The More Abstract the Better? Raising Education Cost for the Less Able when Education is a Signal

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Abstract

More able individuals may over-invest in education when education signals ability. If education directly increases productivity, increasing education cost for the less able *may* increase welfare by reducing over-investment by the more able, but will not do so if such cost is already either too small or too large because no over-investment then occurs. Increasing cost for the less able is most likely to increase welfare when education is relatively unproductive compared to the initial ability difference between more and less able individuals. Our results have implications for online education which may *lower* cost relatively more for the less able.

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1. Introduction

When potential employers are uncertain about individuals' ability, education may be used as a signal of (pre-matriculation) ability (Spence, 1974). Since Spence's early work, there has been much interest in what equilibrium might result in a world of signaling, and on the welfare properties of different equilibria.[1] Of particular interest have been signaling equilibria when education does not affect productivity, either directly, or by allowing sorting of workers when the allocation of individuals to different jobs affects welfare. In that case, education is only obtained by more able individuals, and merely redistributes wealth.

For the moment, ignore any productive effect of education. If education does not affect productivity, the lower the level of education, the greater is social welfare. Spence (2002) discusses how a tax might be employed to reduce excessive education and increase welfare when education is not productive. Alternatively, since the level of the signal is inversely related to the educational cost difference between more and less productive individuals, raising this cost difference can increase welfare. McAfee (2013) suggests the best subjects for signaling are those that are less useful or practical since they may imply the biggest cost difference between more and less able individuals.

> "…interpreting long medieval poems could more readily signal the kind of flexible mind desired in management than studying accounting, not because the desirable type is good at it, or that it is useful, but because the less desirable type is so much worse at it."[2]

The idea of analyzing medieval poems suggested by McAfee (2013) as a good signal is actually supported by some evidence. Bukszpan (2012) reports on the value of seemingly useless

[1] Perri (2013) lists and discusses many of these papers.
[2] McAfee (2013), p.249.

degrees. One individual majored in epic Renaissance literature and works as a financial analyst. She claims her critical skills in analyzing literature are important in making smart investment choices. Of course, her education may have added to her analytical skills. However, some of what potential employers learned from her major is that she had analytical skills in order to master such a subject. This is the signaling role of education.

Given the recent focus in some states (*e.g.* Florida and North Carolina) on the relevance of university degrees, it is of particular interest to examine the welfare effects of making education relatively more costly for the less able, and possibly less useful. As will be demonstrated in the next section, making education relatively more costly for the less able can improve welfare---even if the marginal and average cost of the more able is increased---provided education is not directly productive (useful). However, neither making education more costly for the less able nor making education less useful or productive will necessarily improve welfare when education *is* useful. The reason is, when education is productive, the welfare-maximizing level of education for even less able individuals will be positive. Increasing cost for the less able lowers welfare for them, although welfare increases as the more able over-invest less in education.

One contribution herein is the formal analysis of the gains from raising education cost for the less able. A second contribution is that there are three differences between models when firms are uncertain of the ability of prospective employees and education is productive, versus the same case except when education is not productive, and only the first of these differences seems to have been previously considered. These differences are as follows.

First, the welfare-maximizing choice of education by those who are more able may be high enough that less able individuals find it too costly to mimic the more able. This possibility was mentioned by Spence (2002). In this case, over-investment in education by more able individuals will not occur.

Second, at the other extreme, when the cost difference between the less and more able is relatively small, the more able may prefer a pooling equilibrium in which they receive a lower wage than they would if they were sorted from the less able. This result occurs because the over-investment by the more able required for sorting is prohibitively high. With pooling, the more able choose the level of education they would under costless information regarding individual ability. *However, the less able then would over-invest in education* (see Section 4).

Third, unlike the case when education is not productive, *the less able may not prefer a pooling equilibrium to a signaling equilibrium* because, with the former, although the less able are paid more than with the latter, they obtain a higher level of education with pooling than with signaling. The less able over-investing in education or not desiring to pool with the more able are not features of standard signaling models where the productive effect of education is often assumed away.

With signaling, the net effect of increasing cost for the less able is uncertain. The main focus of this paper is to examine welfare when there is a policy to increase cost for the less able. One question is how cost could be increased for the less able and not for the more able. Note, we could have cost increase for the more able as long as it increases proportionately less than it does for the less able (see Section 2). Riley (1981) discusses the effect of an innovation which increases educational advancement for the less able, implying their time cost of education falls. Posner (2012) notes how online classes have advantages for students who cannot work at as fast a pace as others. Thus, offering online courses may lower cost for the less able more than for those who are more able---the opposite of what we will consider herein.

The outline of the rest of this paper is as follows. In Section 2, as a benchmark, we consider increasing education cost for the less able when education is not productive. In Section 3, we consider productive education, and use a graphical analysis of increasing education cost for the less able. A mathematical model with productive education is analyzed in Section 4. In Section 5, a tax on education is considered. The case when education is more costly for the less able and less useful for all is considered in Section 6. A summary is contained in Section 7.

2. Education is not productive

Suppose there are two types of individuals in terms of productivity, *Highs* (H), and *Lows* (L). Productivity for an $H = a\theta$, productivity for an $L = \theta$, and we assume $a > 1$and $\theta > 0$. Units of education are denoted by y. The total cost of education for an H is $C_H = \frac{y^2}{2}$, and the total cost of education for an L is $C_L = \frac{(1+z)y^2}{2}$. With $z > 0$, z reflects the higher cost of education for an L versus an H. If education is not productive, with costless information to firms regarding individual productivity, all individuals would set $y = 0$.

In the classic Spence (1974) signaling problem, the private return to signaling via education is the productivity and pay difference between Hs *and* Ls, which equals $(a\text{-}1)\theta$ herein. A signaling or separating equilibrium requires this return at least equal education cost for an H, and the return must be less than education cost for an L. Thus, we must have:

$$\frac{y^2}{2} \leq (a\text{-}1)\theta < \frac{(1+z)y^2}{2}\text{, or} \tag{1}$$

$$\left(\frac{2(a-1)\theta}{1+z}\right)^{1/2} < y \leq [2(a\text{-}1)\theta]^{1/2}. \tag{2}$$

Following Riley (1979), the lowest level of y that will allow a signaling/separating equilibrium is the Riley outcome[3] for Hs, y_R:

$$y_R \approx \left(\frac{2(a-1)\theta}{1+z}\right)^{1/2}. \tag{3}$$

All education is a social waste in this case. The social loss is the expenditure by Hs on education = [# of Hs] $\times C_H$, and the social loss is inversely related to z because a higher z lowers y_R. Thus, if education can be made more costly for Ls, welfare will increase.

Even if education is made more costly for all individuals, if cost increases more for Ls, welfare can be increased. Suppose $C_H = \frac{(1+z\varphi)y^2}{2}$, with $0 < \varphi < 1$. Now an increase in z means all have higher cost of education, but the cost for Ls increases more than the cost for Hs. With $y = y_R$ (eq.(3)), $C_H = \left(\frac{1+z\varphi}{1+z}\right)(a-1)\theta$, and $\frac{\partial C_H}{\partial z} = \{+\}(\varphi - 1) < 0$. As z increases, each unit of education costs more for an H, but, with y_R reduced, $C_{H|_{y=y_R}}$ falls and welfare increases.[4]

3. Productive education: a graphical analysis

When education directly adds to productivity, raising educational cost for less able individuals, Ls herein, does not necessarily increase welfare. We first consider the case of productive education graphically. A mathematical analysis of productive education begins in Section 4.

