

Worksheet on Hyperbolic Geometry Part 3

Dr. Sarah's MAT 3610: Introduction to Geometry

Physical Geometry Manipulative: hyperbolic crochet model, missing square puzzle

Goals: • IGS Exploration

I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate that they seem to apply in a wide variety of examples.

- Proof Considerations

I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.

- Geometric Perspectives

I can compare and contrast multiple geometric perspectives.

Welcoming Environment: Actively listen to others and encourage everyone to participate and try to help each other! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss, handwrite and ask me questions during group work time as well as when I bring us back together:

1. **Building Community:** What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

Pythagorean theorem

2. Open up

<https://www.geogebra.org/m/jxejcdy5>

to explore the Pythagorean theorem in hyperbolic geometry. Does the Pythagorean theorem appear to hold in this model of hyperbolic geometry?

3. Drag the vertices but keep them all inside the disc and keep the configuration the same. What seems to happen to the Pythagorean theorem in small triangles that still show the hyperbolic sides bowed in?
4. Sketch at least one picture that relates and identify geometric components.

5. Open up

<https://www.geogebra.org/m/akwvszgu>

–under the first wrench tool, find the **Hyperbolic Segment** tool and create a segment \overline{AB}

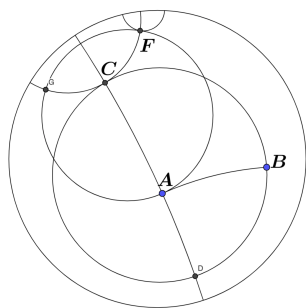
–under the second wrench, apply the **Hyperbolic Perpendicular At Point** by selecting A first and then B . You should now have a hyperbolic perpendicular through A to \overline{AB} .

–under the first wrench, use the **Hyperbolic Circle** to select A first and then B . This is a hyperbolic circle with center A and radius \overline{AB} .

–under the point tool, choose **Intersect** and select the perpendicular and the circle.

–next apply the **Hyperbolic Perpendicular At Point** again, but now select C first and then A .

- apply the **Hyperbolic Circle** and choose C and then A
- Intersect** the new circle with the latest perpendicular.
- Select the intersection point closest to B and use **Hyperbolic Perpendicular At Point** to select that point (F in my diagram) and C .



We have three equal width segments (\overline{AB} , \overline{AC} , and \overline{CF} in my picture) and three 90° angles. In Euclidean geometry, the figure would have already closed up to form a square. Here in this model does this form a square that encloses four 90° angles and four equal sides?

6. Drag A and B —if we can still clearly see the hyperbolic sides bowing/curving, can we form a square as we move A and B around?
7. As a review, sketch a diagram from the Euclidean proof of the Pythagorean theorem from *Euclid's Elements*.
8. As a review, sketch a diagram from the 周髀算經 or *Zhoubi Suanjing* proof and puzzle of the Euclidean Pythagorean theorem.
9. Based on your explorations above, what goes wrong with these proofs in hyperbolic geometry?
10. As time allows, work on the missing square puzzle.
11. **Circle ☺✓**, or, if you received help beyond people present during class, and/or outside resources beyond our course materials 🖱️ **Cite/Disclose**, verify their accuracy and revise so that it is in your own words and based on our course content and language, as required by syllabus policies.
12. **Help each other and PDF responses to ASULearn:** If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit this, continue reviewing and solidifying or discuss upcoming class work. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.