NEW 4TH YEAR HIGH SCHOOL MATH COURSES

A LOT HAS CHANGED!

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WHAT HAPPENED?

In Fall of 2020 *Precalculus*, *Discrete Mathematics*, and *Advanced Functions and Modeling* were replaced by redesigned courses titled *Precalculus*, *Discrete Mathematics for Computer Science*, and *NC Math 4*. This session will provide an overview of the new courses.

~~DISCRETE MATH~~ ↦ ↦ ↦
DISCRETE MATH FOR COMPUTER SCIENCE

- Was: The purpose of this course is to introduce students to the mathematics of networks, social choice, and decision making. The course extends students' application of matrix arithmetic and probability. Applications and modeling are central to this course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.
- Will be: The purpose of this course is to introduce discrete structures that are the backbone of computer science. Discrete mathematics is the study of mathematical structures that are countable or otherwise distinct and separable. The mathematics of modern computer science is built almost entirely on discrete mathematics, such as logic, combinatorics, proof, and graph theory. At most universities, an undergraduate-level course in discrete mathematics is required for students who plan to pursue careers as computer programmers, software engineers, data scientists, security analysts and financial analysts. Students will be prepared for college level algebra, statistics, and discrete mathematics courses.

DISCRETE MATH DIFFERENCES

- Matrices and Graphs: The coverage has been expanded and now includes
  - Specific matrix models such as Leslie Models, Markov Chains
  - Specific graph algorithms such as critical path analysis, nearest neighbor, Kruskal and Prim
- Statistics and Probability: The coverage has been greatly reduced and now includes only permutations, combinations, and the fundamental counting principle (gone are theoretical and experimental probability as well as voting and apportionment
- Recursion and Sequences and Series: Remains largely unchanged, with a slight reduction in emphasis
- Added:
  - Number and Set Theory: sets, subsets, set operations, Venn diagrams, Euclidean algorithm, Fundamental Theorem of Arithmetic, GCD, LCM, prime numbers
  - Logic: truth tables, analysis of logical arguments, Boolean logic

EXAMPLE

DCS.N.4 Understand statements related to number theory and set theory.

DCS.N.4.3 Conclude that sets are equal using the properties of set operations.

Clarification

Students may use a variety of tools to determine the equality of sets, including the listing method, Venn diagram, and logical statements. Sets are equivalent when the sets contain the same elements.
Students are expected to know the basic properties of set operations for unions and intersections.

Properties	**Unions**	**Intersections**
Commutative	If $A \cup B$, then $B \cup A$.	If $A \cap B$, then $B \cap A$.
Associative	If $(A \cup B) \cup C$, then $A \cup (B \cup C)$.	If $(A \cap B) \cap C$, then $A \cap (B \cap C)$.
Distributive	If $A \cup (B \cap C)$, then $(A \cup B) \cap (A \cup C)$.	If $A \cap (B \cup C)$, then $(A \cap B) \cup (A \cap C)$.
Empty Set	$A \cup \varnothing = A$	$A \cap \varnothing = \varnothing$

Checking for Understanding

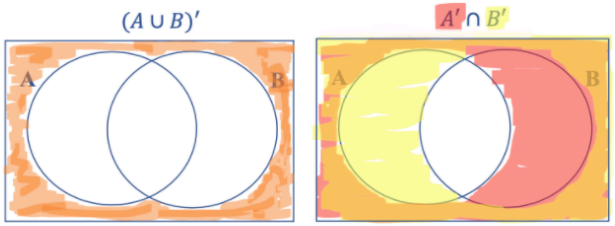
Indicator: Make a conjecture about the following:

a. the number of subsets of a set with five elements.
b. the number of subsets of a set with n elements.

Answer: a. 2^5 subsets, b. 2^n subsets

Indicator: Provide a justification for the following:
If A and B are sets, then $(A \cup B)' = A' \cap B'$

Answer: Using Venn Diagrams to show equivalence.



