Investigations in Teaching Geometry

Lesson Plans for Day One

Purpose: The purpose of the activities during the first two days is to have students begin to be able to visualize a two-dimensional drawing of a three-dimensional shape. In addition, the activities will focus on trying to help students develop visual imagery.

Materials: Polygons, solids, multi-link cubes, and spatial sheets

Description: I will begin by asking students to look at a two-dimensional shape and a three-dimensional shape and tell me what is alike and what is different. I will probably use a square and a cube or a rectangle and a rectangular solid for this activity. It is hoped that the discussions will center around the difference in the number of dimensions. While I do not anticipate that the students will frame the discussions in those terms, I would anticipate such dialogue as: one is solid, one is flat, or you can fill one up. I will not necessarily introduce the term dimension but will hope to achieve some discussion concerning that aspect of the shapes. From there I will ask them how they would draw the cube on a sheet of paper. I will use that as an introduction to the overhead sheets.

With the first shape on the overhead, I will ask the students to make a model with multi-link cubes. I will include questions about whether or not they have enough information to build the shape. I will take the same approach with the second shape. It is likely that some students will have no concept of how to begin. I will encourage them to try to build what they "see." From there, I will begin simply asking students how many cubes are needed to build the shape. If students have difficulty determining the answer, they will be encouraged to build the shape from multi-link cubes. I will have the shape ready to show if necessary.

Discussions will be focused on different ways of arriving at the solutions and on the symbolic representations of their solutions. Solutions will be represented in terms of multiplication where appropriate.

Attached is a proposed sequence of shapes to be used. "Teacher discretion" will be necessary to determine exactly which ones will be used. This will be based on the students' ability or inability to successfully complete the tasks. A tentative outline follows. It is hoped that students will be able to draw conclusions and make conjectures about the number of cubes in the shapes. These types of generalizations will allow me to move more rapidly through the sequence, eliminating some of the shapes.

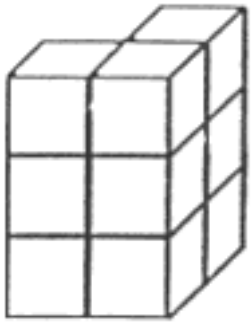

Figure 1
Overhead 1

I will start by showing them **Overhead 1** *(Figure 1)*. I will show the first shape only and ask if they can build it from what they see. Hopefully we will have discussions about the three possibilities that exist. Afterwards, I will show the second view and ask each to build the shape. I will repeat the process with the second shape on the page. After this whole class activity, students will break into groups and work on Spatial Visualizations 2, 3, and 4. They will be asked to determine how many blocks are in the shape. Blocks will be made available and students may use them as necessary. After they finish these three sheets, we will return to whole class and discuss solutions and methods of solution. As a group we will do **Spatial Visualization 2** *(Figure 2)* and then they will return to groups to finish the remainder of the sequence through **Spatial Visualization 5** *(Figure 3)*.

