PATTERNING AND PARTITIONING

Introduction

The instructional activities in this sequence are of three types: *finger patterns, spatial patterns, and partitioning*. In addition, there are *context problems* that do not fall within any one of these three categories. Activities of the three types are designed to run concurrently.

Across all of the activities, the emphasis is on developing grouping, as opposed to counting, methods. Nevertheless, the teacher necessarily takes into account the current conceptual possibilities of each of the children and responds to them in ways uniquely appropriate for them. As noted earlier, if a child's only possible solution method is counting, the teacher adjusts her expectations that methods should be based on grouping and number relationships to accommodate the immediate situation. At the same time, counting does not become a normative activity for the class more generally.

Instructional Activities

Finger Patterns

The purpose of this instructional sequence is to:

- support children's development of flexible finger patterns for numbers up to ten so that they can create such patterns *without counting* individual fingers.
- support children's development of number relationships using grouping methods (e.g., 5 is 3 and 2; 5 is also 4 and 1).

Initially fingers are perceptual items but later they change function and then represent a collection. To accomplish this shift, the types of questions progress as well as the size of the numbers used. Five-referenced and doubles-referenced relationships are encouraged.

The focus here is on the product rather than on how you figure it out since the goal is to develop patterns without having to rely on counting. Further, since the teacher can see what the children are doing there is no need to ask them how they are figuring things out. Initially children may count or may imitate others. (It may be useful for the teacher to hold up her fingers as well in some cases.) Children progress from having difficulty making finger patterns to spontaneously using finger patterns to express their thinking.

The finger pattern activity might be posed in a Simon Says scenario. This activity can be used productively for 10 to 15 minutes a day until children have flexible finger patterns they can use so they can operate without counting.

The following activities indicate a progression in the types of tasks that can be included:

- Show a specified number on each hand and think about how many fingers you have up altogether. How many do you have up?
- Show a specified number. Then show it another way.
- Show a specified number using both hands. Then show it another way.

- Show a specified number. Put up (put down) some fingers to make another specified number. (E.g., Show 3 fingers. Put up enough fingers to make 7.) How many fingers did you put up (put down)?

Number choices can progress from numbers under 5 to 5 as a reference, doubles as a reference, and number combinations up to 10. Further, problems can be sequenced to encourage thinking strategies and inverse relationships.

Examples: 5 and 1; 5 and 2; 5 and 4.

5 and put up two more (makes 7), now take down two (makes 5).

4 and 4; 5 and 3.

Spatial Patterns

The instructional activities in this sequence include:

Spatial flashing (dot patterns)

Tiles activity

Single tens-frame

Double tens-frame

(Spatial patterns games)

The purpose of the spatial patterns activities is to:

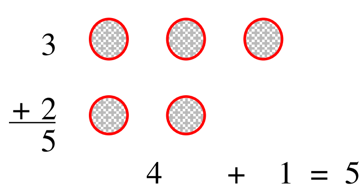
- support children's development of flexible spatial patterns for number.
- support children's development of number relationships using grouping ideas.
- support children's development of number relationships based on five- and ten-referenced strategies and doubles strategies.
- support children's development of thinking strategies, such as +1, -1, and compensation strategies and filling up the tens strategy.
- provide a situation to initiate the interactive constitution of the sociomathematical norm of what counts as mathematically efficient.

- provide situations to introduce conventional addition and subtraction notation through teacher notating.

<u>Spatial Flashing (Dot Patterns)</u>

This instructional activity is productive without being posed in a scenario. This activity supports children's development of flexible spatial patterns for numbers so they can recognize how many are in a pattern without counting. It also supports their development of number relationships using spatial groupings (e.g., 6 is 2 rows of 3 or 3 columns of 2).

This activity involves showing spatial patterns for small numbers ($n \leq 6$) using the overhead projector. Using transparent chips, the teacher can create a spatial pattern to be flashed several times on the overhead screen. The students' task is to figure out how many there are. Some children may attempt to count the chips or count their mental "picture". The approach of flashing the pattern encourages children to develop mental imagery and operate on mental rather than on visual material. By using patterns rather than random arrangements the teacher can encourage children to use grouping strategies and thereby facilitate development of number relationships. Typically, there is a shift in children's thinking from visual items to an increasingly numerical interpretation. Children's descriptions of how they thought about what they saw are useful here and are productive for discussion. Children have different ways of thinking about patterns they see. For example,



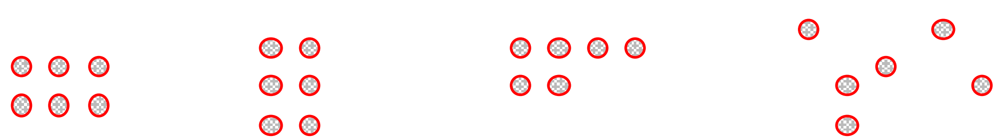
may be thought of as 4 and 1 more or as 3 and 2 more. It might also be thought of as 2 and 2 and 1. The teacher can capitalize on such descriptions and notate them using standard notation, such as $4 + 1 = 5$, $3 + 2 = 5$, $2 + 2 + 1 = 5$. By linking the notation to the children's descriptions and even, if possible, by writing the numbers adjacent to the dots they represent (see examples above) the teacher develops ways of notating naturally. The notation grows out of the children's thinking and is introduced as a means of recording the thinking and of facilitating communication.

