Chapter 12

Choosing Your Voters

Focus Questions

In this chapter, we'll explore the following questions:

- What is gerrymandering, and what are some recent examples of gerrymandering in the United States?
- What are some of the laws and regulations that govern redistricting?
- What is compactness, and how is it related to gerrymandering? What are some different ways of measuring the compactness of a congressional district?
- What are some ways to measure the partisan fairness of a districting plan?
- What is the efficiency gap, and what kind of information does it provide about partisan fairness?
- What are some possible solutions to the gerrymandering problem?

Warmup 12.1. In the image from Figure 12.1, suppose that each circle represents a voter, with the filled circles representing Republicans and the open circles representing Democrats.

(a) If you had to divide the population into five congressional districts, each having an equal number of voters, how would you do it? Where would you draw the district boundaries?

(b) In the plan from your answer to part (a), how many districts would have a majority of Republican voters, and how many would have a majority of Democrats?

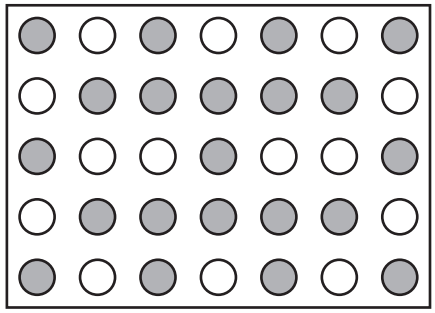


Figure 12.1. A redistricting example

(c) Suppose you are a Republican who wants to make sure that your party has a majority in as many districts as possible. Where would you draw the district boundaries, and how many districts would your party control?

(d) Repeat part (c), but this time assume that you want to maximize the number of districts in which Democrats have a majority.

As we learned in Chapter 11, changes in the U.S. population, as measured by each decennial census, may lead to states gaining or losing seats in the House of Representatives. In addition, shifts in population may cause congressional districts to have unequal numbers of residents. Therefore, each census brings an opportunity (and in most cases, a legal obligation) to redraw congressional district boundaries to create an appropriate number of equal-population districts. This redistricting process seems simple enough on the surface. However, as we saw in Warmup 12.1, there are often many ways to draw the congressional districts in a state, and some districting plans may seem fairer than others.

In the example from Warmup 12.1, 60% of the voters were Republicans, and 40% were Democrats. Since there are five districts total, you might expect that any reasonable division of the state into districts would result in three districts where Republicans hold a majority and two where Democrats are in control—and, of course, there is a way to draw the district boundaries so that this happens. But even some of the simplest solutions do not result in this outcome. For example, using four horizontal lines to divide the state into five rectangular districts results in a 4-1 split between Republican and Democratic districts. It's also possible to draw the boundaries so that Republicans control all five districts. There is an argument to be made for each of these outcomes; in fact, since Republicans do have a fairly substantial majority overall in the state, it makes sense for the state's congressional delegation to be mostly—or even entirely—Republican. But what if that didn't happen? What if, in spite of support from only 40% of the voters, Democrats won three of the five congressional districts? In fact, this is possible with a clever drawing of the district boundaries. Perhaps you even discovered a way to do it in part (d) of Warmup 12.1. If not, look back at the map and try again. Here's a hint: For Democrats to win more seats than they deserve, they'll need to make the most of their votes, winning as many seats as possible by razor-thin margins. Republicans are going to win at least two districts regardless of how the boundaries are drawn (do you see why?), so Democrats will be best served if the Republicans win those two districts by unnecessarily wide margins, with the remaining Republican votes spread out among the three remaining districts.

Gerrymandering

As we've seen, the outcome of congressional redistricting depends not only on the distribution of the voters, but also on where the district boundaries are drawn. The potential to manipulate the system in favor of one party or another is significant. In fact, President Barack Obama put it this way in his book, *The Audacity of Hope* ([**38**], p. 103):

> *"These days, almost every congressional district is drawn by the ruling party with computer-driven precision to ensure that a clear majority of Democrats or Republicans reside within its borders. Indeed, it's not a stretch to say that most voters no longer choose their representatives; instead, representatives choose their voters."*

In light of this observation, it shouldn't surprise us too much to see congressional districts like the ones shown in Figures 12.2–12.4.

