sets up wavelets and head waves...
Huygens’s Principle

• Recall that rays are not real
  – They are just an easy way to understand and quantify waves

• Wave fronts are what is really happening
  – But what causes wave fronts?
    • Huygens’s wavelets explains...
      – Each point along a material is acts like a point source of waves
      – Like a pebble dropped into water
Huygen's Wavelets

- Huygens (a 17th century Dutch physicist) realized that:
  - When any particle oscillates it is a tiny source of waves
    - So, every point on a wave front acts as a small source that generates waves
    - The waves have circular (spherical) wave fronts and are called wavelets
    - Wavelets constructively interact (reinforcement) to produce the wave front
  - Has important implications for diffraction and critical refraction
Wavelets and Diffraction

- If wavelets didn’t occur, we wouldn’t be able to hear around corners.
  - Light doesn’t travel around corners very well because of its very high frequency.

If only there were wavelets... then I could hear you.

What Up Dr. Kate??
Wavelets and Diffraction

• Because of wavelets, a wave front that encounters an obstacle:
  – Will travel through the open space
  – The wave front after the barrier diffracts, or bends into an area that is predicted to be a shadow by ray theory.

• But what about critical refraction?? *(java animation)*

---

New Wave Front

Final Wave Front

Wasuuuup!

Wavelets

What Up Dr. Kate??
Wavelets and Head Waves

• The wave front just above the interface produces a continual stream of critically refracted rays.
• The wave front just below the interface does the same.
• These streams of critically refracted rays form wavelets.
• The wavelets combine to form head waves.
  - The head waves propagate up to the surface and can be recorded.

(a) S • The recorded rays are called the refracted rays.

(b) wavelets
  refracted ray
  head waves
Potential Paths in a Refraction Survey

- When doing a seismic refraction survey, a recorded ray can come from three main paths
  - The direct ray
  - The reflected ray
  - The refracted ray
- Because these rays travel different distances and at different speeds, they arrive at different times
- The direct ray and the refracted ray arrive in different order depending on distance from source and the velocity structure
The Time-Distance (t-x) Diagram

Think about:

- What would a fast velocity look like on this plot?
- Why is direct ray a straight line?
- Why must the direct ray plot start at the origin (0,0)?
- Why is refracted ray straight line?
- Why does refracted ray not start at origin?
- Why does reflected ray start at origin?
- Why is reflected ray asymptotic with direct ray?

Understanding the t-x Diagram is Key!!!
The Direct Ray

- The Direct Ray Arrival Time:
  - Simply a linear function of the seismic velocity and the shot point to receiver distance

\[ t_{direct} = \frac{x}{v_1} \]
The Reflected Ray

- The Reflected Ray Arrival Time:
  - is never a first arrival
  - Plots as a curved path on t-x diagram
  - Asymptotic with direct ray
  - Y-intercept (time) gives thickness
  - Why do we not use this to estimate layer thickness?
The Refracted Ray Arrival Time:

- Plots as a linear path on t-x diagram
  - Part travels in upper layer (constant)
  - Part travels in lower layer (function of x)
- Only arrives after **critical distance**
- Is first arrival only after **cross over distance**
  - Travels long enough in the faster layer

**Critical Distance**

\[ t = \frac{x}{v_2} + 2h_1 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}} \]
Making a t-x Diagram

**Reflected Ray Arrival Time, t**

\[
t = \frac{x}{v_2} + 2h_1 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}
\]

or

\[
t = \frac{x \sin i_c}{v_1} + \frac{2h_1 \cos i_c}{v_1}
\]

**v_2 = 1/slope**

**v_1 = 1/slope**

Y-intercept to find thickness, h_1
Refraction...What is it Good For?

