

The Importance of Herrnstein's Mathematical Statement of the Law of Effect for Behavior Therapy

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ABSTRACT: *Herrnstein's mathematical statement of the law of effect improves on Skinner's view of reinforcement by asserting (a) that response rate varies hyperbolically with reinforcement rate and (b) that responding is governed not only by reinforcement obtained for responding but also by reinforcement obtained from all other concurrent sources. Data from infrahuman organisms and from humans in laboratory and clinical situations support this new view of reinforcement. Clinically, Herrnstein's equation requires a broader environmental approach to conceptualizing cases and to applying reinforcement principles in therapy. The equation can help clinicians design treatment regimens more efficiently, and it suggests four new intervention strategies that may be especially useful in difficult cases, for example, when standard procedures are contraindicated or in environments that already supply large amounts of reinforcement.*

During the past two decades, research in basic behavioral science has become increasingly concerned with formal, that is, mathematical, descriptions of behavior. Very recently, a number of fairly complete formalisms have been proposed that account remarkably well for certain features of behavior under specific environmental conditions (e.g., Killeen, 1979; McDowell & Kessel, 1979; Myerson & Miezin, 1980; Rachlin & Burkhard, 1978). The impetus for much of this work was the empirical success of Herrnstein's (1970) mathematical statement of the law of effect.

It is not difficult to find cases in the history of science where a mathematical treatment has led to a substantially improved, and sometimes radically different, understanding of the phenomenon it was meant to describe. The so-called Merton relation¹ in medieval kinematics, for example, was ultimately responsible for laying to rest the Aristotelian doctrine that motion, like color, was an inherent property of a body (Pedersen & Pihl, 1974). In a similar, although perhaps less radical, way Herrnstein's equation has led to an improved

understanding of the effects of reinforcement on the behavior of organisms.

The new view of reinforcement provided by Herrnstein's equation is especially important for contemporary behavior therapy, which relies on reinforcement principles for many of its techniques. This article will explain Herrnstein's mathematical statement of the law of effect and examine its importance for clinical practice. The basic features of the equation will be presented in the first section, followed by a brief review of confirming evidence, with special attention paid to data from human subjects. In the second section, the clinical relevance of the equation will be discussed.

Herrnstein's Hyperbola

Herrnstein's equation is an ordinary hyperbola,

$$R = \frac{kr}{r + r_e} \quad (1)$$

where R is the rate of responding, r is the rate of reinforcement obtained for that responding, and k and r_e are parameters of the hyperbola. Equation 1 specifies the way in which reinforcement governs behavior: Response rate varies hyperbolically with obtained reinforcement rate.

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¹ This is the now familiar "distance equals rate times time" formula.

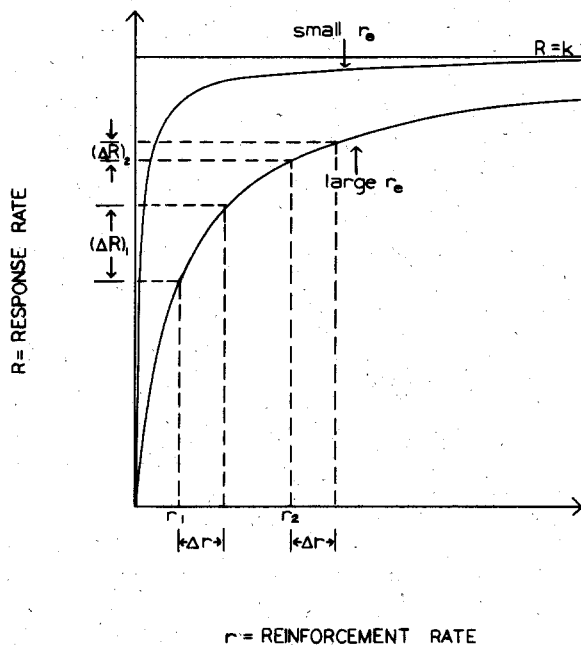


Figure 1. Two plots of the first quadrant branch of Equation 1. Both hyperbolas approach the line $R = k$ asymptotically.

Figure 1 shows two plots of the first quadrant branch of Equation 1. The remaining three quadrants are not shown because the negative rates of responding and (positive) reinforcement that appear in these quadrants have no physical referents. The figure shows that the hyperbola passes through the origin (i.e., when $r = 0$, $R = 0$) and that it increases monotonically and is concave downward in the first quadrant.

The y asymptote is the same for both hyperbolas shown in the figure. This asymptote is, of course, the maximum possible rate of responding, and is given by the parameter k . The line $R = k$ is drawn in the figure. Notice that the units on k are the same as those on R , which typically are responses per minute. From Equation 1 it is clear that response rate approaches its maximum value, k , as reinforcement rate increases. Notice also that if $r_e = 0$, $R = k$. The physical interpretation of k is subject to some dispute (McDowell, 1980), but regardless of its interpretation, this parameter must represent the maximum obtainable response rate for a given response form and a given reinforcer.

The only difference between the two hyperbolas drawn in Figure 1 is the value of r_e , which is larger for the bottom hyperbola. Evidently, this parameter governs the rapidity with which the hyperbola approaches its asymptote. It is generally agreed

that r_e is properly interpreted as "extraneous" reinforcement—that is, all reinforcement that the environment delivers to the organism exclusive of r (de Villiers, 1977). This may include contingent reinforcement that is scheduled for a concurrently available response alternative, "free" or noncontingent reinforcement that is delivered according to some fixed- or variable-time schedule, and any "spontaneous" reinforcement that may accrue to the organism from uncontrolled behaviors such as preening or grooming (Herrnstein, 1970). Notice that the units on r_e must be the same as those on r , which typically are reinforcements per hour. Evidently, the denominator, $r + r_e$, on the right of Equation 1 is the total rate of reinforcement delivered by the environment to the organism. This denominator is sometimes referred to as the environmental context of reinforcement (Herrnstein, 1970).