[3] The Riley outcome is when less able individuals set y equal to the level they would choose with perfect information (zero in the case considered in this section), and more able individuals set $y = y_R$---the lowest level of the signal that induces a signaling equilibrium (Riley, 1979). Using the *intuitive criterion* (Cho and Kreps, 1987), signaling only occurs at the Riley outcome.
[4]Suppose education is not directly productive but has social value because signaling enables output gains as individuals are sorted to jobs appropriate to their skills, as in Perri (2013). Then we have a clear gain to increasing z, provided $C_{H|_{y=y_R}}$ falls as $dz > 0$, because the same sorting is attained at a lower cost.

Spence (2002) has a graphical analysis of signaling with productive education, and Figures One and Two follow his presentation. Consider Figure One with high ability individuals (*H*s) and low ability individuals (*L*s). With perfect information, an *H* would maximize (net) income by choosing $y = y_H$, and an *L* would do likewise by choosing $y = y_L$. An *L* who mimicked an *H* would have higher income because the individual would be viewed as an *H*, but would not have income as high as an *H* because of the greater education cost for an *L*. The maximum education level an *L* would obtain to mimic an *H* is y_R. In Figure One, the education cost difference between an *L* and an *H* is sufficiently large so $y_R < y_H$. Thus, the income-maximizing choose of education for an *H* will not be mimicked by an *L*, and signaling occurs naturally as individuals choose their income-maximizing levels of education. Hence welfare is maximized.

In Figure Two, the education cost difference between an *L* and an *H* is sufficiently small that $y_R > y_H$. Now an *H* must overinvest in education by the amount $y_R - y_H$. Welfare is thus reduced by the amount [# of *H*s]×[Height at P1 minus Height at P4]. The only difference between the case when education is not productive and when education is productive and $y_R > y_H$ is that, in the former case, the welfare-maximizing *y* is zero for both types of individuals, and, in the latter case, the welfare-maximizing *y* is positive for both types. As in the case when education is not productive, in the signaling equilibrium in Figure Two, each *H* over-invests in education, and each *L* chooses the level of *y* that maximizes welfare, y_L.

When education is productive and $y_R > y_H$, consider what happens when the cost of education is increased for an *L*. We think of this as both a real cost increase for *L*s (and not a tax), but one that is avoidable so the initial level of *y* and income for *L*s (y_L and the height of P2 in Figures Two and Three) are potentially available should policymakers choose not to make education more costly for *L*s.

Figure One. No excessive education with signaling.

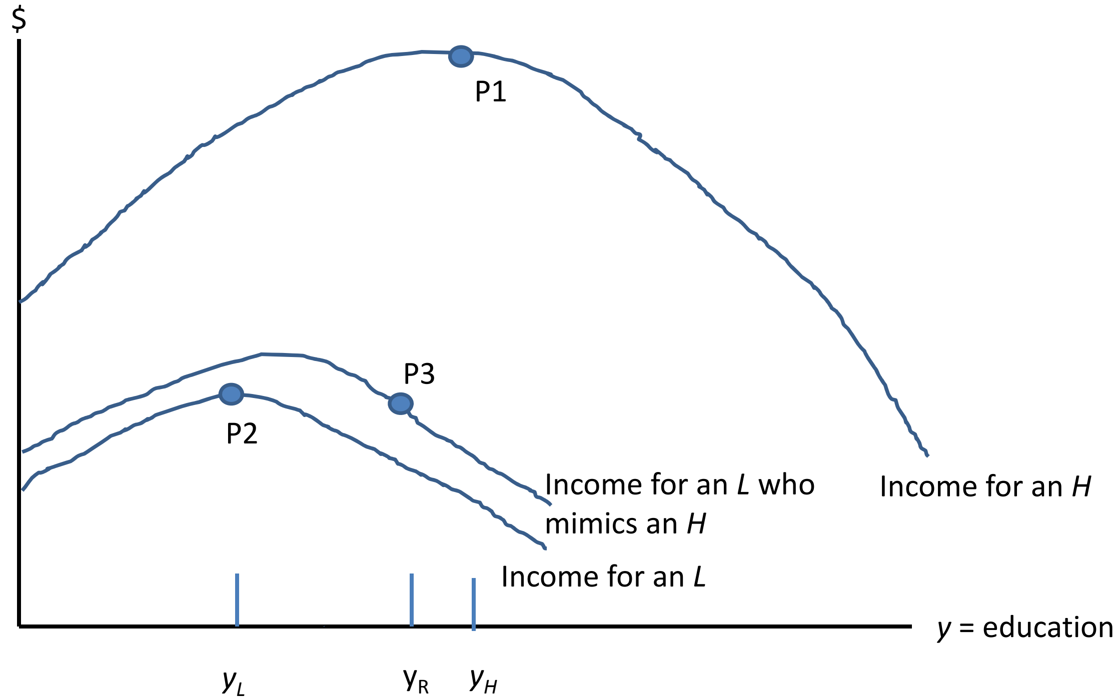
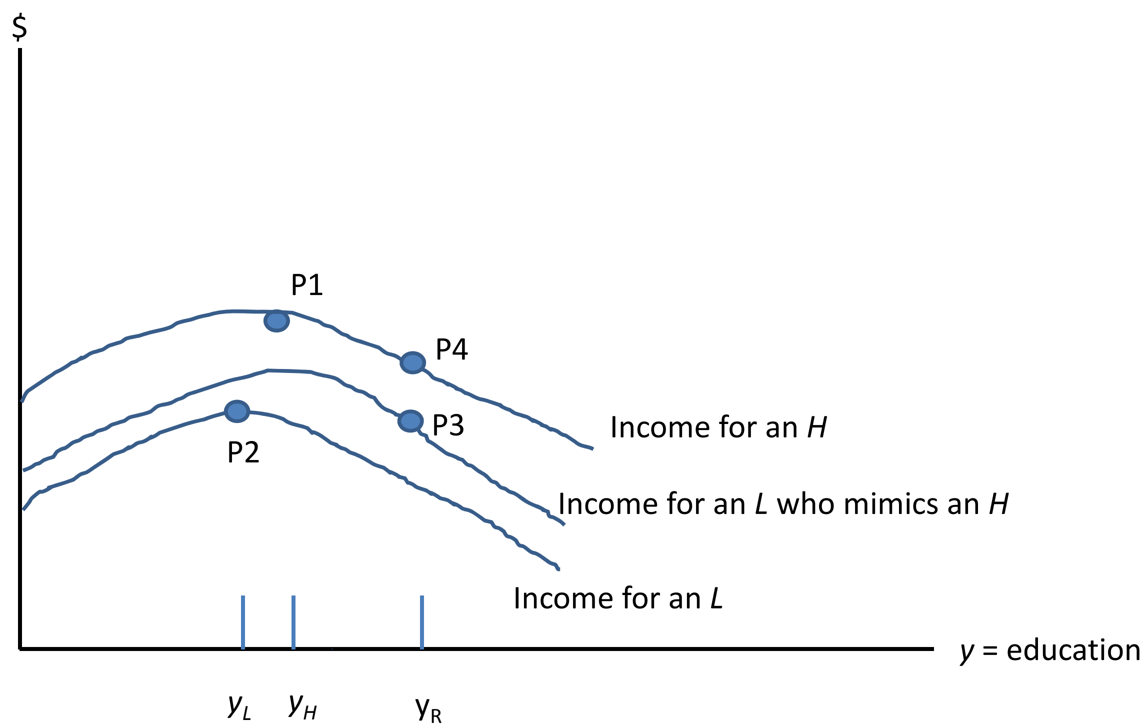
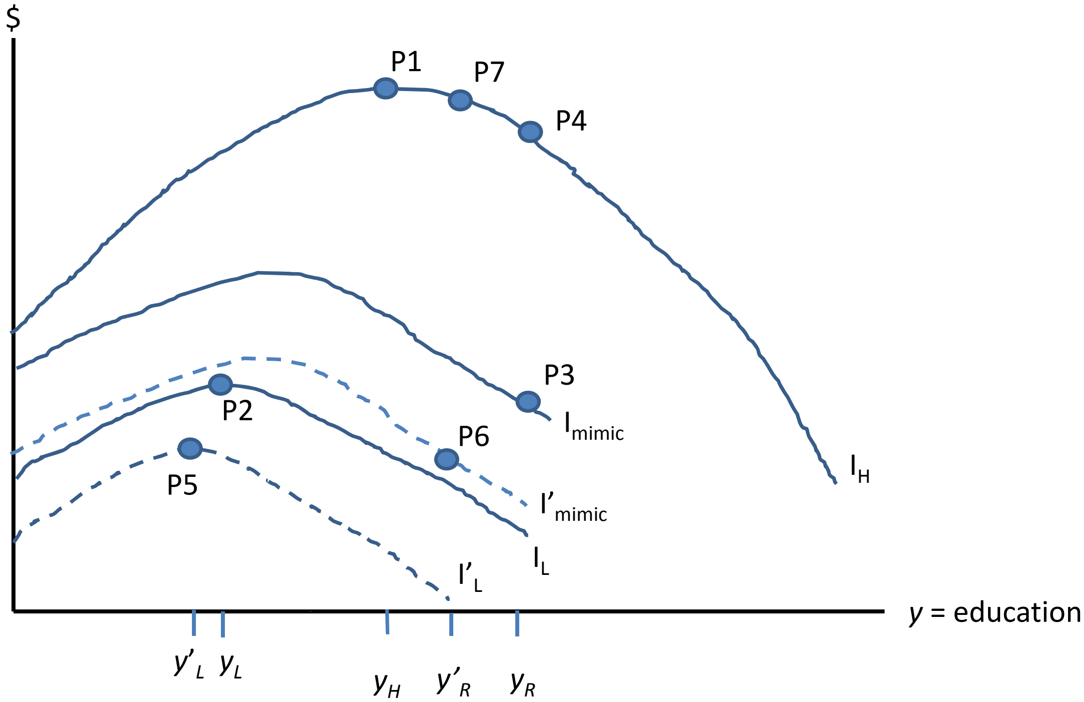