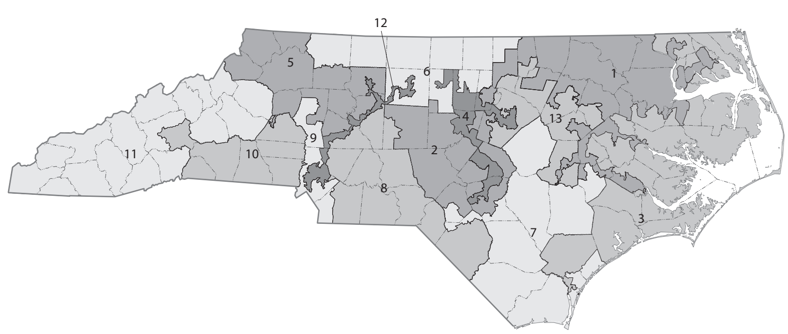


FIGURE 12.2. North Carolina congressional districts, 2013–2016

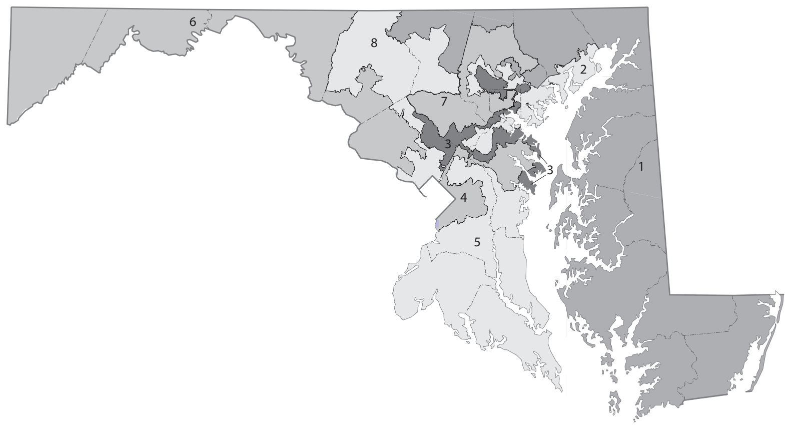


Figure 12.3. Maryland congressional districts, as of 2013

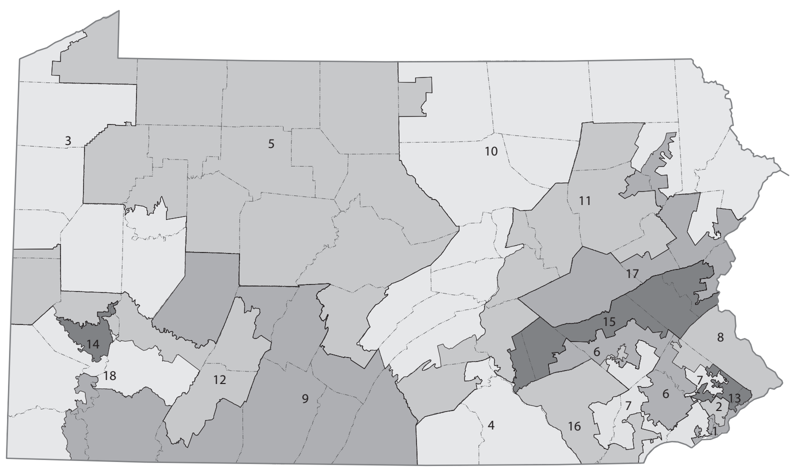
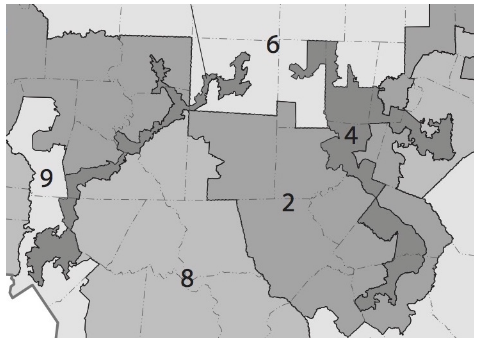


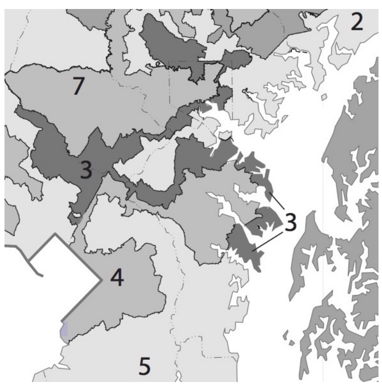
Figure 12.4. Pennsylvania congressional districts, 2013–2018

Notice that in each of these examples, several congressional districts have been drawn with strange, convoluted shapes. If we zoom in on a few areas, as shown in Figure 12.5, we can get an even better look. In each of these cases, districts have been drawn in a way that gives an advantage to one party or the other by concentrating like-minded voters into non-competitive districts and/or dividing like-minded voters among multiple districts, thereby diluting their power. These techniques are called *packing* and *cracking*, respectively.

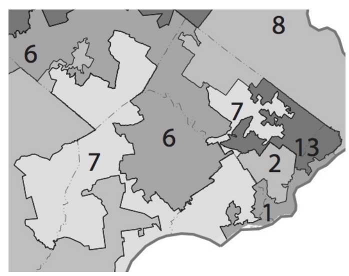
Question 12.2.* In the districting plan shown in Figure 12.6, Republicans earn only 12 of the 25 votes, but win 4 of the 5 seats. Explain how this plan uses packing and cracking to give Republicans an advantage.



(A) Winston-Salem, Charlotte, Greensboro, Raleigh, Durham



(B) Baltimore / Annapolis region



(C) Philadelphia region

FIGURE 12.5. Redistricting closeups

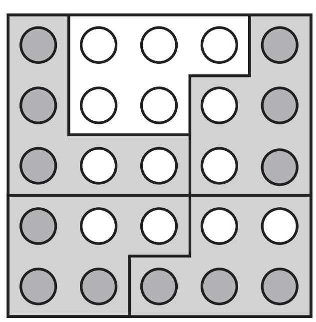


FIGURE 12.6. An example of packing and cracking

Oddly-shaped congressional districts, like those in Figures 12.2–12.6, are often viewed as evidence of *gerrymandering*, which is the practice of drawing congressional districts to give an advantage to a political party or class of voters. The word *gerrymandering* was coined in 1812 when Massachusetts Governor Elbridge Gerry passed a bill to redraw the State Senate election districts in order to favor his party. The resulting map included a district that looked to some like a salamader. The *Boston Weekly Messenger* subsequently printed an editorial cartoon, shown here in Figure 12.7, that stylized the district in question as a mythical dragon, dubbed the "Gerry-Mander".