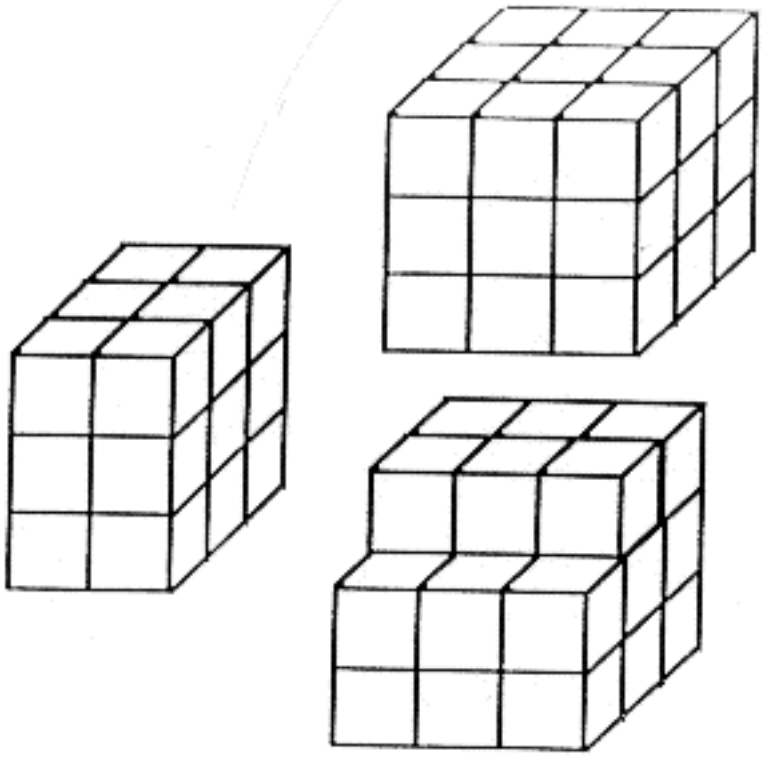

Figure 2
Spatial Visualization 2

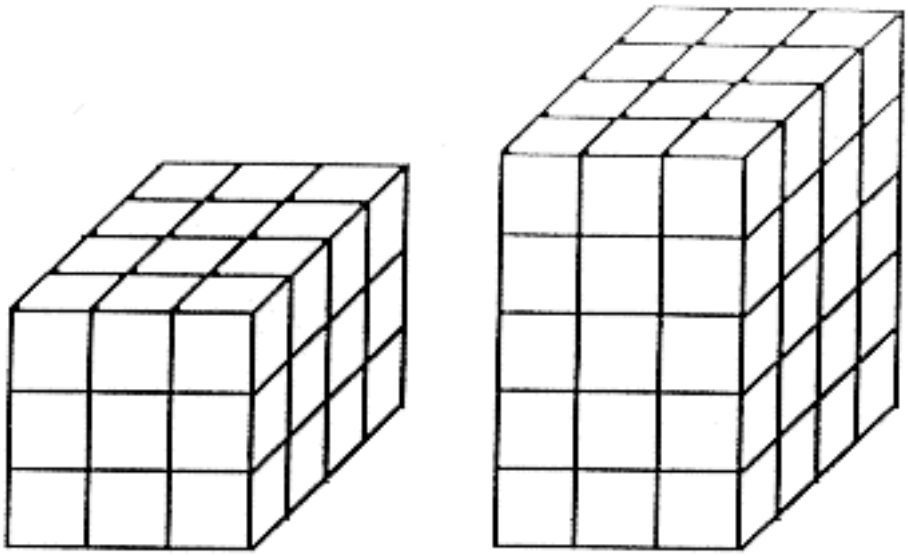

Figure 3
Spatial Visualization 5

We will discuss this sequence, and then I will introduce a couple with the smudges in whole class. Afterwards, they will break back into groups to work on the remainder of the smudge sheets.

We will discuss solutions to these in whole class. Then I will introduce the shapes with only partial blocks drawn. We will do **Spatial Visualization 13** *(Figure 4)* in whole class and then break into groups where they will do Spatial Visualizations 17 and 18. Afterwards, we will go back to whole class to discuss and summarize.

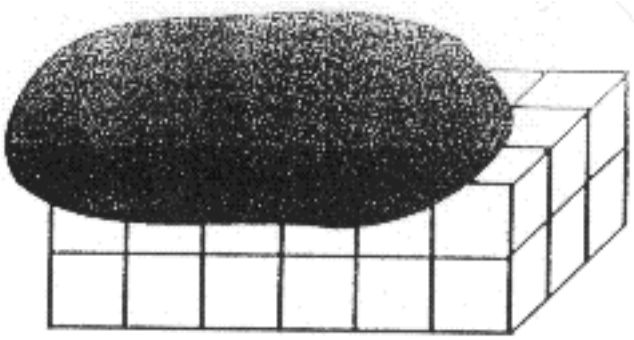

Figure 4
Spatial Visualization 13

The exact flow and format will be contingent upon student response. Lack of ability to visualize will create the need for concrete models. An inability to "imagine" the cubes under the smudge will result in my having to build the shape and cover it to create a model of that activity. Ease of transition from the concrete to the two dimensional will allow better opportunity to challenge the students to make conjectures about the total number of cubes. I hope that student interactions will facilitate students' being able to

develop the imagery necessary to hypothesize and generalize about the number of cubes in the shapes.

Questions that I anticipate utilizing include:

1. What did you see?
2. How did you arrive at that conclusion?
3. What did you see that helped you get the answer?
4. What might you tell a classmate who was having trouble to help him or her see the solution?
5. What made this difficult/easy for you?

Lesson Plans for Day Two

Purpose: The purpose of these activities is to help students develop the concept of "filling a box." It is hoped that students will be able to build on the knowledge developed in the previous two days to be able to make some generalizations about the way to determine the total number of cubes necessary to fill the box.