Figure Two. Excessive education with signaling.



In Figure Three, I_H shows (net) income for an H with perfect information, I_L shows income with perfect information for an L, and I_{mimic} shows income for an L who mimics an H in choosing y. From Figure Three, in a signaling equilibrium, initially an L sets $y = y_L$ and has income at Point 2 (P2). An H sets $y = y_R$ and has income at P4. Income for an H is reduced from that with perfect information (P1).

Figure Three. An increase in education cost for lows.



An increase in education cost for an L lowers income for an L both with perfect information and when an L mimics an H (see the dashed income curves). As shown in the next section, at least for the cost functions $C_H = \frac{y^2}{2}$ and $C_L = \frac{(1+z)y^2}{2}$, y_R falls as cost increases for an L. Since an H now sets $y = y_R' < y_R$, income for an H has increased from P4 to P7. However, an L now sets $y = y_L'$, lowering income for an L from P2 to P5. Except for the curves reflecting income to an L from mimicking an H, the income curves reflect (social) welfare. With education cost increased for an L, welfare increases by [# of Hs]×[Height at P7 minus height at P4] as overinvestment in education by Hs is reduced. However, welfare declines by [# of Ls] × [Height at P2 minus height at P5] because Ls reduce their education level from the amount that maximizes welfare for them absent the cost increase (y_L), and because their cost of education is higher at all levels of education. Thus, the effect on welfare of an increase in education cost for the less able is ambiguous.

If, in the process of making education more costly for Ls, education is made less useful, (net) income for Hs and Ls would be reduced, lowering welfare. Additionally, if $y_R > y_H$, since y_R would be reduced, this effect would raise welfare. Since the impact of making education less useful is (roughly) the same as making education more costly for Ls, for simplicity, we shall mainly focus on a model in which education becomes more costly for Ls, but does not become less useful. In Section 6, we briefly discuss a model in which education becomes less useful.

4. A model of productive education

A. Outline of the model

From Section 2, more able individuals, Hs, have innate productivity of $a\theta$, and less able individuals, Ls, have innate productivity of θ. Again, we assume $a > 1$ and $\theta > 0$. For either type of individual, let productivity increase due to education, y, by the amount ky, where k is a positive constant. The total cost of education for an H is $C_H = \frac{y^2}{2}$, and the total cost of education for an L is $C_L = \frac{(1+z)y^2}{2}$. For simplicity, it is assumed education cost can be increased for Ls only; z is initially positive, and can be increased by some policy.

With perfect information, an L maximizes (net) income:

$$\max_{y}\left\{\theta + ky - \frac{(1+z)y^2}{2}\right\}, \text{ yielding} \tag{4}$$

$$y_L = \frac{k}{1+z}\,. \tag{5}$$

Similarly, we have for an *H*:

$$\max_{y}\left\{a\theta + ky - \frac{y^2}{2}\right\}, \text{ yielding} \tag{6}$$

$$y_H = k. \tag{7}$$

Note the income of an *L* with $y_L = \frac{k}{1+z}$ is:

$$\theta + \frac{k^2}{2(1+z)}\,, \tag{8}$$

with the amount in *eq*.(8) representing both the social and private net income for an *L*, with the former referred to as welfare for an *L*. To determine when signaling occurs, we must derive the lowest level of *y* that would prevent an *L* from mimicking an *H*, y_R. Using *eq*.(8), an *L* will not mimic an *H* for any *y* if an *L* is better off correctly viewed as an *L* and setting $y = y_L$, or if:

$$a\theta + ky - \frac{(1+z)y^2}{2} < \theta + \frac{k^2}{2(1+z)}\,, \text{ or} \tag{9}$$

$$(1+z)^2y^2 - 2k(1+z)y - 2\theta(a-1)(1+z) + k^2 > 0. \tag{10}$$

The lowest level of y that will deter an L from mimicking an H is when the LHS of *ineq*.(10) is approximately zero. We then have:

$$y = \frac{k \pm [2\theta(a-1)(1+z)]^{1/2}}{1+z}. \tag{11}$$

The lower root of *eq*.(11) has $y < y_L = \frac{k}{1+z}$, which will not deter an L from mimicking an H. Thus, we have:

$$y_R = \frac{k + [2\theta(a-1)(1+z)]^{1/2}}{1+z}. \tag{12}$$

As noted in Section 3,

$$\frac{\partial y_R}{\partial z} = \{+\}\{-k - ½[2\theta(1+z)(a-1)]^{1/2}\} < 0. \tag{13}$$

Thus, making education more costly for the less able does have the beneficial effect of reducing over-investment in education by the more able. Whether such over-investment would occur, and the overall effect on welfare from raising z will be considered in the rest of this section.

B. Do the more able over-invest in education in a signaling equilibrium?

As shown in Figure One, if the difference between Ls and Hs in the marginal cost of education, z, is large enough, signaling does not lead to over-investment in education by Hs. We have overinvestment in education if $y_R > y_H$ (when a signaling/separating equilibrium exists), which, using *eqs*.(7) and (12), occurs only if:

$$z < \frac{\theta(a-1) \pm \{\theta[a-1][\theta(a-1)+2k^2]\}^{1/2}}{k^2}. \quad (14)$$

Since the $\{\bullet\}^{1/2}$ term on the RHS of *ineq*.(14) exceeds $[\theta(a\text{-}1)]$, the larger root is required for the RHS > 0. Thus, $y_R > y_H$ only if $z < z^{**}$:

$$z^{**} = \frac{\theta(a-1) + \{\theta[a-1][\theta(a-1)+2k^2]\}^{1/2}}{k^2}. \quad (15)$$

If $z > z^{**}$, the marginal cost of education is sufficiently larger for an L than for an H that the former would not mimic the latter at an H's desired (and welfare-maximizing) choice of y, $y_H = k$ (Figure 1). Thus, one condition for excessive education to occur by Hs in a signaling equilibrium is $z < z^{**}$.