FIGURE 12.7. The "Gerry-Mander", from the *Boston Weekly Messenger*, 3/26/1812

While Elbridge Gerry may have been the first to engage in gerrymandering, he certainly was not the last. In fact, there have been numerous recent court cases alleging unconstitutional violations of voter rights as a result of gerrymandering. While a detailed discussion of the legal issues surrounding gerrymandering is beyond the scope of our investigations here, it is important to note the two main types of claims that are typically brought in redistricting cases.

First, courts have regularly ruled redistricting plans to be unconsitutional when district boundaries are drawn to disadvantage or marginalize a particular racial or ethnic group. The 2013–2016 redistricting of North Carolina is one of the more recent examples of an unconstitutional racial gerrymander. In *Cooper* v. *Harris* (2017), the U.S. Supreme Court ruled that the plan violated the Equal Protection Clause of the 14th Amendment by packing African American voters into the the 1st and 12th districts. (See Figure 12.8.)

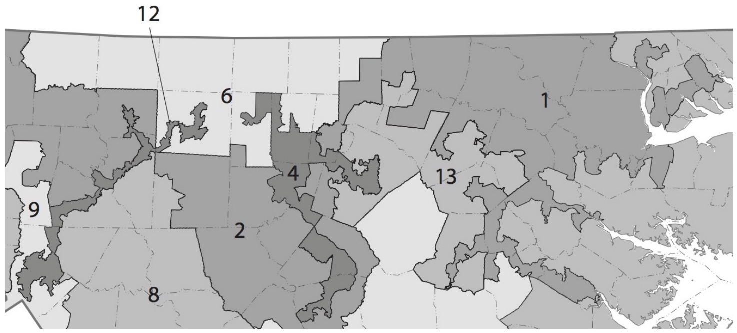


FIGURE 12.8. North Carolina's 1st and 12th districts, 2013–2016

But what about when districts are gerrymandered with partisan, but not racial, motivations? On this issue, the law remains unclear—or at least that was the case when this book was written. By the time you read it, the situation may have changed due to several recent and pending lawsuits. For example:

- In 2015, a lawsuit was brought against the State of Wisconsin after the Republican-controlled legislature drew districts that led to Republicans winning 60 out of the 99 seats in the 2012 State Assembly election—in spite of receiving less than half of the vote statewide! In 2016, a three-judge panel ruled 2-1 that the plan was unconstitutional, which prompted an appeal by the State of Wisconsin to the U.S. Supreme Court. Oral arguments for the case, now known as *Gill* v. *Whitford*, took place in 2017, with a ruling expected in 2018.
- In January 2018, the Pennsylvania Supreme Court ruled the state's congressional districts to be in violation of the State Constitution by unfairly favoring Republicans. The U.S. Supreme Court declined to block the Pennsylvania court's order to redraw the districts, but this decision did not establish legal precedent for other cases since the original decision was based on Pennsylvania's *State* Constitution, rather than federal law.
- The Maryland map has been the subject of a protracted legal battle since 2013 and is now awaiting an appeal from the U.S. Supreme Court.
- After the previous redistricting of North Carolina was struck down due to racial gerrymandering, a new map was drawn—and contested on the basis of unconstitutional *partisan* gerrymandering! In January 2018, a three-judge panel ruled that the new map was unconstitutional and ordered that the districts be redrawn. The U.S. Supreme Court subsequently blocked the order, pending the decisions on other cases such as *Gill* v. *Whitford*.

What distinguishes these cases from previous ones is the fact that they are based entirely on allegations of partisan—not racial—gerrymandering. So that will be our primary focus in this chapter. And, of course, since this is a book on the *mathematics* of voting and elections, we'll look at several different ways that mathematics can be used to identify gerrymandered districts.

Rules for Redistricting

Courts have held that the 14th Amendment to the U.S. Constitution requires congressional districts to have equal populations as much as is practical. Apart from that requirement and the requirements of the Voting Rights Act—which we'll discuss later—other rules for redistricting are determined on a state-by-state basis, normally by the state legislatures that are in charge of drawing the districts. A few common requirements include:

- **Contiguity**, meaning that all the parts of a congressional district must be physically connected in some way. This requirement can lead to some creative uses of unpopulated areas such as roads, bridges, and even exit ramps to connect parts of a district that would otherwise be geographically separated from each other.
- **Compactness**, meaning that districts should be…compact. If you think that's not a very good definition, you're right. Many states don't specifically define what compactness means, but the general sense is that the residents in a district should live relatively close to one another, and that congressional districts should have relatively normal-looking shapes.
- **Preserving political boundaries**, meaning that, whenever possible, district boundaries should not cut through the middle of counties, cities, or other municipal entities.
- **Preserving communities of interest**, meaning that, whenever possible, district lines should not separate groups of voters who live near one another and have common political, social, or economic interests.

Geometry and Compactness

Although most state laws governing redistricting allow a lot of room for interpretation, the requirement of compactness is one that is particularly nebulous.

Question 12.3. The laws in the state of Idaho specify that "to the maximum extent possible, [redistricting plans] should avoid drawing districts that are oddly shaped." Using this incredibly precise definition of compactness, put the districts in Figure 12.9 in order from "least oddly shaped" to "most oddly shaped."

