Worksheet on Hyperbolic Geometry Part I

Dr. Sarah's MAT 3610: Introduction to Geometry

Physical Geometry Manipulative: hyperbolic crochet model

Goals:

- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate that they seem to apply in a wide variety of examples.
- Geometric Perspectives
  I can compare and contrast multiple geometric perspectives.

Welcoming Environment: Actively listen to others and encourage everyone to participate and try to help each other! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss, handwrite and ask me questions during group work time as well as when I bring us back together:

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

Shortest Distance Paths

2. Open up `https://www.geogebra.org/m/d845j9a3`. We have two paths between A and B. Notice that the path through C looks like the intrinsically straight paths that cut angels and demons in half in Escher's *Circle Limit IV: Heaven and Hell* that we looked at in the measurements and angle sum interactive video. Sketch a picture of the two paths.

3. Which path is shorter using the hyperbolic metric in this model? The path from A to B through C or through D?

4. Drag B up toward A (but keep the configuration the same with C to the left of D and B below them). What happens to the difference between the distances?

5. Open up `https://www.geogebra.org/m/akwvszgu`. Under the first wrench tool, find the **Hyperbolic Segment** tool. Create a hyperbolic segment. Then select the (usual) move symbol (pointer) and drag the endpoints around but inside the disc. Can you obtain the different types of paths that Escher represented as cutting angels and demons in half? Sketch pictures here:

Sum of the Angles

6. What was the sum of the angles at the three points we explored in Escher's *Circle Limit IV: Heaven and Hell* in the measurements and angle sum interactive video?

7. How did we compute the sum of the angles there?

8. Open up `https://www.geogebra.org/m/svywsx3r` to explore the sum of the angles in hyperbolic geometry more generally. Drag A, B, and C but keep the same configuration (so that you don't end up with an exterior angle measurement—i.e. don't let A cross $\overline{BC}$ or its extension, and similar for the other points) and keep them all inside the disc. How small can the sum of the angles get in this IGS exploration? What kind of triangle must we form in this model to get a small angle sum?

9. How large can the sum of the angles get in this IGS? What kind of triangle must we form in this model to get a large angle sum?

10. Sketch pictures that illustrate your prior two responses.

Euclid's 5th Postulate

11. Write down the statement of the 5th postulate from *Euclid's Elements* Book I

12. Open `https://www.geogebra.org/m/qmbmeas9`. Do the intrinsically straight paths in the sketch seem to satisfy the assumptions/conditions as well as the conclusion of Euclid's 5th Postulate?

13. Sketch a picture and identify the components in Euclid's 5th, including the transversal, the angles, and the point of intersection.

14. Drag point E so that the assumptions of Euclid's 5th still show as holding like the same side interior angles appear less than 180°. Does the conclusion about meeting on that side always seem to hold?

15. If not, sketch a picture showing what appears to be a contradiction to Euclid's 5th.

16. Based on #12–15, is Euclid's 5th postulate true in hyperbolic geometry? A postulate holds in a geometry if it holds in all cases.
17. **Circle** ☺✓, or, if you received help beyond people present during class, and/or outside resources beyond our course materials **Cite/Disclose**, verify their accuracy and revise so that it is in your own words and based on our course content and language, as required by syllabus policies.
18. **Help each other and PDF responses to ASULearn**: If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit this, continue reviewing and solidifying or discuss upcoming class work. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.