C. Will the more able prefer signaling to pooling?

Mailath *et al*. (1993) use the concept of *undefeated equilibrium* and argue signaling would occur only if more able individuals are better off in a signaling equilibrium, with $y = y_R$, than in the pooling equilibrium in which both types choose the level of education the more able would choose with perfect information. With $\frac{\partial y_R}{\partial z} < 0$, if z is small enough, y_R is sufficiently large that an H would prefer pooling at $y = y_H$ to a signaling/separating equilibrium. Note, in a world in which education does not increase productivity ($k = 0$), pooling would occur at $y = 0$. With the pooling wage equal to $[\alpha a + (1-\alpha)]\theta + ky$, where α is the known fraction of Hs in the population, an H would choose y to maximize $\{[\alpha a + (1-\alpha)]\theta + ky - \frac{y^2}{2}\}$, yielding $y = y_H = k$. Thus, with productive education, pooling involves the less able over-investing in education, setting $y = k$ rather than their perfect information (and signaling equilibrium) choice of $y = \frac{k}{1+z}$.

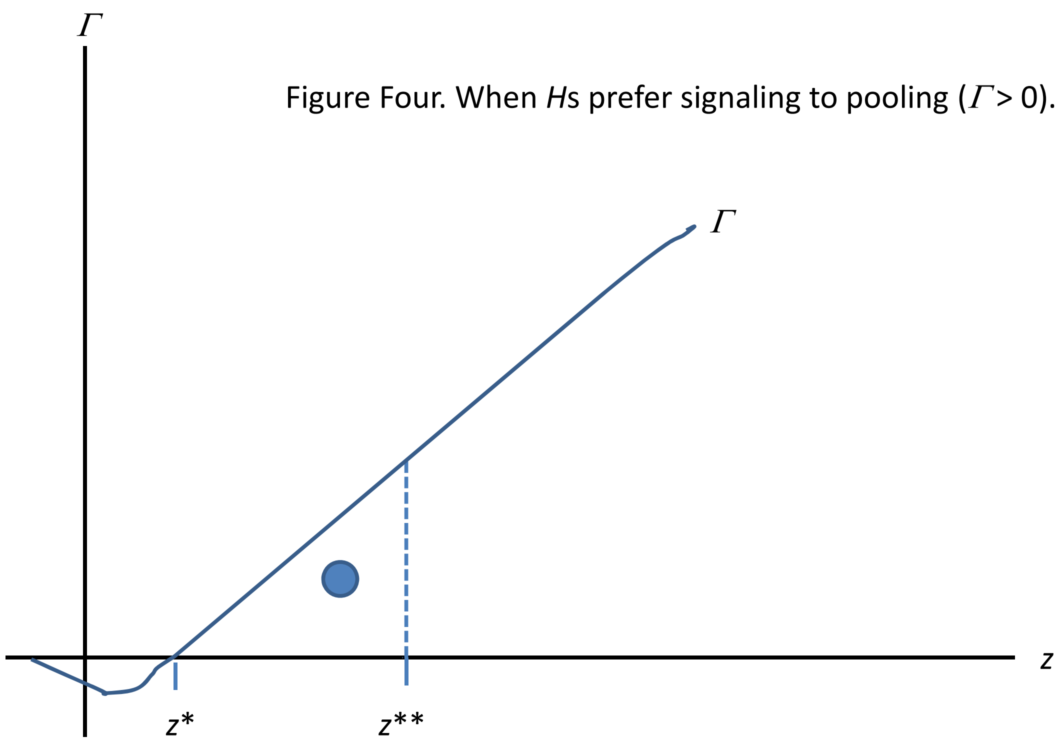
Clearly, whether an H prefers signaling or pooling depends on the share of Hs in the population, α. Since there are enough variables with which to be concerned (z, k, a, and θ), and since the role of α has been considered in great detail with signaling in Perri (2013), we assume an identical number of Hs *and* Ls, normalized to one each. Thus, the pooling wage is now $\frac{(a+1)\theta}{2} + ky_H = \frac{(a+1)\theta}{2} + k^2$. Given education cost of $\frac{y^2}{2}$, an H's net income in a pooling equilibrium is $\frac{1}{2}[(a+1)\theta + k^2]$. Net income for an H with signaling is $a\theta + ky_{Riley} - \frac{y_R^2}{2}$. Using *eq*.(12), an H prefers signaling ($y = y_R$ and a wage of $a\theta + ky_R$) to pooling ($y = y_H$ and a wage of $\frac{(a+1)\theta}{2} + ky_H$) if:

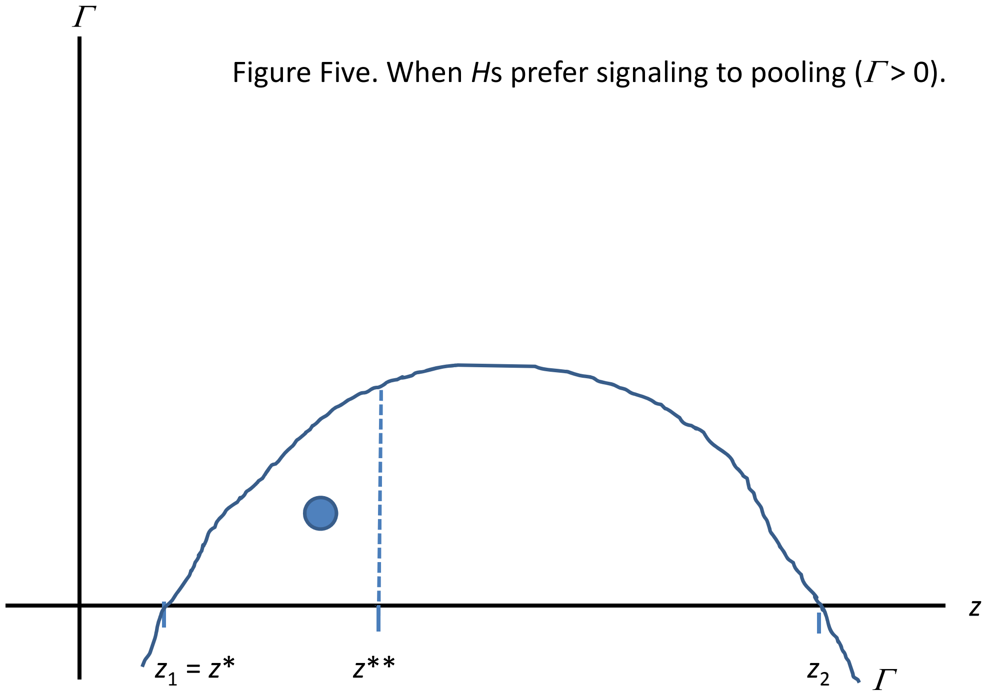
$$2kz[2\theta(a-1)(1+z)]^{1/2} + \theta(a-1)(1+z)(z-1) - k^2z^2 \geq 0. \tag{16}$$

Let z^* be the solution to the equality in i*neq*.(16). If $z > z^*$, y_R is small enough that an H prefers signaling to pooling. To this point, we have seen that a signaling equilibrium with over-investment in education requires $z^* < z < z^{**}$. Because *ineq*.(16) cannot be explicitly solved for z^*, it is not necessarily the case that $z^* < z^{**}$ and over-investment in education would occur. However, the different parameter values we tried have $z^* < z^{**}$, and some of these cases will be considered in Sub-section E below.