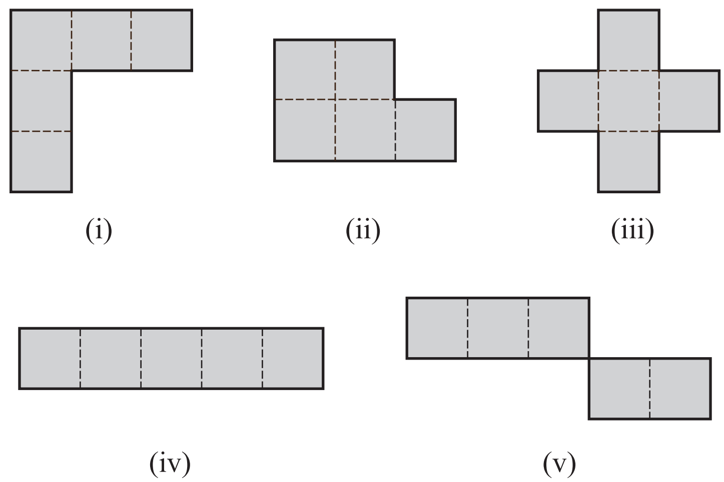


FIGURE 12.9. Some hypothetical district shapes

Question 12.4.* The state of Iowa considers compact districts to be those which are "square, rectangular, or hexagonal in shape, and not irregularly shaped, to the extent permitted by natural or political boundaries." When comparing two redistricting plans, Iowa state law prescribes two specific measures of compactness: length-width compactness and perimeter compactness.

(a) Length-width compactness assumes that a district is most compact when its length and width are equal. A district's compactness under this measure is the absolute value of the difference between its length and its width. Using this definition, order the districts in Figure 12.9 from most compact to least compact.

(b) Perimeter compactness assumes that the most compact districts are those with the shortest perimeter. (Recall that the perimeter of a shape is the length of its boundary.) Using this definition, order the districts in Figure 12.9 from most compact to least compact.

Question 12.5.* In the state of Michigan, districts are required to be "as compact as possible, measured by drawing a circle around the district, and assessing the area within the circle (and within the landmass of the state) but outside the district lines." In other words, the tighter a circle can be made to fit around the district, and the more the district fills up this circle, the more compact the district is. Using this definition of compactness, order the districts in Figure 12.9 from most compact to least compact. (Hint: All of the districts in Figure 12.9 have the same area, so Michigan's compactness measure will be completely determined by the size of the smallest circle that encloses each region. The diameter of this smallest circle will be the distance between the two points in the district that are the farthest away from one another. Or, if all else fails, just draw some circles and eyeball it.)

As you may have seen in Questions 12.3–12.5, different measures of compactness can tell different stories about which districts are more or less compact. There are a lot of other measures of compactness that we haven't looked at, but most of them measure characteristics like indentation or convexity, jaggedness, and elongation or dispersal (how far the area of the district is dispersed from its center). In past rulings, courts have viewed odd shapes as "persuasive circumstantial evidence" of racial gerrymandering and have ruled maps unconstitutional on the basis of shape. However, there is no universally accepted standard for compactness—and at times, no standard at all other than the "eyeball test" suggested by laws like those in Idaho. In addition, it's important to note that not every district that fails to meet compactness standards is the result of an unconstitutional gerrymander. This is where the law gets even murkier, with competing and sometimes even contradictory requirements.

As an example, consider Illinois' 4th congressional district, which consists of two separate neighborhoods in the Chicago area connected by a thin strip of Interstate 294, as shown in Figure 12.10.

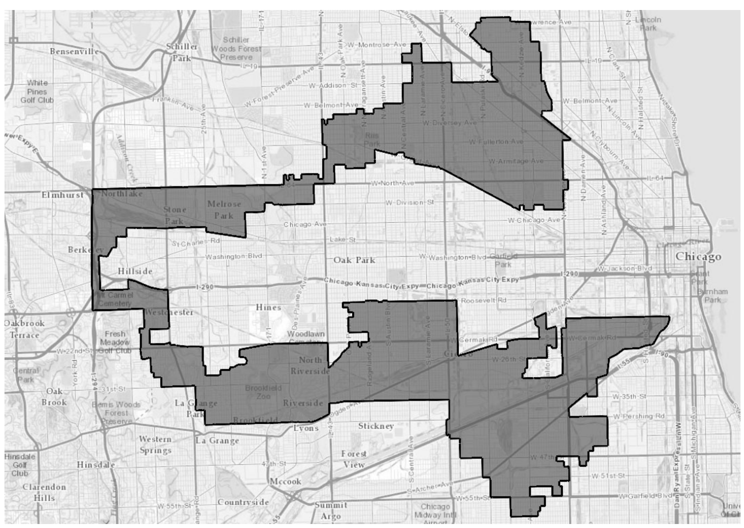


FIGURE 12.10. Illinois' 4th congressional district

Question 12.6. How does Illinois' 4th congressional district measure up using the definitions of compactness from Idaho, Iowa, and Michigan? Explain your answer.

At first glance, you may think that the strange shape of Illinois' 4th congressional district is clear evidence of gerrymandering, and in one sense you'd be right. This district *is* in fact gerrymandered, but not for nefarious purposes. In fact, it was created as a result of a lawsuit to join two Hispanic communities for the purpose of *strengthening* their voting power. In essence, Hispanic voters were *packed* into one district in order to prevent them from being *cracked* into two separate districts—an action that would have diluted their power in violation of the 1965 Voting Rights Act.[1] So while courts have ruled that district lines cannot be drawn on the basis of race, the Voting Rights Act sometimes requires race to be a factor in order to create majority-minority districts that allow racial and ethnic minorities an opportunity to elect a candidate of their choosing. This tension is what some have referred to as the *Goldilocks rule*: those charged with drawing district boundries must think about race and enthnicity—but not too much.