Materials: Boxes made of inch grid paper and 1-inch cubes

Description: I will begin by passing out the following challenge. I will read it with the class and make sure that the student pairs are clear about the assignment.

The Choc Full Chocolate Company has been selling so many candies that the management has decided to start placing candies in pre-packaged boxes and making the boxes of candies available to their customers. They have ordered several different sized boxes and are now trying to determine how many candies will fit in each sized box.

Mr. I.M. Fattening, the owner of the company, has asked you to determine how many candies will fit in each box so he can price the boxes accordingly. He wants you to prepare a report which explains how you determined the number of candies for each box. Prepare a written report for Mr. Fattening.

Working in groups, students will be given a box and asked to determine how many candies will fit. The interior of the box will be made of inch grid paper. The box size will be 3" x 4" x 6". Each group will also be given a handful of "candies" which measure 1" x 1" x 1". They can use these to determine the number that would fill the box. Since the students will not have enough 1-inch cubes to fill the box, they will need to use the ones they have to try to develop a plan or pattern for determining the total number. I would anticipate that strategies would involve trying to make layers and trying to use the

dimensions. It is possible that pairs may combine resources in order to have enough 1-inch cubes to give them a clearer picture of the results. If pairs are unable to approach the task, I anticipate increasing the number of one-inch cubes available to them so that they could build a layer.

After this activity, students will return to whole group to present their findings. Discussions will center around the processes used to determine the number of candies. Groups will be encouraged to present their findings to the class. After discussions, students will go back into groups and work on boxes which are 3" x 5" x 2" and 4" x 4" x 5". Students will be asked to prepare a written report of their work and their findings. For these next boxes, students will be given additional candies so that they are able to make the longest dimension.

Whole class will center on discussions about process and solutions. Maybe someone will even make a conjecture about the relationship between dimensions and the "answer."

1. How did you get started?
2. What process/method did you use?
3. How do you know what you did works?
4. What did you discover?

Lesson Plans for Day Three

Purpose: The purpose of this activity is to have the students experiment with the impact of dimension size on volume. While it will not be presented in such sophisticated terms, students will be conducting investigations into the result of changing the dimensions with the goal of trying to find a maximum volume.

Materials: sheets of inch grid paper, scissors, tape, 1-inch cubes

Description: I will begin by passing out the following challenge. I will read it with the class and make sure that the student pairs are clear about the assignment (adapted from "Cutting Corners," an activity contained in *Interactive Mathematics: Activities and Investigations*, published by Glencoe).

Candy sales at Choc Full Chocolate Company have skyrocketed. As a result, Choc Full Chocolate Company is designing fancy new boxes to be used in selling their candies. The boxes will be made from a piece of cardboard that is 10 inches by 10 inches. The box company will simply cut sections out of each corner and fold up the sides to make this new box. Mr. I. M. Fattening thinks this is a great idea and has sent an order off to the local box company.

The box company is faced with a serious problem - they do not know how much to cut out of each corner. When they called Mr. Fattening of Choc Full he replied, "Just make the biggest box possible!"

You are an employee of the box company. You have been assigned the task of determining the dimensions of the box for the Choc Full Chocolate Company. Use the sheets of grid paper, scissors, and tape that have been provided to experiment and find the box that will hold the most. Prepare a report that explains your process and supports your findings.

I will begin by taking a sheet of inch grid paper and demonstrating the process of making the box. I will cut one-inch squares off of each of the four corners. I will also use this time to explain that the box has no lid and that they must cut on the lines. From there, students will work in pairs, trying to make the boxes from the grid paper and determine the volume. Inch cubes will be available for the students to use in determining the volume, if necessary. I would anticipate that students will not have a sense of what should happen. The shape of the box may lead them to anticipate incorrect solutions. For instance, the tall box will not hold as much as they might think. I think that these counterintuitive sizes may work against the process by causing students to doubt their conclusions.

Since only four different boxes can be made, I would anticipate that students would not know to stop after the volume begins to decrease and would actually make all four. After they have completed all four, it is hoped that they will be able to look at them and make some type of conjectures about the relationship of dimensions and volume.

Class discussions would include methods for determining the largest box. It is hoped that they would develop a pattern of increasing and then decreasing sizes. Questions might include:

1. How did you know when you had the largest box?
2. What process did you use?
3. How did your group begin?
4. Present an argument for your solution.
5. How might you convince Mr. Fattening that your solution is correct?