1. A monopolist can segment its market into two sub-markets, call them 1 & 2. \( C = 200 + 5Q \), with \( Q = q_1 + q_2 \). The demand in the sub-markets is:

\[
P_1 = 20 - q_1/2 \quad \text{&} \quad P_2 = 35 - q_2.
\]

a) Find the profit-maximizing \( q_1, q_2, P_1, P_2 \), and find \( \pi \), and \( E_p^D \) in each sub-market at the profit-maximizing \( P \& q \).

b) Which sub-market gets the lowest \( P \)? Why?

c) What happens if there is a capacity constraint \( Q \leq 20 \)?

2. In Figure 1, is \( \pi \) maximized when \( q_A = 30 \) & \( q_B = 20 \)? Explain.

3. Stars have a value of $60 & lemons have a value of $30 to firms. Firms are unable to cheaply identify who is a star. Education, \( y \), is cheaper for stars than for lemons because stars exert less effort than lemons. For a star, education costs \( y/2 \), & for a lemon, education costs \( 2y/3 \). Let \( y \) be a continuous variable (that is, it can be a non-integer).

a) Show algebraically & explain the lowest & highest values for \( y \) for which signaling could occur.

b) Assuming \( y = y_{Riley} \), if the fraction of stars in the population is known to equal \( s \), when will stars prefer signaling to pooling?

4. Suppose utility = \( U = 10\sqrt{I} \), where \( I \) = income. \( I = 100 \) (probability = .25) or $900 (probability = .75).

a) Find \( E(I) \) & \( E(U) \).

b) Find the risk premium (\( RP \)).
Answers

1. a) & b) \( MC = $5. \ TR_1 = 20q_1 - \frac{q_1^2}{2} \), & \( TR_2 = 35q_2 - q_2^2 \). Thus \( MR_1 = 20 - q_1 \) & \( MR_2 = 35 - 2q_2 \).
   
   Set \( MR_1 = MC \) & \( MR_2 = MC \): \( 20 - q_1 = 5 \) & \( 35 - 2q_2 = 5 \), so \( q_1 = q_2 = 15 \).
   
   Insert \( q_1 \) into the demand for sub-market 1 & do likewise for sub-market 2 & get \( P_1 \) & \( P_2 \):
   
   \( P_1 \approx $15.83 \) & \( P_2 \approx $23.33 \); both \( Ps \) \( \uparrow \) & both \( q_s \) \( \downarrow \) due to the capacity constraint.

2. The relevant \( MC \) is \( MC \) for \( Q = 50 \), which is clearly > $15. Thus, the \( \pi \)-maximizing \( Q < 50 \).
   
   Given the firm sells \( Q = 50 \), it should sell more in sub-market A & less in sub-market B since, with \( q_A = 30 \) & \( q_B = 20 \), \( MR_A = $15 \) & \( MR_B = $10 \). If the firm sells 1 more unit in sub-market A & 1 less unit in sub-market B, \( \Delta R = $5 \) ($15-$10 \), & \( \Delta C = 0 \), so \( \Delta \pi = $5 \). The firm should continue to sell more in sub-market A & less in sub-market B until \( MR_A = MR_B \), which \( \Rightarrow q_A > 30 \), \( q_B < 20 \), & \( $10 < MR_A = MR_B < $15 \).

3. a) If employers believe those with \( y \geq y^* \) are stars, then the conditions for a star to signal & a lemon to not mimic a star (given those who signal will be paid 60, & others will be paid 30) are:
   
   \[
   60 - y/2 \geq 30,
   
   60 \geq y. \tag{1}
   
   60 - 2y/3 < 30,
   
   45 < y. \tag{2}
   
   \Rightarrow 45 < y \leq 60.
   
   Thus, \( 45 < y^* \leq 60 \). Competition by firms for workers will drive \( y^* \rightarrow 45 = y_{Riley} \). Technically, \( y^* \) must be slightly greater than 45 for lemons not to mimic stars, but we can use \( y^* = 45 \).
   
   b) If all set \( y = 0 \) (pooling), then the pooling wage is \( W_{pool} = 60s + 30(1-s) = 30(1+s) \). The payoff to a star from signaling = \( 60 - y_{Riley}/2 = 37.5 \). Stars prefer signaling to pooling if \( 37.5 > 30(1+s) \), or \( s < .25 \).
4. \( E(I) = \text{[probability } I = \$100]\times\$100] + \text{[probability } I = \$900]\times\$900] = .25\times\$100] + .75\times\$900] = \$700. \)

\( E(U) = \text{[probability } I = \$100]\times[U($100)] + \text{[probability } I = \$900]\times[U($900)] = \)

\[.25\times[10\sqrt{100} + .75\times[10\sqrt{900} = 25 + 225 = 250. \]

To find \( RP \), find the certain \( I \) that yields \( U = 250 \):

\[ 10\sqrt{I} = 250 \]

\[ \sqrt{I} = 25 \]

\[ I = 25^2 = 625. \]

Thus \( RP = \$75. \)