In the same way that strange, non-compact shapes should not be taken as conclusive evidence of unconstitutional gerrymandering, we must also be careful to not assume that a districting plan is fair simply because its districts are compact. The next question gives an example to illustrate this point.

Question 12.7. Consider a square state with voters distributed as shown in Figure 12.11. Note that there are 72 Republican voters (represented by filled circles) and 90 Democrats (represented by open circles).

(a) If this state was to be divided into nine districts, and the number of seats won by each party was perfectly proportional to the number of votes they received, how many seats would each party win?

(b) Use the tick marks on the boundary of Figure 12.11 to divide the state into nine equal-sized, square-shaped districts. Using these districts, how many seats will each party win?

(c) Now try to draw nine equal-population districts that would result in each party winning the number of seats you specified in part (a).

(d) In which of your plans—part (b) or part (c)—were the districts more compact? Explain your answer.

Partisan Symmetry

Since compactness can't tell us the whole story—particularly when it comes to identifying partisan gerrymandering—we need to consider other ways of identifying when a districting plan gives an unfair advantage to one party or the other. One solution, which seems simple on the surface, would be to use proportional representation as an ideal standard. For example, we could

[1] Interestingly, one of the attorneys originally involved in the creation of the district is reported to have said that they may have gone "a little too far," since the same representative, Congressman Luis Gutiérrez, has won the district in every election since 1992, capturing between 77 and 100 percent of the vote.

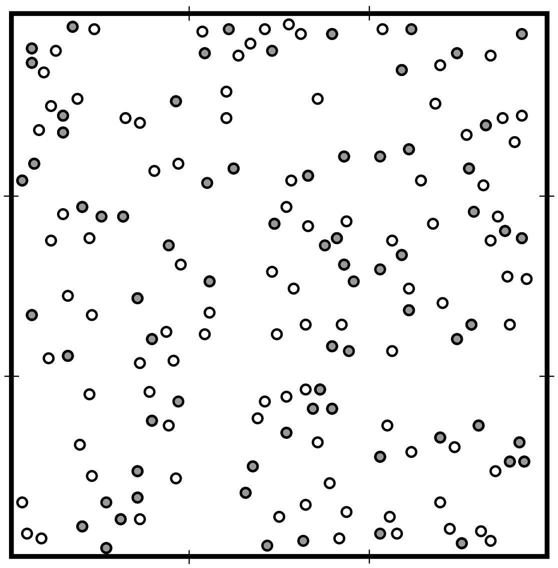


FIGURE 12.11. A square state

say that if Republicans win 57% of the vote, then they should win 57% of the seats.

Question 12.8. Suppose that eight seats are up for grabs, and Republicans win 57% of the vote. How many seats should they win?

Apart from the fact that achieving exact proportional representation may be numerically impossible—at least not without fractional seats—the U.S. Supreme Court has held that "the mere lack of proportional representation will not be sufficient to prove unconstitutional discrimination" (*Davis* v. *Bandemer*, 1986). The idea of a *winner's bonus*—that is, the winning party receiving more seats than they would be entitled to under a purely proportional system—is fairly widely accepted. The bigger question, at least in regards to fairness, concerns the notion of *symmetry*. To illustrate, suppose Republicans won 57% of the vote but won 75% of the seats. They earned a pretty hefty winner's bonus, but was the districting plan unfairly biased against Democrats? To answer this question, we could consider what would happen if the tables were turned and Democrats won 57% of the vote. Would they now win 75% of the seats? If so, the districting plan shows evidence of symmetry: each party has a chance to receive a winner's bonus, but they do have to be a winner in order to get it. If, on the other hand, Republicans won only 43% of the vote but retained their winner's bonus—still winning more than half of the seats—then we would suspect that something was amiss.

In order to test a plan's symmetry, we need to consider hypothetical questions about what might happen if voters switched their votes from one party to the other. To answer these kinds of questions, we'll need to make assumptions about how the votes in each district change when the statewide vote totals change. The simplest and most common assumption is one called *uniform partisan swing*—meaning that, in each district, the percentage of voters who change their votes from one party to the other is the same as the percentage of voters who switch statewide. So, for example, if 10% of Democratic voters *statewide* change their votes to the Republican candidate, then we'll assume that 10% of the Democrats *in each district* change their votes. In other words, we'll assume that the voters who switch from one party to the other are distributed proportionally among the various districts.

Question 12.9.* In the districting plan shown in Figure 12.12, assume that each circle represents 100 voters, with the filled circles representing Republican voters and the unfilled circles represented Democrats. Notice that Democrats earn 1300 of the 2500 votes statewide (52%) but win 4 out of 5 districts (80%—a significant winner's bonus!).

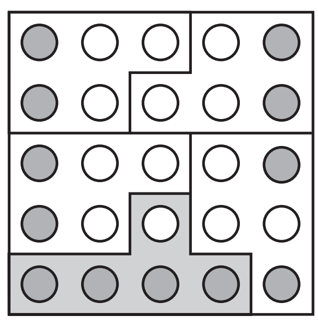


FIGURE 12.12. Measuring partisan fairness

(a) How many Democrats would need to switch their votes in order for Republicans to win 52% of the vote statewide? What percent of the total number of Democrats is this?

(b) In each of the four districts won by Democrats, there were 300 Democratic voters and 200 Republicans. Suppose that the same percentage of voters you identified in part (a) switched their votes from Democrat to Republican. Who would now win each of these four districts, and what would the vote totals be?