Let Γ represent the LHS of *ineq*.(16). When $\Gamma > 0$, *H*s prefer signaling to pooling. Depending on the values of θ and k (see Table One), Γ can look as shown in either Figure Four or Figure Five.

When $\Gamma > 0 \ \forall \ z > z^*$ (Figure Four), we find $z^* < z^{**}$, so the range $z^* < z < z^{**}$ is when over-investment in education by *H*s occurs. When $\Gamma > 0$ for some range of z, $z_1 < z < z_2$ (Figure Five), we find $z_1 < z^{**} < z_2$. Thus, $z_1 = z^*$ in this case, and we again have a range when over-investment in education by *H*s occurs.





D. Will the less able prefer pooling to signaling?

When education is not productive (k = 0), Ls set y = 0 in a signaling equilibrium. In a pooling equilibrium, both Hs and Ls set y = 0.[5] In that case, Ls clearly prefer pooling to signaling. As long as there are any Hs in the population, the pooling wage, $[\alpha a + (1-\alpha)]\theta$, exceeds the wage paid to an L with signaling, θ.

When education is productive, pooling involves all choosing $y = y_H = k$, the perfect information education level of education for Hs, which exceeds the level of education Ls obtain with signaling,[6] $y_L = \frac{k}{1+z}$. Thus, with pooling as opposed to signaling, Ls receive a higher wage, but incur greater cost because of the additional education they obtain.

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[5] Since education is not productive, the Pareto superior pooling equilibrium is for both types to set y = 0.Neither has any incentive to set y > 0.

[6] Suppose the return to education were multiplicative in ability, say kjy, with j = ability. Then, with pooling, perceived ability is $(a+1)\theta/2$, and $y = k(a+1)\theta/2$. With signaling, Ls would set $y = k\theta/(1+z)$, and Hs would set $y = ka\theta$. In this case, relative to the

It is necessary to determine when *L*s would prefer pooling to signaling because, when we consider when an *H* would prefer signaling to pooling ($z > z^*$), it must be the case pooling would occur if *H*s do not signal. Otherwise (see below), an *H* would not have pooling as the alternative to signaling. From *eq*.(8), we have income for an *L* with signaling. With pooling and $y = k$, an *L*'s income is:

$$(a+1)\theta + k^2 - \frac{(1+z)k^2}{2}. \tag{17}$$

Using *eqs*.(8) and (17), an *L* prefers pooling to signaling if:

$$\frac{2a\theta}{k^2} \geq \frac{z^2}{1+z}. \tag{18}$$

The RHS of *ineq*.(18) is increasing in *z*. Thus, there is a level of *z*, call it z^{***}, such that, for $z < z^{***}$, *L*s prefer pooling to signaling. Solving for the equality in *ineq*.(18):

$$z = \frac{a\theta}{k^2} \pm \frac{1}{k}\left[\frac{a^2\theta^2}{k^2} + 2a\theta\right]^{1/2}. \tag{19}$$

The larger root of *ineq*.(19) is the only one greater than zero, so:

$$z^{***} = \frac{a\theta}{k^2} + \frac{1}{k}\left[\frac{a^2\theta^2}{k^2} + 2a\theta\right]^{1/2}. \tag{20}$$

costless information case, *H*s would choose too little education, and, as in the case we consider in the text, *L*s would choose too much education (since $a > 1$).

If $z^{**} < z^{***}$, then, when over-investment in education occurs ($z^* < z < z^{**}$), *Ls* would indeed prefer pooling to signaling. Using *eqs*.(15) and (20), $z^{**} < z^{***}$ if:

$$-\theta + \{\theta[a-1][\theta(a-1) + 2k^2]\}^{1/2} < k\left\{\frac{a^2\theta^2}{k^2} + 2a\theta\right\}^{1/2}. \tag{21}$$

Ineq.(21) holds if the RHS exceeds the {•} term on the LHS, which requires:

$$\theta < 2(k^2 + a\theta), \tag{22}$$

which, with $a > 1$, is true. For different functional forms for cost and the productive value of education, it is possible $z^{***} < z^{**}$. Since that is not the case herein, we do not consider further in the text what the equilibrium might be if $z^{***} < z^{**}$. That discussion is in Appendix A.

E. Increasing education cost for the less able.

We now consider the impact on social welfare if it is possible to make education more costly for the less able (*L*s), that is, if z is increased. We have the following propositions:

Proposition One. If education cost for the less able is either relatively low ($z < z^*$) or high ($z > z^{**}$), an increase in z must reduce welfare.

Proposition Two. If education cost for the less able is neither relatively low nor high ($z^* < z < z^{**}$), an increase in z may increase welfare, but is not likely to do so when the productive value of education (k) is large relative to the difference in innate productivity between more and less able individuals (reflected by θ).

Proof of Proposition One. With $z^* < z^{**}$, if $z < z^*$, the marginal cost of education for an L is low enough relative to that for an H so y_R is relatively large. Thus, Hs prefer pooling to signaling, as do Ls since $z^{**} < z^{***}$. If education were not productive ($k = 0$), with pooling, Hs and Ls would both set $y = 0$---the perfect information levels for both. There would be no welfare loss from the perfect information case. As discussed in sub-section C of this section, with productive education and pooling, both Hs and Ls set $y = y_H = k$. Unlike the case when education is not productive, we still have over-investment in education with pooling, but by Ls who set $y = k$ rather than the social welfare-maximizing level for them of $\frac{k}{1+z}$.

Consider an increase in the marginal cost of signaling for Ls, d$z > 0$, when $z < z^*$. As discussed before, the increase in z does not occur exogenously, but is the result of a conscious policy. In other words, the increase in z does not have to occur. Now, d$z > 0$ does not change the pooling equilibrium (unless z increases so much that $z > z^*$). Welfare would increase if Ls lowered y, but they prefer pooling to the separating equilibrium and continue to set $y = y_H = k$. All that happens as z is increased is the cost of education for Ls increases, unambiguously lowering social welfare.

When $z > z^{**}$, signaling occurs, but does so at the perfect information levels of y since $y_R < y_H$. An increase in z again does not affect the choice of y by Hs; $y_H = k$. However, $y_L = \frac{k}{1+z}$ falls as z increases. Social welfare is lower as Ls choose a lower y and attain that y at a higher cost. ☐

Proof of Proposition Two. With an identical number of both types (normalized to one), social welfare in a signaling equilibrium is:

$$\theta(a+1) + \frac{3k^2}{2(1+z)} + \frac{k}{1+z}[2\theta(a-1)(1+z)]^{1/2} - \frac{1}{2}\left\{\frac{k+[2\theta(a-1)(1+z)]^{1/2}}{1+z}\right\}^2. \tag{23}$$

Choosing the welfare-maximizing level of *z* does not yield a manageable expression for *z*. Thus, to see if welfare, Ω, can be increased by an increase in *z*, we compare welfare when *z* is slightly above *z** and slightly below *z***. If the latter exceeds the former then we know that increasing *z* increases welfare for some values of *z*. Of course, even if $\Omega_{z \approx z^*} > \Omega_{z \approx z^{**}}$, it is possible there are values of *z* in the range $z^* < z < z^{**}$ that may involve higher welfare than $\Omega_{z \approx z^*}$.[7] Rather than try many values of *z*, we mainly focus on the extreme values of *z*, *z** and *z***, within which over-investment in education in a signaling equilibrium occurs, to see if welfare rises or falls as we move from *z** to *z***.

For simplicity, we fix $a = 2$ and focus on the variables θ and *k*. An increase in *k* means education is more valuable in production. Thus, if *k* increases, it is less likely welfare increases if we adopt a policy of increasing *z*. The good result is over-investment in education by *H*s is reduced as y_R falls, but, when *k* is relatively large, this effect is more than offset by the lower amount of education chosen by *L*s (see the proof of Proposition One).