Notating serves the dual purpose of introducing conventional symbolism and initiating the children into the practice of recording and symbolizing their thinking. Further, it encourages grouping strategies and discourages counting. As this example illustrates, it is important to have students describe how they thought about what they saw as well as indicate how many they saw.

This activity lends itself to discussions of ways to figure out *without counting*. Consequently, the classroom sociomathematical norm of what counts as efficient can emerge from this activity. The latter can be initiated by showing random as well as patterned arrangements for a given number. Children can discuss which was easier for them to figure out and why. Those arrangements that can be figured out without counting are easier. It is more efficient to use grouping strategies than to count.

By keeping the numbers in the pattern small, counting is discouraged. Larger numbers are dealt with more productively in the subsequent ten-frame activities.

Examples of patterned and random arrangements for six are shown here.



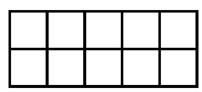
<u>Tiles (or chips)</u>

In this activity children work individually to create their own spatial patterns for numbers using tiles or chips. They are asked to show a specified number in a way that they think makes it easy to figure out how many there are and in a way that they think makes it hard. This activity can be used in conjunction with the teacher showing patterned and random ways to show a number (as described in the spatial flashing activity above). The discussion of easy versus hard might be initiated during the spatial flashing and continue during the individual activity. Children can be encouraged to consult with those sitting nearby to see if they think the creation is easy or hard.

There are two main purposes for this activity. The first is that it gives rise to discussions about easy/hard and therefore to notions of grouping and efficiency. Second, it gives children the experience of creating spatial patterns themselves. The teacher can select some of the children's patterns and display them for the class. However, there is likely to be little whole-class discussion. One teacher who has used this activity commented that it is more internal than social. Nevertheless, in her view, it has been very beneficial.

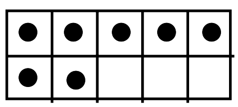
Single Tens-Frames

The single tens-frame is a rectangular arrangement of two columns (or rows) of five squares as shown below.

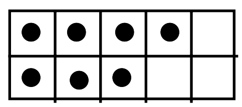


The tens-frame is an extension of the dot pattern activity. It provides a spatial organization for the dots that supports children's development of five-referenced, ten-referenced, and doubles-referenced conceptions of numbers up to ten and the development of mental imagery for such numbers. It also supports development of partitions of ten.

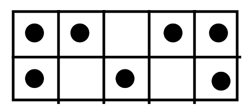
This activity involves showing a number on the tens-frame. Using transparent chips, the teacher can create an arrangement of a number ten or under. The students' task is to figure out how many there are. The approach of flashing patterns encourages children to develop mental images and operate on their mental images rather than on the visual material. By arranging the chips to encourage five-referenced, ten-referenced, or doubles-referenced strategies, the teacher can facilitate children's development of number relationships. The tens-frame lends itself to a variety of solution methods. For example, in the first frame shown below, children might think of seven as five (the top row) and two more, or as four (the four on the left) and three more. They might also think of it as three empty boxes so its three less than ten. In the second frame shown below, children might think of it as four (the top row) and three more (the bottom row), as two, two, two and one more (focusing on the columns), or as three and three and one more (using a doubles-references strategy). Discussions in which children explain their strategies provide occasions for children to reflect on their solution methods in light of the methods others describe. Students then have the opportunity to attempt to make sense of solution methods that they might not have used previously themselves without feeling obliged to use them. As a result, children develop alternative solution methods and strategies that are in keeping with their current understandings.



five-referenced seven



doubles-referenced seven



random seven

Children's descriptions of how they thought about what they saw are productive for discussion. As with the dot pattern activity, the teacher can capitalize on children's descriptions and notate them using standard or pedagogical notation. This further supports number relationships and discourages counting. As with the spatial dot patterns, random arrangements can be used to foster discussions about easy/hard. Easy arrangements are those that can be figured out without counting by ones. In addition, questions about the number of empty squares in the tens-frame encourages the development of partitions of ten.

One productive way to pose the tens-frame tasks is to develop a scenario that children can use as the basis for developing imagery. For example, the tens-frame might be a crate at Earl's Fruit Stand. Earl keeps his pumpkins in the crates. When the crate is full it has ten pumpkins in it. A scenario such as this creates a forum for posing questions in a variety of ways. By thinking about the scenario, children can use their imagery of the situation to help them think about how they might solve the problem. For example, if a tens-frame with seven chips is shown the teacher can ask a variety of questions such as:

How many pumpkins does Earl have in his crate?