- Seismic refraction surveys reveal two main pieces of information
  - Velocity structure
    - Used to infer rock type
  - Depth to interface
    - Lithology change
    - Water table

<table>
<thead>
<tr>
<th>Rock type</th>
<th>$v_p$ (km/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unconsolidated sediments</strong></td>
<td></td>
</tr>
<tr>
<td>clay</td>
<td>1.0–2.5</td>
</tr>
<tr>
<td>sand, dry</td>
<td>0.2–1.0</td>
</tr>
<tr>
<td>sand, saturated</td>
<td>1.5–2.0</td>
</tr>
<tr>
<td><strong>Sedimentary rocks</strong></td>
<td></td>
</tr>
<tr>
<td>anhydrite</td>
<td>6.0</td>
</tr>
<tr>
<td>chalk</td>
<td>2.1–4.5</td>
</tr>
<tr>
<td>coal</td>
<td>1.7–3.4</td>
</tr>
<tr>
<td>dolomite</td>
<td>4.0–7.0</td>
</tr>
<tr>
<td>limestone</td>
<td>3.9–6.2</td>
</tr>
<tr>
<td>shale</td>
<td>2.0–5.5</td>
</tr>
<tr>
<td>salt</td>
<td>4.6</td>
</tr>
<tr>
<td>sandstone</td>
<td>2.0–5.0</td>
</tr>
<tr>
<td><strong>Igneous and metamorphic rocks</strong></td>
<td></td>
</tr>
<tr>
<td>basalt</td>
<td>5.3–6.5</td>
</tr>
<tr>
<td>granite</td>
<td>4.7–6.0</td>
</tr>
<tr>
<td>gabbro</td>
<td>6.5–7.0</td>
</tr>
<tr>
<td>slate</td>
<td>3.5–4.4</td>
</tr>
<tr>
<td>ultramafic rocks</td>
<td>7.5–8.5</td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td></td>
</tr>
<tr>
<td>air</td>
<td>0.3</td>
</tr>
<tr>
<td>natural gas</td>
<td>0.43</td>
</tr>
<tr>
<td>ice</td>
<td>3.4</td>
</tr>
<tr>
<td>water</td>
<td>1.4–1.5</td>
</tr>
<tr>
<td>oil</td>
<td>1.3–1.4</td>
</tr>
</tbody>
</table>

Ranges of velocities, which are from a variety of sources, are approximate.
Multiple Layers

- Seismic refraction can detect multiple layers
- The velocities are easily found from the slopes on the t-x diagram
Multiple Layers: Refracted Ray Travel Times

- Seismic refraction can detect multiple layers
- Each subsequent refracted ray has a different travel time equation

1st Refracted Ray (top of layer 2)
\[ t_1 = \frac{x}{v_2} + 2h_1 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}} \]

2nd Refracted Ray (top of layer 3)
\[ t_2 = \frac{x}{v_3} + 2h_1 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}} + 2h_2 \sqrt{\frac{1}{v_2^2} - \frac{1}{v_3^2}} \]

3rd Refracted Ray (top of layer 4)
\[ t_3 = \frac{x}{v_4} + 2h_1 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}} + 2h_2 \sqrt{\frac{1}{v_2^2} - \frac{1}{v_3^2}} + 2h_3 \sqrt{\frac{1}{v_3^2} - \frac{1}{v_4^2}} \]

And so on for subsequent layers...
• The layer thicknesses are not as easy to find

Recall layer 1 thickness:

\[ \begin{align*}
  t_1 &= \frac{x}{v_2} + 2h_1 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}} \\

  t_{int1} &= 2h_1 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}
\end{align*} \]

Then solve for \( h_1 \)...

Use y-intercept of best fit line... 

\[ \begin{align*}
  2^{nd} \text{ Refracted Ray Travel Time Equation} \\
  t_2 &= \frac{x}{v_3} + 2h_1 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}} + 2h_2 \sqrt{\frac{1}{v_2^2} - \frac{1}{v_3^2}}
\end{align*} \]

Then solve for \( h_2 \)...

But BEWARE: \( h_1, h_2 \), are layer thicknesses, not depth to interfaces.

So, depth to bottom of layer 2 (top of layer 3) = \( h_1 + h_2 \)
Caveats of Refraction

• Only works if each successive layer has increasing velocity
  – Cannot detect a low velocity layer

• May not detect thin layers

• Requires multiple (survey) lines
  – Make certain interfaces are horizontal
  – Determine actual dip direction not just apparent dip
Dipping Interfaces

• What if the critically refracted interface is not horizontal?

• A dipping interface produces a pattern that looks just like a horizontal interface!
  – Velocities are called “apparent velocities”

• What do we do?