~~PRECALCULUS~~ ↦ ↦ ↦ PRECALCULUS

- Was: the purpose of Precalculus is to provide students with an honors-level study of trigonometry, advanced functions, analytic geometry, and data analysis in preparation for calculus. Applications and modeling should be included throughout the course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.
- Will be: The purpose of Precalculus is to build upon the study of algebra, functions, and trigonometry experienced in previous high school mathematics courses. This course will build on students' algebraic skills and understanding of functions to delve into real world phenomena and to deepen understanding of the functions in the course. This course is designed for students pursuing careers in STEM-related fields. Students will be prepared for Calculus, AP Calculus and any entry-level college course.

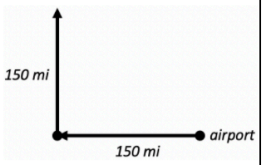
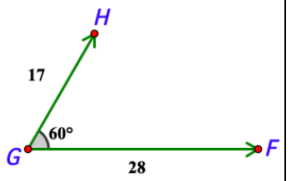
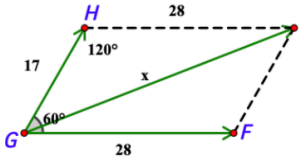
PRECALCULUS DIFFERENCES

- General focus: Studying families of functions (linear, exponential, power, rational, logarithmic, trigonometric)
- Added/Increased:
  - Complex number properties and arithmetic
  - Matrix arithmetic properties
  - Inequalities that include rational and polynomial expressions
- Emphasized less:
  - Polar coordinates (although parametric equations are still included)
- Gone: Fitting functions to data and evaluating goodness of fit

EXAMPLE

PC.N.3 Understand properties and operations with vectors.
PC.N.3.2 Execute sum and difference algorithms to combine vectors.

Clarification	Checking for Understanding

Students have not previously learned how to add and subtract vectors. In Precalculus students should be familiar with three different methods for adding and subtracting vectors:
• Adding vectors end-to-end - positioning the vectors (without changing their magnitudes and directions) so that the initial point of one vector coincides with the terminal point of the other vector
• Adding/subtracting corresponding components - add or subtract the corresponding components
• Using the parallelogram rule - a graphical method used for:
  ○ addition of two vectors,
  ○ subtraction of two vectors, and
  ○ resolution of a vector into two components in arbitrary directions.
 	
Students should understand the process of finding the sum/difference of two vectors using any of the methods mentioned. They are not expected to know the name of the methods.
Students are not expected to execute procedures for vectors beyond 2-dimensional vectors.
 	
Please note the sum and difference algorithms refer to adding and subtracting, respectively, vectors using the methods explained above. Students are not expected to know how to compute the dot product or find a unit vector.
	Indicator: Given the vectors $u = \langle -10, 12 \rangle$ and $w = \langle 5, -10 \rangle$
	a. Find $u + w$.
	b. Find $u - w$.
	Answers: a. $\langle -5, 2 \rangle$, *b.* $\langle -15, 22 \rangle$
	
	Indicator: A pilot flies a plane that takes off from an airport and travels due west for 150 miles. The pilot then turns due north and travels 150 miles. Use vectors to describe the location of the plane from the airport.
	Answer: $\langle -150, 0 \rangle + \langle 0, 150 \rangle = \langle -150, 150 \rangle$
	
	 
	
	Indicator: Given vector GH and vector GF with an angle of 60 degrees between them, find the magnitude of the resultant vector.
	Answer:
	 
	 
	$x^2 = 17^2 + 28^2 - 2(17)(28) * cos(120)$
	$x = 39.36$

Return to: Standards

~~ADVANCED FUNCTIONS AND MODELING~~ ↦ ↦ ↦ NC MATH 4

- Was: AFM provides students with an in-depth study of modeling and applying functions. Home, work, recreation, consumer issues, public policy, and scientific investigations are just a few of the areas from which applications should originate. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.
- Will be: The primary focus of this course is on functions and statistical thinking, continuing the study of algebra, functions, trigonometry and statistical concepts previously experienced in NC Math 1-3. The course is designed to be a capstone to introductory statistical concepts. Additionally, the course intentionally integrates concepts from algebra and functions to demonstrate the close relationship between algebraic reasoning as applied to the characteristics and behaviors of more complex functions. In many cases, undergraduate students majoring in non-STEM fields will take an entry-level Algebra or Introductory Statistics course. Students will be prepared for college level algebra and statistics or as a bridge to prepare students for Precalculus or other advanced math courses.