At the beginning of the day Earl's crate was full. How many has he sold?

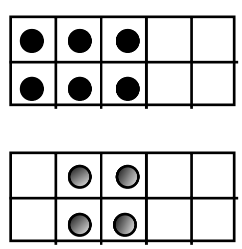
How many more pumpkins does Earl need to fill up his crate?

The last two questions can contribute to children's development of partitions of ten and filling up the tens strategy.

Double Tens-Frame

The double tens-frame is useful to support children's development of number relationships based on five- and ten -referenced strategies and doubles strategies for numbers up to 20. In addition, it is productive for developing thinking strategies, such as the +1, -1, and compensation strategies and filling up the tens strategy. Comparisons, including more, less, how many more, how many less can also be facilitated by use of the double tens-frame.

The activity involves showing a number in each of two adjacent tens-frames. The children's task is to figure out how many there are. Alternatively, the task can be to figure out how many are missing or how many more (or fewer) there are in one frame than in the other. Posing the tasks using chips of two different colors, one color for each frame, facilitates the children's explanations as well as the posing of tasks. For example, if the following display shows red chips in the upper frame and blue chips in the lower frame, a child might explain that s/he moved one blue chip to the frame with the red chips to make five and five. Because five and five is ten, there are ten chips in all.



Another child might explain that s/he (mentally) moved the four red chips beside the blue chips to fill up the upper tens-frame. So there are ten in all. By capitalizing on solutions such as these which make spontaneous use of thinking strategies the teacher can make these strategies explicit topics of discussion. Furthermore, the teacher can actually move the chips to visually show the compensation or filling up the ten. In this way some children might develop visual imagery for some of the strategies that they hear described verbally.

Typically, children's thinking shifts from thinking about visual items and counting-based solution methods to numerical interpretations based on five- or ten referenced and strategy-based interpretations. Increasingly, children develop number relationships for number combinations up to 20 and come to "just know" the addition facts up to 20. At the same time, those children for whom visual imagery is still necessary have a way to participate in the activity. Thus, the task accommodates the individual needs of the children while at the same time providing opportunities for all of them to advance in their interpretations.

The double tens-frame can be used to support children's understanding of comparisons and inequality by asking questions relating the quantities shown in the two frames. For example, questions based on the above display might include:

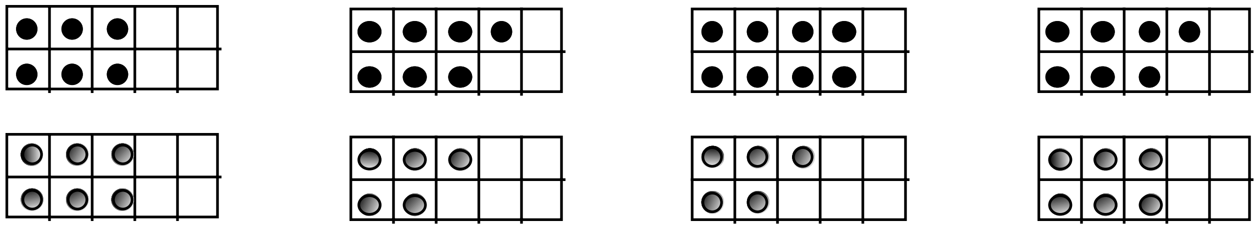
Are there more red chips or more blue chips?

How many more (fewer) red are there than blue?

How many more red chips do we need to have the same number as blue?

Children might think about the visual material to develop answers to these questions. For example, in the double tens-frame shown above some children might think of putting two more red chips in the upper frame to make it "the same" as the lower frame. The teacher can notate such descriptions using standard notation such as $4 + \underline{2} = 6$. As noted in the discussion of the spatial flashing (dot patterns) activity, in this way the teacher develops ways of notating naturally. The notation emerges from the children's thinking and is introduced as a means of recording thinking and facilitating communication.

