Worksheet on Measurement

Dr. Sarah's MAT 3610: Introduction to Geometry

Physical Geometry Manipulatives: bucket with water or sand and power solids set

Goals:
- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate that they seem to apply in a wide variety of examples.
- Geometric Perspectives
  I can compare and contrast multiple geometric perspectives.

Welcoming Environment: Actively listen to others and encourage everyone to participate and try to help each other! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss, handwrite and ask me questions during group work time as well as when I bring us back together:

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

2. Archimedes (c. 287–212 BCE) is considered to be one of the greatest mathematicians and scientists. According to Plutarch (c. 45–120) in *Parallel Lives: Marcellus*, Archimedes had requested that a pictorial representation of a sphere and a cylinder appear on his tombstone. From this, we might infer that he could have considered his work on a sphere and a cylinder to be one of his greatest accomplishments. Cicero (c. 106-43 BCE) in *Tusculan Disputations* Book V Sections 64–66, states that he went to Syracuse and indeed found the grave which contained the pictorial representation along with text verses.

   The formulas for the volume and surface area of a cylinder were known before Archimedes' time, but those for a sphere were not known. Archimedes wanted to find exact expressions for the volume and surface area of a sphere. Fill up the sphere from the power solids set with sand or water and pour it into the cylinder. Eyeball this: approximately what fraction of the cylinder does the sphere take up?

3. How many cones of sand or water does it take to fill up the cylinder? Test this.

4. What fraction of the cylinder does the cone take up?

5. Use only your answers in 2. and 4. to make (and write down) a conjecture relating the cylinder to the cone plus the sphere.

6. Test your conjecture with sand or water and explain your results.

7. Sketch the figures and label all the dimensions in terms of r, the radius of the sphere, including the height of the cone and cylinder as $2r$—a function of r.

8. Next, test your conjecture by using formulas for the volume of these three objects, in terms of r. Show work.

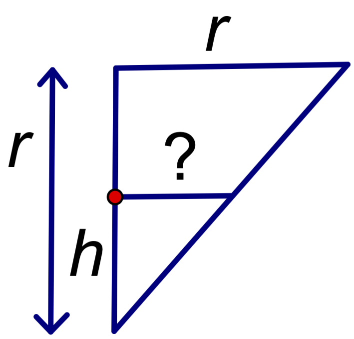
9. Archimedes was trying to derive the formula for the volume of a sphere, so he could not assume this formula anywhere in his work. Instead he used slices to argue they were the same area, giving the same volume overall—much like we do now in integration, but not in that terminology. Slide the red point on the sphere, the one at the right angle, to see the cross sections dynamically on Walter Fendt's Volume of a Sphere (Cavalieri's Principle) app at
https://www.walter-fendt.de/html5/men/volumesphere_en.htm
Stop the slide somewhere in the middle and sketch the top figures and their labels here—of the hemisphere and the cone inside the cylinder. Note that Walter Fendt is a retired teacher from the Paul-Klee-Gymnasium Gersthofen, a secondary school in Germany.

10. Solve for s on the sphere in terms of h and r. Show work.

11. What familiar theorem is needed here?

12. What is the area of the shaded disk on the sphere, in terms of h and r?

13. For the annulus between the cone and the cylinder, I have labeled a side with a question mark, which Walter Fendt has as h, but why is it so? Assume that we have right angles between r and r and between h and ?. Explain/show reasoning for a theorem and then apply it to show ? = h.



14. What familiar theorem is needed here?

15. Back from Walter Fendt's dynamic visualization, sketch the annulus between the cone and the cylinder and label h and r on it, as in the bottom right figure.

16. To find the area of the annulus between the cone and the cylinder, in terms of h and r, we can subtract the area of the inner disk from the area of the outer disk. Show work.

17. Compare the area of the disk on the sphere in #12 to the area of the annulus between the cone and the cylinder. Show this.

18. If we add up the cross sectional areas, then we form the volume (that's integration in our Calculus I with Analytic Geometry and Calculus II with Analytic Geometry terms!), so what would that show us about the volume of the sphere compared to the volume of the region between the cone and the cylinder?

19. **Circle** ☺✓, or, if you received help beyond people present during class, and/or outside resources beyond our course materials 🤝 **Cite/Disclose**, verify their accuracy and revise so that it is in your own words and based on our course content and language, as required by syllabus policies.
20. **Help each other and PDF responses to ASULearn**: If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit this, continue reviewing and solidifying or discuss upcoming class work. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.