In this case, velocity of lower layer is underestimated
Dipping Interfaces

- To determine if interfaces are dipping...
- Shoot lines forward and reversed
- If dip is small (< 5°) you can take average slope
- The intercepts will be different at both ends
  - Implies different thickness

Beware: the calculated thicknesses will be perpendicular to the interface, not vertical
Dipping Interfaces

- **If you shoot down-dip**
  - Slopes on t-x diagram are too steep
    - Underestimates velocity
  - May underestimate layer thickness
- **Converse is true if you shoot up-dip**
- **In both cases the calculated direct ray velocity is the same.**
- **The intercepts $t_{int}$ will also be different at both ends of survey**
The Hidden Layer

• There are two cases where a seismic interface will not be revealed by a refraction survey.
  
  – The Hidden Layer (book calls it “Hidden Layer Proper”)
  
  – The Low Velocity Layer

This one is straightforward, so we will look at it first.
The Low Velocity Layer

- If a layer has a lower velocity than the one above...
  - There can be no critical refraction
    - The refracted rays are bent towards the normal
  - There will be no refracted segment on the t-x diagram
  - The t-x diagram to the right will be interpreted as
    - Two layers
    - Depth to layer 3 and Thickness of layer 1 will be exaggerated
The Low Velocity Layer

- **Causes:**
  - Sand below clay
  - Sedimentary rock below igneous rock
  - (sometimes) sandstone below limestone

- **How Can you Know?**
  - Consult geologic data!
    - Boreholes / Logs
    - Geologic sections
    - Geologic maps
The Hidden Layer

- Recall that the refracted ray eventually overtakes the direct ray (cross over distance).
- The second refracted ray may overtake the direct ray first if:
  - The second layer is thin
  - The third layer has a much faster velocity
Geophone Spacing / Resolution

- Often near surface layers have very low velocities
  - E.g. soil, subsoil, weathered top layers of rock
  - These layers are likely of little interest
  - But due to low velocities, time spent in them may be significant

- To correctly interpret data these layers must be detected
- Decrease geophone spacing near source
- This problem is an example of...?
Undulating Interfaces

- Undulating interfaces produce non-linear t-x diagrams
- There are techniques that can deal with this
  - delay times & plus minus method
  - We won’t cover these techniques...

![Graph showing undulating interfaces](image-url)
Detecting Offsets

- Offsets are detected as discontinuities in the t-x diagram
  - Offset because the interface is deeper and D’E’ receives no refracted rays.
Fan Shooting

- Discontinuous targets can be mapped using radial transects: called “Fan Shooting”
  - A form of seismic tomography
Ray Tracing

• All seismic refraction techniques discussed thus far are inverse methods

• One can also fit seismic data to forward models using Snell’s law, geometry, and a computer
  – Initial structure is “guessed” and then the computer uses statistical versions of “guess and check” to fit the data.
  – Model generates synthetic seismograms, which are compared to the real seismograms
Survey Types

• The simplest (and cheapest) type of survey is called a hammer seismic survey
  – A sledgehammer is whacked into a steel plate
  – Impact switch tells time=0
  – First arrivals are read digitally or inferred from seismogram
  – Because swinging a hammer is free, only one geophone is needed
    • More can be used, but single geophones must be along a linear transect
Survey Types

• The maximum workable distance depends on:
  – The sensitivity of the system
  – The strength of the sledgehammer whacks
  – The amount of noise
    • Wind shakes trees, etc...
    • Cars, footsteps, HVAC, traffic, etc...
    • Surveys may be done at night to minimize noise
Survey Types

• Often the signal to noise ratio is very poor:
  – Stacking is often used to help delineate first arrivals

• General rule of thumb:
  – Geophone array should be about 10x the depth to interface
  – 100 meters is the typical upper limit on length of hammer seismic transect
    • So hammer seismics are best for shallow interfaces (< 10 m)
Other Survey Types

• Explosion seismics
  – Offers a much stronger signal
    • Can detect deeper features
    • Often involves water explosions (much cheaper)
    • Geophones / Seismometers are often linked wirelessly (RF / radio waves)

• Marine Surveys
  – Sometimes use explosives, compressed air, high voltage charges, or many other types.
  – Usually use hydrophones
    • Respond to pressure changes (p-waves)
    • Surveying is often done while the ship is moving, so very long transects can be done at a lower cost