"THE OTHER MATH" DIFFERENCES

- Still included:
  - Using functions to solve problems: log, piece-wise, power, trig, recursively defined
  - Fitting functions to data and determining goodness of fit
- New:
  - Complex number, matrix, and vector operations
  - An overview of statistical inference and investigations of large sets of real-world data with technology
- Enhanced:
  - Understanding composition of functions
  - More focus on the normal distribution in probability
- De-emphasized: modeling and applying functions from areas such as home, work, recreation, consumer issues, public policy, and scientific investigations

EXAMPLE

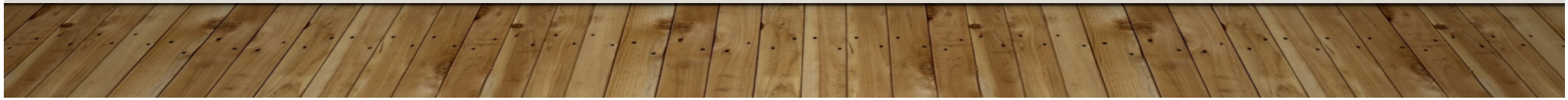
NC.M4.AF.1 Apply properties of function composition to build new functions from existing functions.

NC.M4.AF.1.1 Execute algebraic procedures to compose two functions.

Clarification	Checking for Understanding
Students will extend their understanding of function notation and evaluating functions to working with compositions of two functions. This includes evaluating a composition of functions for specific values in the domain. Students will understand that the net effect of $f \circ g$ is $a \to g(a) \to f(g(a))$ for any real value, a. Students understand that a is in the domain of g and that $g(a)$ is in the domain of f. There is no limit as to the type of functions to be used when composing functions. In problems involving roots in the denominator, it is not necessary to rationalize the denominator.	**Indicator**: Find $(f \circ g)(x)$ if $f(x) = -6x + 11$ and $g(x) = 3x - 5$. *Answer:* $-18x + 41$ **Indicator:** Given the two functions, $f(x) = 2x - 3$ and $g(x) = x^2 + 1$, find the following: a. $f(g(x))$ b. $g(f(x))$ c. $(f \circ f)(x)$ d. $(g \circ g)(x)$ *Answers: a.* $2x^2 - 1$, *b.* $4x^2 - 12x + 10$, *c.* $4x - 9$, *d.* $x^4 + 2x^2 + 2$ **Indicator:** Given $f(x) = \frac{1}{x^2+4}$ and $g(x) = \sqrt{x+1}$. Find $f(g(x))$ and $g(f(x))$. *Answers:* $f(g(x)) = \frac{1}{(\sqrt{x+1})^2 + 4}$ *and* $g(f(x)) = \sqrt{\frac{x+5}{x+4}}$

MY TAKE ON THE CONSEQUENCES…

- Precalculus has changed in subtle but substantial ways. How does that impact our algebra, trig, and precalculus courses?
- Our students will have MORE experience with vectors and matrices. What should we do with that information if anything?
- Our students will have MORE experience with complex numbers. What should we do with that information if anything?
- Some of our students headed for computer science BS degrees (needing calculus and linear algebra) will have LESS algebraic and trigonometric skills because they will opt for "discrete math for computer science" as their 4^{th} math course. This worries me the most…



WHERE TO GET MORE INFO

Directly from DPI: https://sites.google.com/dpi.nc.gov/k-12-mathematics/resources/hs-mathematics/4th-maths?authuser=0