(c) In the remaining district, which was won by Republicans, there were 400 Republican voters and 100 Democrats. What would happen if the same percentage of Democratic voters you identified in part (a) switched their votes. Would the district still be won by Republicans?

(d) Combine your answers to parts (a)–(c) to determine how many of the five districts Republicans would win if they won 52% of the vote statewide.

(e) Does your answer to part (d) seem fair? Does this districting plan treat Democrats and Republicans equally? Why or why not?

You may have noticed a lack of symmetry in your answers to Question 12.9. When Democrats earned 52% of the vote statewide, they won four of the five districts. And when Republicans earned 52% of the vote statewide, Democrats *still* won four of the five districts. This is evidence that the districting plan is biased in favor of Democrats. In fact, if you take a closer look at the plan, you'll see several examples of packing and cracking.

The term *partisan bias* has a precise definition in the context of redistricting, based on the idea that when the statewide vote is split evenly, each party should win the same number of districts.

Definition 12.10. Suppose that, under the assumption of uniform partisan swing, a party wins $x\%$ of the districts when they earn 50% of the statewide vote. The **partisan bias** with respect to this party is equal to $(x - 50)\%$, where a positive result indicates bias in favor of the party, and a negative result indicates bias against the party.

Question 12.11.

(a) What is the partisan bias with respect to Democrats in the districting plan from Figure 12.12?

(b) Assuming uniform partisan swing, what is the minimum percentage of the statewide vote that Democrats could earn and still win four of the five districts?

Although partisan bias and related measures can help us identify when a districting plan gives one party an unfair advantage, they do require us to consider hypothetical scenarios or *counterfactuals*, using simplifying assumptions like uniform partisan swing to complete the required calculations. Courts have not looked favorably on the use of such counterfactuals. In fact, in a case involving a redistricting plan in Texas (*LULAC* v. *Perry*, 2006), U.S. Supreme Court Justice Anthony Kennedy wrote the following in the Court's majority opinion:

> *"The existence or degree of asymmetry may in large part depend on conjecture about where possible vote-switchers will reside. Even assuming a court could choose reliably among different models of shifting voter preferences, we are wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs."*

With that in mind, we'll now turn our attention to a new method for identifying partisan gerrymandering—called the *efficiency gap*—that has played a prominent role in the *Gill* v. *Whitford* case currently before the U.S. Supreme Court.

The Efficiency Gap

The *efficiency gap*, introduced by Nicholas Stephanopolous and Eric McGhee in 2015 [**49**], captures the idea that gerrymandering results in wasted votes. In a packed district, the winning party often earns significantly more votes than they would need to win the district. These excess votes are essentially wasted. In a cracked district, the losing party may earn a significant number of votes, but not enough to win. These votes are also wasted. In each case, the wasted votes could potentially make a difference in other, more competitive districts; however, because of where they are located—due to the way the district boundaries are drawn—their impact is neutralized. From each party's perspective, the most efficient use of their votes is to win as many districts as possible by the smallest possible margin, and to have hardly any votes left to waste in the districts they lose.

The efficiency gap formalizes this idea by comparing the number of votes wasted by each party across all of the districts. In a perfectly fair districting plan, each party would waste the same number of votes. Therefore, a large difference in the number of votes wasted between the two parties can be viewed as evidence of partisan gerrymandering.

The precise definition of the efficiency gap is as follows:

Definition 12.12. For each district in a districting plan, we consider the following votes to be wasted:

- All of the votes cast by the losing party
- All of the votes cast by the winning party in excess of the number needed to win the district by a simple majority

Let w_A and w_B denote the number of wasted votes for parties A and B, respectively, and let v denote the total number of votes cast. The **efficiency gap** is defined to be

$$EG = \frac{w_A - w_B}{v}.$$

Question 12.13.* Suppose the efficiency gap, as defined above, is positive. Which party has the advantage?

Question 12.14.*

(a) Calculate the number of votes wasted by each party in the districting plan from Figure 12.12. (Again assume that each circle represents

100 voters, and remember that it takes 251 votes to win a district by a simple majority.)

(b) Use your calculations from part (a) to determine the efficiency gap.

(c) Which party is favored by the plan, and how is this reflected in your calculation from part (b)?

We noted earlier that an efficiency gap of zero would—at least in theory—represent a perfectly fair and neutral districting plan. It can also be shown that the efficiency gap is always between -0.5 and 0.5. But how big of an efficiency gap is too big? Based on historical analysis, Stephanopolous and McGhee suggest a threshold of 0.08 for state house plans, and the equivalent of two seats for congressional plans. Under some simplifying assumptions, it can be shown that the two-seat threshold is equal to $2/n$, where n is the number of districts. So, for a state with five congressional districts, the threshold would be $2/5 = 0.4$. This actually doesn't rule out a lot; in fact, it's fairly hard to come up with a plan for five districts that has an efficiency gap greater that 0.4. The efficiency gap is more useful in states with a larger number of districts. For example, the 2016 maps for North Carolina and Pennsylvania (which, as noted earlier, are both the subject of recent or pending court cases) both violate the two-seat efficiency gap threshold, with gaps of 20% and 16%, respectively.

Question 12.15. Consider the districting plan shown in Figure 12.13, which you first considered in Question 12.7.