The double tens-frame can be useful to facilitate children's development of thinking strategies by posing tasks in sequence which provide opportunities for children to relate the tasks to each other. For example, consider the following sequence of four double tens-frame tasks.



task 1 task 2 task 3 task 4

Some children might solve task 2 by relating it to task one using the compensation strategy, that is, they might mentally move one blue chip from the upper frame to the lower frame to make six and six and reason that it is the same problem as task 1. For task 3, some children might relate it to task 2 by noticing that one more blue chip was added and so reason that the number of chips is one more than in task 2. Some children might solve task 4 by relating it to task 3 and reason using the compensation strategy. Others might say that there is one more red chip than in task 2 and reason using the +1 strategy. Still others might use the +1 compensation strategy but relate it to task 1 noticing that there is one more blue chip. Other children might not relate the problems but solve each one independently. However, by posing the tasks in a sequence such as this the teacher can create the opportunity for thinking strategy solutions. Typically, some children will use thinking strategies when tasks are posed in sequence. As the teacher capitalizes on these solutions in whole class discussions, solutions based on relating problems and on number relationships become the norm. As this happens, children begin to spontaneously use thinking strategies by relating the number combinations to some that they already know. The example given above for the problem that showed four in the first frame and six in the second frame is illustrative. A child might solve this task by relating it to the known fact 5 + 5 = 10, by thinking of moving one from the frame with six to the frame with four.

Partitioning Activities

The purpose of partitioning activities is to:

- support children development of partitions of small numbers (up to 10, possibly 20).
- provide opportunities for children to think about questions such as finding all possible partitions of a number and developing arguments for how they know they have all of the possibilities. These questions are higher level questions than those that ask for only one possible partition of a given number. These questions provide an occasion for the students to reflect on their solution activity and reconceptualize and reorganize it and to develop mathematical arguments that that are not based on arithmetical calculations.

Two instructional activities are described in this sequence. The first is an overhead activity that uses small numbers and visual material. The second is based on the imagery of a double-decker bus and itself forms the imagistic basis for the Arithmetic Rack instructional sequence which will follow.

<u>Overhead Partitioning Activity: Monkeys in the Trees</u>

The initial purpose of this activity is to give students an opportunity to develop various ways to partition a given number. An additional purpose is to make explicit the issue of figuring out a rationale for how to know you have all the partitions. This activity is intended to be used early in the instructional sequence and can be used in the same lessons as the initial finger pattern activities.

In this activity the teacher poses a scenario that involves partitioning a number of visible items. For example, the teacher might pose a scenario of a number of monkeys, say six, playing in two trees, a small tree and a large tree. The scenario is developed so that the children understand that all of the monkeys are always in one of the two trees. The question is, How many monkeys might be in each of the trees? How many monkeys might be in the small tree and how many in the big tree? Children are encourages to come up with more than one way to answer the question. The display on the overhead screen shows the two trees and the six monkeys below the trees (see Appendix). This display is left on the screen for the children to refer to as they think about how to solve the problem.

For some children, figuring out one or more ways to partition six may be quite challenging. They might use their fingers using one hand to indicate number of monkeys in the small tree and the other hand to indicate the number of monkeys in the big tree. From the observer's perspective the task is the same as using two hands to show the number six. As the children give their responses, the teacher can record them and, in doing so, create a table showing the various possibilities. By recording the responses in a table, the teacher introduces this conventional way of recording and notating multiple solutions in a natural way. The table can then itself become the focus of discussion as the children attempt to figure out if all of the possibilities are listed in the table and how they know when they have all of the possibilities. Children might argue from patterns they see in the data, such as, that each number pair can occur twice. For example, there can be 4 monkeys in the small tree and 2 in the big tree or 2 monkeys in the small tree and 4 in the big tree. Others might reason by figuring out the various possibilities for the number of monkeys in the small tree and checking to see if they are all listed. Still others might not be able to reason beyond generating individual pairs of numbers and reject any that are repetitions of pairs that are already recorded. Class discussions that focus on the children's thinking rather than on developing generalizable schemes for producing all solutions will prevent children's development of rote procedures which they cannot explain and justify. The goal of the activity is not that all children can develop all partitions of a given number on request. Rather it is to provide an occasion for thinking about the activity of partitioning and the result of that activity.

As a footnote we remark that the development of the scenario is critical to the success of the activity. The purpose of the scenario is that the students can use it as a basis for developing imagery. In this example, children could think about one way the monkeys might be in the trees and then imagine one (or more) of them jumping to the other tree. This could then provide them with a means of thinking about how to generate other ways.

It is important that children find the scenario credible. An earlier attempt was to use a scenario with birds in the trees. However, the children found this scenario problematic. As some of them said, some of the birds might be flying around and not sitting in either tree. In the case of the monkeys, all of the children agreed that the monkeys would all be in the trees.

The mathematical level of the task can be raised again by posing questions such as, Would there be more or fewer ways if there were 10 monkeys? two monkeys? Questions such as this are useful after children in the class have been able to give arguments for how they know they have all the ways for a given number of monkeys.

Double-Decker Bus Partitioning Activity

This activity is adapted from the work of Jan van den Brink (1989). In this activity the teacher introduces a scenario of a double-decker bus. Passengers can ride either on the upper deck or on the lower deck of the bus. Children are asked to think about different ways a given number of passengers might be seated on the bus. How many might be on the upper deck? How many might be on the lower deck? As the discussion progresses, children might be asked if they have all of the possibilities and how they know they have them all.