Modeling Scenarios

See mathsci2.appstate.edu/~hph/modeling/ for implementations of some of these models

Spread of Disease.

Consider an island population and suppose that some small number of people leave the island and come back, bringing with them an infectious disease. If we want to predict the number of persons at any time who have the disease, we might assume that the change in the number of persons who catch the disease is some fraction of the number of possible encounters between susceptible and infected people. If the disease is one for which recovery results in immunity and it just takes some set amount of time to recover, then we could view the change in the recovered population as some fraction of infected individuals.

Here is a data set taken from a (mythical) island of 5000 inhabitants. Do the data support this model, i.e., can you find parameters that make the model come close)? For the model that comes closest, how long until everyone recovers and is immune?

t (days)	0	2	6	10
sick people	5	1887	4087	3630

Drug Dosage.

Clinical studies have shown that a reasonable model is to assume that the rate at which the concentration of a drug in the blood stream is decreasing at any given time is proportional to the concentration of the drug at that time (the constant of proportionality can be thought of as the "elimination rate" for the drug). In addition, it is common for the same amount of a drug to be administered at regular intervals.

Suppose we know that a certain drug's elimination rate is 4% per hour and the minimum effective dosage is 0.1 mg/ml and the maximum safe dosage is 0.3 mg/ml. Determine what size initial dose to deliver via injection and then how often a repeated dosage of 0.1 milligrams per milliliter should occur.

Pollution through a Chain of Lakes.

Suppose we have a chain of lakes the connect to each other via rivers (like the Great Lakes) where there is a pollution source in the inner most lake. If we consider only flow of water between the lakes, we could model the flow of pollutant between the lakes if we know the average volume of water in each lake and the water flow rates between the lakes.

Suppose we have the following information about flow rate / volume: Lake 1 is spring fed with fresh water; 20% of lake 1 flows into lake 2 each month, 18% of lake 2 flows into lake 3 each month, and 16% of lake 3 is flushed into the ocean each month. If 100 kg/month of pollutant is dumped into Lake 1 for five months before the leaky pipe is found, will pollution levels ever exceed 200 kg in any of the lakes? How long before the pollution is essentially washed out to sea?

Harvesting in a Shrimp Farm.

Suppose we are raising shrimp in an enclosed area for sustainable farming, and that we know that there is a maximum sustainable quantity of shrimp for the amount of nutrients and the size of the area. Thus the rate of growth of the shrimp population is proportional to the number of shrimp times a limiting factor that approaches to 0 as the population of shrimp approaches the maximum sustainable population. The rate of decrease of the shrimp population will depend on how many we harvest, and we plan to harvest a constant amount each month.

Suppose we have the following parameters: maximum sustainable population = 77000; initial population = 5000. Also we have done some testing, and for small numbers of shrimp the population doubles each month. How does the population grow if we harvest 5000 shrimp over the course of a month? Can we harvest 10000 each month without having the population crash? What if we restock with 5000 shrimplets over the course of the month?

Simple Chemical Reaction.

A simple chemical reaction can be thought of as follows: We have a "reactant" that reacts with an "intermediary" compound to produce a "product." The key ideas behind the kinetics:

- The rate at which the reactant changes to the intermediary during the first reaction is proportional to the amount of both the reactant and the intermediary compounds – both are needed for the reaction to occur.
- The rate at which the intermediary then converts to the product during the second reaction is proportional to the amount of the intermediary compound that is present.
- The two reaction rates – usually referred to as k_1 and k_2 depend on the specific compounds involved.

Suppose we have 1000 moles of the reactant and 1 mole of the intermediary in the beaker initially, and that reaction rate constants are 0.005 and 0.05 per second, respectively. How quickly does the reaction occur?