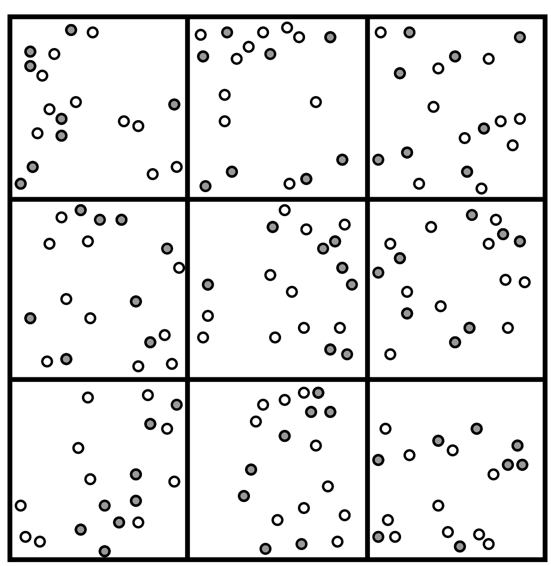


FIGURE 12.13. Gerrymandered or not?

(a) Calculate the efficiency gap of this plan.

(b) For a state with nine congressional districts, Stephanopoulous and McGhee's recommended threshold is 2/9, or approximately 0.22. How does the plan from Figure 12.13 compare to this threshold?

(c) Suppose that two voters in each district changed their votes from Democrat to Republican. What would be the effect of this change on the efficiency gap?

While the efficiency gap is a useful measure that can detect gerrymandering strategies like packing and cracking with a single number, Question 12.15 illustrates that it is not without limitations. First, a districting plan can have an abnormally high efficiency gap without exhibiting any of the telltale signs of gerrymandering—such as non-compact districts. In the case of the state in Figure 12.13, it is the distribution of the voters themselves, rather than any obvious partisan gerrymandering, that leads to a Democratic sweep of the state's congressional districts. While this outcome may seem unfair, we should also view it in light of our earlier discussions of partisan symmetry. With a reasonable shift in voter preferences and party affiliation—and, importantly, no change in the district boundaries—the tables could easily turn, with the map now favoring Republicans over Democrats, and the efficiency gap changing accordingly. So the important question is not only whether a districting plan favors one party over another, but also whether this bias is likely to persist over time and endure natural changes in voter behavior. Since the efficiency gap can be sensitive to small changes, Stephanopolous and McGhee recommend carrying out "sensitivity analysis" when using the efficiency gap to evaluate a plan. Unfortunately, such analysis involves considering hypothetical scenarios—an approach that, as we have discussed, is not viewed favorably by the courts.

Concluding Remarks

Gerrymandering can be a significant barrier to democratic representation in elected bodies such as the U.S. House of Representatives. The problem is exacerbated by the fact that, in most states, it is the state legislatures themselves that draw the district lines—a role that provides both opportunities and incentives for partisan manipulation. Some states have tackled the issue by appointing independent or bipartisan commissions, and Iowa uses a unique model where an advisory commission proposes a plan to the state legislature for an up-or-down vote. These approaches can be a step in the right direction, but they likely won't completely eliminate gerrymandering or lead to universally accepted solutions.

Complicating all of this is the fact that the law, particularly as it pertains to partisan gerrymandering, is still very much in flux. However, as we've seen, mathematics plays a key role in helping courts to interpret important concepts such as compactness and partisan bias. Judges are relying on the work of mathematicans (and other mathematically-inclined scholars in fields like law and the social sciences) to more precisely define standards that can be used to determine when a districting plan should be upheld or struck down. As we've seen in this chapter, none of the measures for detecting gerrymandering are perfect. They all have limitations, and the nuances of any particular districting plan are probably best understood by considering a variety of different measures. With that said, experts who are well versed in these methods can provide valuable testimony—particularly in response to claims that a gerrymandered plan arose naturally or for legitimate reasons. When a defendant claims that it is not possible to come up with a plan that is more compact or has a lower efficiency gap, you can count on mathematicians—armed with computer simulations and pages of careful analysis—to say, "Sure it is—and here are 500 examples to prove it."

Questions for Further Study

Question 12.16. Revisit the example shown in Figure 12.1, but this time assume that you must divide the state into seven districts. What is the least number of districts that Republicans can win? What is greatest number of districts they can win? What do you think the most fair outcome would be? Give specific plans, with explanation, to justify each of your answers.

Question 12.17. Research each of the following compactness measures, describe how they work, and apply them to the shapes in Figure 12.9.

(a) Polsby-Popper

(b) Schwartzberg

(c) Reock

(d) Convex hull

Question 12.18. The *convexity coefficient* is a compactness measure that assigns a score to a district based on the probability that a line segment drawn between two randomly selected points within the district will remain entirely within the district. In other words, the more lines that cross the district's boundaries, the lower the convexity coefficient will be.

(a) Based on this informal definition, which shapes from Figure 12.9 do you think have the highest convexity coefficient? Which have the lowest convexity coefficient?

(b) Learn more about the convexity coefficient by reading the article "Gerrymandering and convexity" in the *College Mathematics Journal* [**28**], and write a summary of your findings.

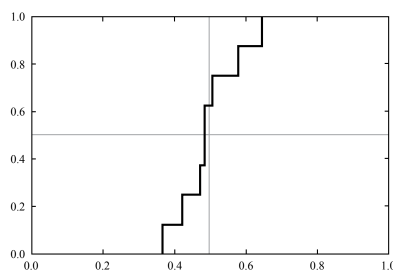
(c) If you have experience with probability and statistics, try to calculate the exact convexity coefficient for as many of the shapes from Figure 12.9 as you can. (Hint: For most of the shapes, you'll have to consider several cases. However, some are easier than others.)

Question 12.19. Research the *coastline paradox*, and explain how it is related to certain compactness measures.