An increase in θ means the productivity difference between *H*s and *L*s, $(a-1)\theta$, increases, which results in an increase in y_R (*eq*.(12)). Thus, a larger value for θ suggests it is more likely raising *z* increases welfare because the over-investment in education by *H*s is larger.

From Table One, we see there are indeed values for θ and *k* for which $\Omega_{z \approx z^*} < \Omega_{z \approx z^{**}}$. We also see, given θ, a larger *k* does make it more likely $\Omega_{z \approx z^*} > \Omega_{z \approx z^{**}}$. Thus, it is possible making education more costly for the less able increases welfare, but this is not true if 1) such cost is already large ($z > z^{**}$), 2) such cost is small ($z < z^*$), or 3) education is relatively

[7] However, see the discussion at the end of this section.

productive compared to the difference in initial ability between more and less able individuals (k is large relative to θ). □

Note, in the case when $\Omega_{z \approx z^*} > \Omega_{z \approx z^{**}}$, it appears that Ω decreases monotonically in z. For example, we use z that is approximately the average of z^* and z^{**}. With $\theta = k = 4$, we use $z = .6$; with $\theta = k = 5$, we use $z = .5$; and with $\theta = k = 10$, we use $z = .33$. We find $\Omega_{z\ =\ .6, \theta = k = 4} = 24.73$, $\Omega_{z\ =\ .5, \theta = k = 5} = 35.41$, and $\Omega_{z\ =\ .33, \theta = k = 10} = 116.62$. In all three cases, welfare, Ω, at approximately the mean of the range $z^* \leq z \leq z^{**}$ is such that $\Omega_{z \approx z^*} > \Omega > \Omega_{z \approx z^{**}}$.

Table One. Social welfare when over-investment in education by the more able would just occur.

θ	a	k	z^*	z^{**}	Social Welfare $z \approx z^*$	Social Welfare $z \approx z^{**}$
4	2	1	.481	8.899	10.86 (z = .49)	**12.55** (z = 8.89)
4	2	2	.309	2.732	13.43 (z = .31)	**14.54** (z = 2.73)
4	2	4	.1784	1	**24.79** (z = .18)	24.02 (z = .99)
5	2	1	.51	10.916	13.36 (z = .52)	**15.54** (z = 10.91)
5	2	2	.335	3.266	16.02 (z = .34)	**17.47** (z = 3.26)
5	2	5	.162	.863	**35.77** (z = .17)	34.22 (z = .86)
10	2	1	.599	20.954	25.82 (z = .6)	**30.52** (z = 20.95)
10	2	2	.42	5.854	28.47 (z = .43)	**32.29** (z = 5.85)
10	2	10	.119	.558	**119.67** (z = .12)	112.26 (z = .55)

5. Taxing education.

An alternative to making education more costly for the less able is to simply tax education. Assuming it is not possible to distinguish individuals (taxing the less able but not the more able), we consider a tax that applies to everyone. Spence (2002) analyzed a tax on education when education is not productive. Herein, we consider the optimal tax when education is productive.

Since a tax does not represent a social cost, although the (net) income of Hs is reduced by a tax, (social) welfare for Hs is increased, provided the tax does not cause them to reduce y below the level that maximizes welfare absent a tax, $y = k$. In a signaling equilibrium, with no tax, an L would set $y = \frac{k}{1+z}$. With a tax, an L will reduce y, lowering welfare for them.

We have already considered how increasing the cost of education for the less able (dz > 0) affects welfare, so, to allow us to solve for the optimal tax, we now fix z and a, and focus on the tax, θ, and k. Let t equal a tax (in dollars) per unit of education obtained. Further, let $z = 1$ and $a = 2$. Thus, private (not social) cost for an H (that is, including the tax) is $y^2/2 + ty$, and private cost for an L is $y^2 + ty$. Note, except for the fact education is productive ($k > 0$), we now have the simple Spence (1974) model in which the more able are innately twice as productive as the less able, with the latter having social cost of education twice that of the former, y^2 versus $y^2/2$.

Now, with costless information, an H maximizes $\{2\theta + (k-t)y - y^2/2\}$, and an L maximizes $\{\theta + (k-t)y - y^2\}$ yielding:

$$y_H = k - t,\ y_L = \frac{k-t}{2}\,. \qquad (24)$$

As before, we assume an equal number of *H*s and *L*s (one each). Net income for an *L* with $y = \frac{k-t}{2}$ is $\theta + \frac{(k-t)^2}{4}$. An *L* will not mimic an *H* at any *y* if:

$$\theta + \frac{(k-t)^2}{4} \geq 2\theta + (k\text{-}t)y - y^2. \tag{25}$$

The equality in *ineq*.(25) yields:

$$y_R = \frac{k-t}{2} + \theta^{1/2}. \tag{26}$$

Now $y_R > y_H$ if $\theta > \frac{(k-t)^2}{4}$, and it is easy to see *H*s prefer signaling to pooling if $\theta > \frac{(k-t)^2}{16}$.

We also must be sure *L*s prefer pooling to signaling. Pooling is at $y = y_H = k - t$. An *L*'s net income is then $\frac{3}{2}\theta + (k-t)^2 - (k-t)^2 = \frac{3}{2}\theta$. From before, with signaling, an *L* has net income of $\theta + \frac{(k-t)^2}{4}$. We find *L*s prefer pooling to signaling if $\theta > \frac{(k-t)^2}{2}$.

Thus, a signaling equilibrium occurs with over-investment in education by *H*s ($y_R > y_H$) if $\theta > \frac{(k-t)^2}{2}$. In this case (social) welfare for an H simplifies to:

$$\frac{3}{2}\theta + \frac{k(k-t)}{2} + k\theta^{1/2} - \frac{(k-t)^2}{8} - \frac{(k-t)\theta^{1/2}}{2}. \tag{27}$$

From above, welfare for an *L* is:

$$\theta + \frac{k^2-t^2}{4} . \tag{28}$$

With one H and one L, total welfare is the sum of the amounts in *eqs*.(27) and (28). Maximizing total welfare with respect to t, we have:

$$\frac{\partial(total\ welfare)}{\partial t} = \frac{\theta^{1/2}}{2} - \frac{(k+3t)}{4} = 0, \tag{29}$$

$$\frac{\partial^2(total\ welfare)}{\partial t^2} = - ¾ < 0. \tag{30}$$

Solving *eq*.(29), we have the welfare-maximizing t, t^*:

$$t^* = \frac{2\theta^{1/2}-k}{3} . \tag{31}$$

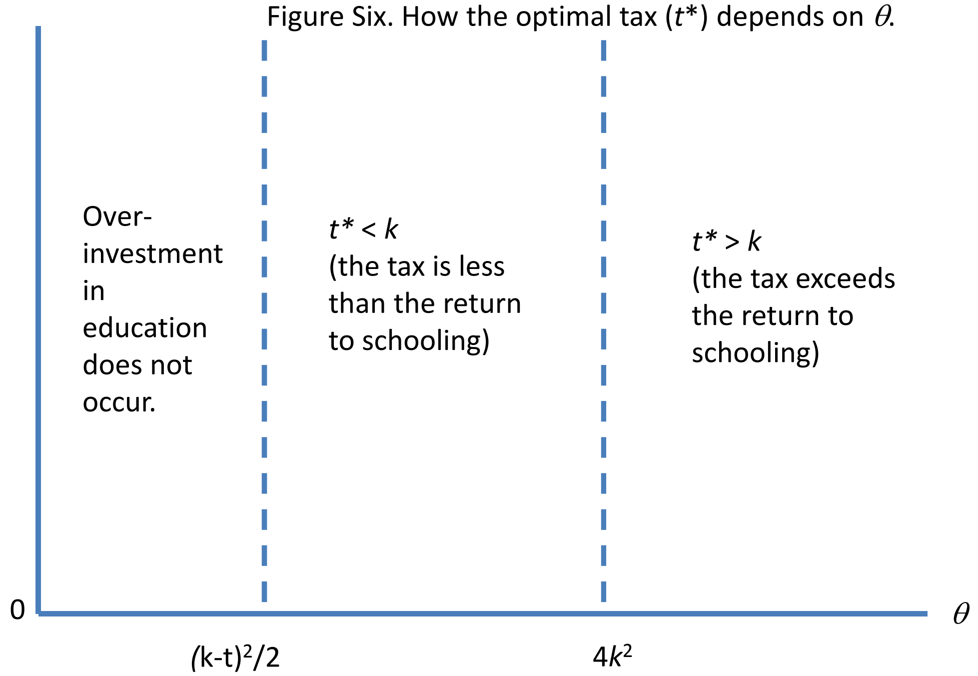
From *eq*.(31), the optimal tax is larger the larger is θ. A larger θ implies the productivity difference between Hs and Ls, $(a\text{-}1)\theta = \theta$, is larger so y_R is larger. Even though a larger k also results in a larger y_R, a larger k means a greater productivity increase from increasing y. The latter effect dominates, so the optimal tax is lower as k increases. For $t^* > 0$, $\theta > \frac{k^2}{4}$ (given $a = 2$ and $z = 1$). If $\theta < \frac{k^2}{4}$, it pays society to not tax education. However, as seen above, for signaling with over-investment in education by Hs, and Ls preferring pooling to signaling (which is necessary for signaling; see *Appendix A*), we must have $\theta > \frac{(k-t)^2}{2}$. If $t = 0$, $k^2/4 < \frac{(k-t)^2}{2}$, so we cannot have signaling (with over-investment in education by Hs) and also have $t^* = 0$.

Now, is $t^* < k$ so the tax does not take more than the gain from education? We have $t^* < k$ if $\theta < 4k^2$. If $\theta > 4k^2$, the optimal tax exceeds the social return from education, and we would have $y_L = y_H = 0$.

Finally, we know a signaling equilibrium with over-investment in education by *H*s occurs, $y_R > y_H = k - t$, if $\theta > \frac{(k-t)^2}{4}$. However, with $t^* = \frac{2\theta^{1/2}-k}{3}$, is $y_R \gtreqless k$ = the welfare-maximizing *y* for an *H*? With $y_R = \frac{k-t}{2} + \theta^{1/2}$ and $t = t^*$, $y_R \gtreqless k$ if $\theta \gtreqless \frac{k^2}{4}$. As shown above, $t^* > 0$ if $\theta > \frac{k^2}{4}$. Thus, if $t^* > 0$, we optimally still have some excessive education, $y_R > k$. Figure Six illustrates the possibilities just discussed.

Note, if $t = t^*$, $y_R = \frac{2}{3}\left(\theta^{1/2} + k\right)$. An increase in θ raises t^*, thus lowering y_R, but the direct effect of θ on y_R dominates the former effect. An increase in *k* directly raises y_R, and indirectly raises y_R by lowering t^*. Thus, y_R is positively related to both θ and *k*, given $t = t^*$.

Figure Six. How the optimal tax (t^*) depends on θ.



6. Making education less useful

Although we have considered making education more costly for the less able, what if, in doing so, education became less useful? For example, suppose the productive return to education is now $\frac{ky}{1+z}$, with no other changes in our model. It is easy to see we then have[8] the perfect information levels of y for Ls and Hs: $y_L = \frac{k}{(1+z)^2}$ and $y_H = \frac{k}{1+z}$. The differences between this case and when the productive value of education is ky are that y_L, y_H, and y_R are smaller, and $y_R - y_H$ is larger. The last result suggests we are less likely to have a situation when signaling occurs without over-investment in education.

Using the same θ, k, and a values as in Table One, in Table A1, we find $z^{**} \rightarrow \infty$ and $\Omega_{z \approx z^*} < \Omega_{z \approx z^{**}}$ for all of these cases. However, in order to have over-investment in education in the first place, we must have $z^* < z < z^{**}$, and, for many of these cases, the required values for z^* are implausibly large, requiring Ls (who, with a equal to two, have productivity one half that of Hs) to naturally have a marginal cost of education that is tremendously larger than that for Hs. Such large for values z^* imply we are likely to have $z < z^*$, in which case pooling would occur. With our focus on the general signaling result of potential over-investment in education by the more able, we will not spend more time on the case when education becomes less useful and not just more costly.

7. Summary

When education is a signal of inherent ability, it is possible over-investment in education by the more able may occur (Spence, 1974, 2002). When over-investment occurs, it is because it is necessary to prevent the less able from mimicking the educational choices of those who are

[8] See Appendix B for proofs of arguments in this section.

more able. Consequently, welfare may be improved if education becomes relatively more costly for the less able, something that may be more likely if education becomes more abstract. We showed herein it is possible increasing education cost for the less able will increase welfare, but it will not do so if such cost is already either 1) too small or 2) too large because no over-investment then occurs by more able individuals. With the first case, we would have a pooling equilibrium at the welfare-maximizing level of education for the more able, y_H. In the second case, y_H is large enough that the less able will not mimic the more able. Signaling then occurs naturally, with each type choosing the level of education that maximizes welfare. Increasing cost for the less able is most likely to increase welfare when education is relatively unproductive compared to the initial ability difference between more and less able individuals.

The movement towards more online courses may be a case of education cost becoming relatively *cheaper* for the less able, the opposite of what we considered herein.[9] Also, it is not clear if such courses provide as much learning as do traditional courses. In terms of the model herein, if online education courses are less valuable, this would be the case when welfare is more likely to be enhanced if education were made relatively *more* costly for the less able (Proposition Two), provided over-investment in education by the more able occurs. Given the possibility for significant growth in online education, the signaling role of traditional and online education, and the possible value of abstract education in sorting individuals are topics that should receive increasing attention.

[9] There are many issues concerning online education. One is the importance of direct personal contact (lacking online) in education (Becker, 2012). A second issue is how much the internet will replace traditional university courses. Weissmann (2012) argues campuses involve more than teaching, and mentions the signaling value of education. It is possible online courses provide worse signals because some of what is required in other classes, such as attendance and group projects that require personal contact, is missing. Roth (2012) suggests that elite universities will survive the online class revolution, at least in part because of the signaling that occurs at such schools. Since there is a continuum of universities in terms of quality, Roth's argument implies schools that are not elite, but that are not at the lowest end of the continuum, may also survive the spread of online education.

Appendix A

Equilibrium when the less able (Ls) prefer signaling to pooling.

When Hs prefer signaling to pooling ($z^* < z$), it must be the case that pooling would occur absent signaling since an H's decision whether to signal is based on pooling being the alternative to signaling. Further, unless $z^{***} > z^{**}$, Ls do not prefer pooling to signaling for the full range when Hs prefer signaling to pooling and over-investment in education by Hs occurs. Thus, we must have $z^* < z < z^{**} \leq z^{***}$.

Suppose $z^{***} < z < z^{**}$ (when $z > z^*$). If Ls do not set $y = y_H$, and instead set $y = y_L$, then Hs set $y = y_H$ and get paid $a\theta + ky_H$. Ls *do* prefer to mimic with the wage = $a\theta + ky_H$ and $y = y_H$ (since $y_H < y_R$), and, of course, prefer this to pooling with $y = y_H$ and the wage = $(a+1)\theta + ky_H$. What then is the equilibrium?

