Block Matrix Exploration

Goals: Create two "rules"; one for the determinant of a block lower triangular matrix, and one for the eigenvalues of a block lower triangular matrix.

1. Recall each:

    (a) The determinant of a lower triangular matrix is _____.

    (b) The eigenvalues of a lower triangular matrix is _____.

2. Hypothesis: The determinant of a block lower triangular matrix is _____.

    Test your hypothesis on the following block matrices $A = \begin{pmatrix} B & 0 \\ C & D \end{pmatrix}$:

    (a) $A1 = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 5 & 6 & 0 & 0 \\ 3 & 4 & 9 & 10 \\ 7 & 8 & 11 & 12 \end{pmatrix}$

        i. What is the determinant of $A1$?
        ii. What is the determinant of B?
        iii. What is the determinant of D?

    (b) $A2 = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 6 & 7 & 0 & 0 & 0 \\ 3 & 4 & 11 & 12 & 13 \\ 5 & 8 & 14 & 15 & 16 \\ 9 & 10 & 17 & 18 & 19 \end{pmatrix}$

        i. What is the determinant of $A2$?
        ii. What is the determinant of B?
        iii. What is the determinant of D?

3. Hypothesis: The eigenvalues of a block triangular matrix are _____.

    Test your hypothesis on the block matrices in problem 2.

    (a) Matrix A1:

        i. What are the eigenvalues of $A1$?
        ii. What are the eigenvalues of B?
        iii. What are the eigenvalues of D?

    (b) Matrix A2:

        i. What are the eigenvalues of $A2$?
        ii. What are the eigenvalues of B?
        iii. What are the eigenvalues of D?