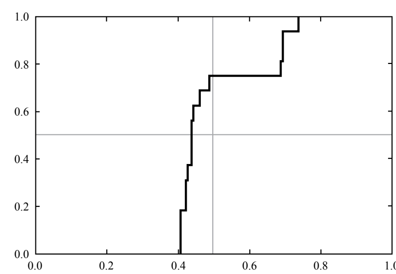
Question 12.20. Compare and contrast the idea of partisan symmetry to the neutrality criterion we considered for voting systems.

Question 12.21. Do you think the assumption of uniform partisan swing is reasonable? Why or why not?

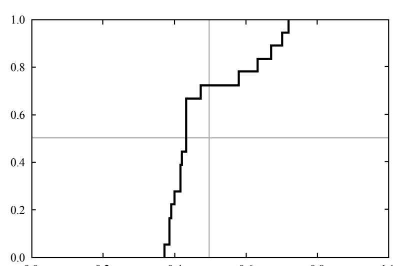
Question 12.22. One way to investigate partisan symmetry is to calculate the number of seats that would be won by a party for a variety of possible vote percentages (again using the assumption of uniform partisan swing). The graphs in Figure 12.14 (called *seats-votes curves*) show the results of these calculations for four different districting plans. The x-axis displays the proportion of votes won by the Republican party, while the y-axis displays the number of seats (districts) won by Republicans. So, for example, if the point $(0.4, 0.6)$ is on the graph, this means that when Republicans earn 40% of the vote, they will win 60% of the seats.



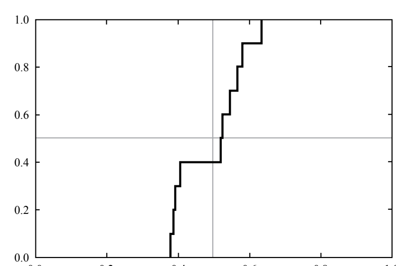
(A) Minnesota, 2016



(B) Ohio, 2016



(C) Pennsylvania, 2012



(D) Washington, 2016

FIGURE 12.14. Seats-votes curves

(a) For each plan, use the seats-votes curve to decide whether the plan is biased in favor of Republicans, Democrats, or neither. Explain your reasoning.

(b) How do these graphs show evidence of packing and/or cracking? Which features correspond to each of these gerrymandering strategies?

Question 12.23. Research some critiques of the efficiency gap as a measure of partisan fairness, and write a summary of your findings.

Question 12.24. Explain why the efficiency gap is always between -0.5 and 0.5.

Question 12.25. Under the assumptions that the number of votes cast in each district is identical, and the vote is split between exactly two parties, it can be shown that the efficiency gap is equal to $2V - S - \frac{1}{2}$, where V is the proportion of votes received statewide by one of the parties and S is the proportion of seats (districts) won by that party. Use algebra to show why this formula holds. Then explain why, to maintain an efficiency gap of zero, every 1% increase in the number of votes received by a party should lead to a 2% increase in the number of seats that party receives.

Question 12.26. How must the vote in a district be split in order for each party to waste the exact same number of votes? Do you think a districting plan in which each district had exactly this split would be fair and/or desirable? Why or why not?

Question 12.27. On February 24, 2018, President Donald Trump tweeted: "Democrat judges have totally redrawn election lines in the great State of Pennsylvania. @FoxNews. This is very unfair to Republicans and to our country as a whole. Must be appealed to the United States Supreme Court ASAP!" Research the history behind this tweet, and explain whether you agree or disagree with Trump's position.

Question 12.28. Research one of the court cases mentioned in this chapter—ideally one that had not been resolved when the book was written. Write a detailed summary of your findings, and explain the impact of the case on the law regarding gerrymandering.

Question 12.29. Look up a recent news article on gerrymandering, and use what you learned in this chapter to either critique the article or respond to the points made in it.

Answers to Starred Questions

12.2. The one district won by Democrats is extremely packed: it doesn't have a single Republican voter! The other Democratic votes are cracked among the remaining four districts, with two Democrats per district—just shy of what would be needed to win.

12.4. (a) Using length-width compactness, shapes (i) and (iii) are tied for being most compact, and shape (iv) is the least compact.

(b) Using perimeter compactness, shape (ii) is the most compact, and shape (v) is the least compact.

12.5. The ordering of the shapes, from most compact to least compact, is: (iii), (ii), (i), (iv), (v).

12.9. (a) If 100 Democrats (about 7.69%) switched their votes, the new statewide totals would be 1300 Republicans (52%) and 1200 Democrats (48%).

(b) 23 Democrats in each of these four districts would change their votes, but Democrats would still have a majority.

(c) 8 Democrats would change their votes, increasing the Republican majority in this already packed district.

(d) Republicans would still only win one of the five districts.

(e) This doesn't seem fair. When Democrats had 52% of the vote statewide, they won four of the five districts. But if Republicans were to earn 52% of the statewide vote, they would only win one of the five districts.

12.13. As it is defined here, a positive efficiency gap would indicate that Party A wasted more votes than Party B, meaning that Party B has the advantage.

12.14. (a) In the four districts that Democrats win, they waste 49 votes, while Republicans waste 200. In the remaining district, Democrats waste 100 votes, while Republicans waste 149.

(b) In total, Democrats waste 296 votes, whereas Republicans waste 949. The efficiency gap is $\frac{949-296}{2500} \approx 0.26$.

(c) The positive efficiency gap from part (b) indicates that the plan favors Democrats.