Ls would *not* deviate from an equilibrium with $y = y_L$ to a pooling equilibrium ($y = y_H$ and paid the pooling wage). Following the logic of *undefeated equilibrium* (Mailath *et al.*, 1993), Ls will not deviate from $y = y_L$ and a wage = $\theta + ky_L$ when they realize the outcome at $y = y_H$ is for the pooling wage and *not* a wage of $a\theta$. Undefeated equilibrium essentially allows commitment to wage offers by firms to be endogenous (Koufopoulos, 2011, Perri, 2013). Thus, if all choose $y = y_H$, firms realize these individuals are not all Hs, and the pooling wage replaces a wage equal to the productivity of Hs.[10]

The equilibrium in this case should be the one with perfect information. By not signaling with $y = y_R$, Hs would get their best possible outcome. Ls would not deviate from this equilibrium and set $y = y_H$ because they know wage offers of $a\theta + ky_H$ would not result.

Note: if $z^* < z < z^{***}$, Hs (who prefer signaling to pooling) would not set $y = y_H$ since Ls prefer pooling to $y = y_L$ and a wage = $\theta + ky_L$ in this case.

Appendix B

The return to education is a function of z.

Suppose the return to education is $\frac{ky}{1+z}$. Thus dz > 0 means education is less useful. Now the perfect information case yields $y_L = \frac{k}{(1+z)^2}$ and $y_H = \frac{k}{1+z}$. Wealth for a low with perfect information is $\theta + \frac{k^2}{2(1+z)^3}$.

[10] Mailath *et al.* (1993) use the idea of *undefeated equilibrium* to find when a pooling equilibrium would be broken by a signaling equilibrium when the latter is preferred to the former by the more able in a world in which education is not productive. The *intuitive criterion* (Cho and Kreps, 1987) rules out all pooling equilibria in such a situation. Herein, we use undefeated equilibrium to show that a pooling equilibrium would not survive when the less able prefer signaling to pooling.

An L who mimics an H has a payoff of $a\theta + \frac{ky}{1+z} - \frac{(1+z)y^2}{2}$. For signaling, for an L, the payoff from mimicking an H must be less than the payoff with perfect information:

$$0 < (1+z)^4y^2 - 2k(1+z)^2y + k^2 - 2(a\text{-}1)\theta(1+z)^3. \tag{A1}$$

When the RHS in the above inequality = 0, we have y_R:

$$y = \frac{k \pm \left[2(a-1)\theta(1+z)^3\right]^{1/2}}{(1+z)^2}. \tag{A2}$$

Since we must have $y_R > y_L$,

$$y_R = \frac{k + \left[2(a-1)\theta(1+z)^3\right]^{1/2}}{(1+z)^2}. \tag{A3}$$

We now derive z^{**}. For $y_R > y_H$ (so excessive education is obtained in a signaling equilibrium by Hs) we have:

$$1 + 3z + \left[3 - \frac{k^2}{2(a-1)\theta}\right] z^2 + z^3 > 0. \tag{A4}$$

Now z^{**} is found when the LHS of the above inequality = 0. A necessary condition for the LHS = 0 is $k^2 > 6(a\text{-}1)\theta$. Otherwise, the LHS > 0 $\forall z$, which means we always have $y_R > y_H$. In this case, $z^{**} \rightarrow \infty$: all signaling equilibria involve over-investment in education by Hs.

Using *eqs*.(12) and (A3), y_R is lower when educational productivity equals $\frac{ky}{1+z}$ than when it equals ky if;

$$\frac{k + \left[2(a-1)\theta(1+z)^3\right]^{1/2}}{(1+z)^2} < \frac{k + [2(a-1)\theta(1+z)]^{1/2}}{1+z}, \text{ or if}$$

$$[2\theta(a\text{-}1)(1+z)^3]^{1/2} < zk + [1+z][2\theta(a\text{-}1)(1+z)]^{1/2}, \text{ or if}$$

$$0 < zk. \tag{A5}$$

When educational productivity is *not* a function of z, using *eqs*.(7) and (12), we have:

$$y_R - y_H = \left[\frac{2\theta(a-1)}{1+z}\right]^{1/2} - \frac{kz}{1+z}. \tag{A6}$$

When educational productivity *is* a function of z, using *eq*.(A3) and $y_H = \frac{k}{1+z}$, we have:

$$y_R - y_H = \left[\frac{2\theta(a-1)}{1+z}\right]^{1/2} - \frac{kz}{(1+z)^2}. \tag{A7}$$

Comparing *eqs*. (A6) and (A7), clearly $y_R - y_H$ is larger when educational productivity is a function of z than when it is not a function of z. Thus, we are more likely to have over-investment in education when educational productivity is a function of z than when it is not a function of z.

We now derive z^*. We want to find when an H is indifferent to signaling or pooling at $y = y_H$. With pooling, wealth for an H is:

$$\frac{1}{2}\left[\frac{k^2}{(1+z)^2} + (a+1)\theta\right]. \tag{A8}$$

Wealth for an H with signaling is $\frac{ky_R}{1+z} + a\theta - \frac{y_R^2}{2}$. Hs prefer signaling to pooling if:

$$0 < \theta(a\text{-}1)(1\text{+}z)^3(z\text{-}1) - 2zk[2(a\text{-}1)\theta(1\text{+}z)^5]^{1/2} - k^2z^2. \tag{A9}$$

When the RHS of the above inequality = 0, we have z^*. For $z < z^*$, Hs prefer pooling to signaling.

Now we derive z^{***}. An L must prefer pooling to separating (or else it will affect an H's decision on separating versus pooling). With pooling at $y = y_H$, an L's wealth simplifies to:

$$\frac{1}{2}\left[(a+1)\theta + \frac{k^2(1-z)}{(1+z)^2}\right]. \tag{A10}$$

From before, with a signaling equilibrium, an L's wealth is: $\theta + \frac{k^2}{2(1+z)^3}$.
For an L to prefer pooling to separating:

$$0 < \theta(a\text{-}1)(1\text{+}z)^3 - z^2k^2. \tag{A11}$$

When the RHS of the above inequality = 0, we have z^{***}. For the values of θ, k, and a used in Table A1, L*s* prefer pooling to signaling when $z' < z < \infty$. For each of these cases, $z^* > z'$. Thus, for $z^* < z < z^{**}$, H*s* prefer signaling to pooling when the alternative to signaling *would* be pooling.

Welfare (with one of each type) is:

$$(a\text{+}1)\theta + \frac{k^2}{2(1+z)^3} + \frac{ky_{Riley}}{1+z} - \frac{y_{Riley}^2}{2}. \tag{A12}$$

As suggested in Section 6, in all of the examples in Table A1 (and in others we have tried), welfare increases as z goes from z^* to z^{**}.

Table A1 shows welfare when $z \approx z^*$ and $z \approx z^{**}$.

Table A1. Social welfare when over-investment in education by the more able would just occur.

θ	a	k	z^*	z^{**}	Social Welfare $z \approx z^*$	Social Welfare $z \approx z^{**}$
4	2	1	3.274	∞	10.49 (z = 3.28)	**12** ($z = \infty$)
4	2	2	9.253	∞	11.77 (z = 9.26)	**12** ($z = \infty$)
4	2	4	33.245	∞	11.94 (z = 33.25)	**12** ($z = \infty$)
5	2	1	2.876	∞	14.04 (z = 2.88)	**15** ($z = \infty$)
5	2	2	7.655	∞	14.65 (z = 7.66)	**15** ($z = \infty$)
5	2	5	41.249	∞	14.94 (z = 41.25)	**15** ($z = \infty$)
10	2	1	2.072	∞	27.36 (z = 2.08)	**30** ($z = \infty$)
10	2	2	4.466	∞	28.78 (z = 4.47)	**30** ($z = \infty$)
10	2	10	81.249	∞	29.94 (z = 81.25)	**30** ($z = \infty$)

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