Figure 1 illustrates two important features of Equation 1. The first is that a given rate of contingent reinforcement supports a higher response rate in a "barren" environment (small r_e) than in a "rich" environment (large r_e). The second is a kind of law of diminishing returns—a given increment in reinforcement rate (Δr) produces a larger increment in response rate (ΔR) when the prevailing rate of contingent reinforcement is low (r_1) than when it is high (r_2).

According to Skinner (1938), "If the occurrence of an operant is followed by presentation of a reinforcing stimulus, the strength is increased" (p. 21). For Skinner, the "strength" of an operant is indicated by its rate of occurrence. In other words, Skinner's view is that contingent reinforcement increases response rate. This view is the basis of every reinforcement procedure used in behavior therapy today. It is evident from Equation 1, however, that Skinner's view does not tell the whole story about the effects of reinforcement on behavior. For example, the equation asserts not merely that response rate increases with reinforcement rate but that it does so hyperbolically. More important for behavior therapy is the equation's assertion that response rate depends not only on the rate of reinforcement obtained for the responding but also on the rate of extraneous reinforcement provided by the environment. Put another way, Herrnstein's hyperbola asserts that responding is governed by contingent reinforcement evaluated relative to all reinforcement provided by the environment.

This important idea of reinforcement relativity may be clarified by rewriting Equation 1 as

$$R = k \left(\frac{r}{r + r_e} \right) \quad (2)$$

In this form, the observed response rate is seen to be some fraction of the maximum possible response rate (k), the fraction being determined by the ratio of contingent reinforcement rate to total reinforcement rate ($r/(r + r_e)$). This means that a given rate of contingent reinforcement may produce a high or a low response rate depending on the context of reinforcement in which it occurs.

It is interesting to note that the view of reinforcement implied by Herrnstein's hyperbola is considerably more "naturalistic," and hence more clearly applicable to naturally occurring human behavior, than is Skinner's earlier view. This is because the hyperbola takes into account concurrently available sources of reinforcement, which are always present in human environments no matter how restricted, whereas Skinner's view does not.

EVIDENCE SUPPORTING THE HYPERBOLIC FORM

The hyperbolic form of Equation 1 has been extensively confirmed and is widely accepted (Herrnstein, 1979). De Villiers (1977) and de Villiers and Herrnstein (1976) have reviewed the ev-

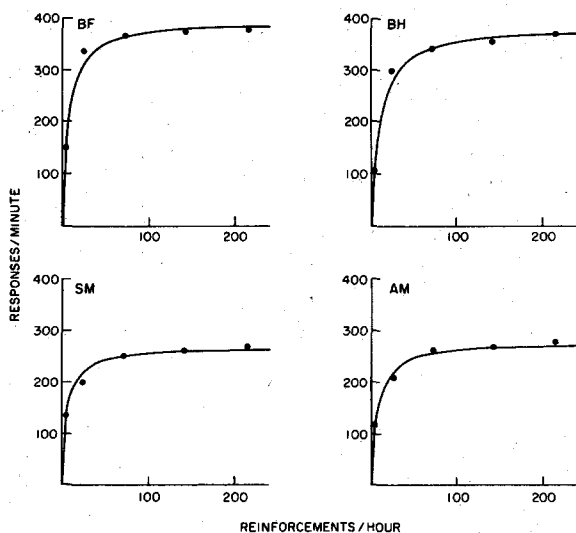


Figure 2. Equation 1 fitted to data from Bradshaw et al.'s (1976) human subjects. See Footnote 2. From "A Multivariate Rate Equation for Variable-Interval Performance" by J. J. McDowell and Robert Kessel, *Journal of the Experimental Analysis of Behavior*, 1979, 31, 267-283. Copyright 1979 by the Society for the Experimental Analysis of Behavior, Inc. Reprinted by permission.

TABLE 1
Fits of Herrnstein's Hyperbola to Figure 2 Data

Subject	k	r_e	Proportion of variance accounted for
BF	399	7	.96
BH	392	10	.98
SM	277	6	.97
AM	287	8	.99

Note. k = maximum response rate, expressed in responses per minute; r_e = extraneous reinforcement rate, expressed in reinforcements per hour. Values are from "Behavior of Humans in Variable-Interval Schedules of Reinforcement" by C. M. Bradshaw, E. Szabadi, and P. Bevan, *Journal of the Experimental Analysis of Behavior*, 1976, 26, 135-141. Copyright 1976 by the Society for the Experimental Analysis of Behavior, Inc. Reprinted by permission.

idence for its applicability to infrahuman behavior. The hyperbolic form for human behavior has been confirmed by Bradshaw, Szabadi, and Bevan (1976, 1977, 1978) in a series of studies of button pressing for money on variable-interval (VI) schedules of reinforcement. Figure 2 shows fits of Equation 1 to data from four of Bradshaw et al.'s (1976) subjects.² The equation was fitted by the method described by McDowell (1981c). The values of the parameters of the hyperbola and the proportion of variance accounted for by the equation are given in Table 1. The small values of r_e are consistent with the barrenness of the experimental environment, which was a small room. Notice that the proportion of variance accounted for by the equation was .96 or higher for every subject.

The fits shown in Figure 2 are typical of Bradshaw et al.'s subjects. Altogether, Equation 1 was fitted to 20 sets of human data (button pressing for money on VI schedules) and was found to account for between 89% and 99% of the individual subjects' data variance, with a mean of 96% of the variance accounted for by the equation. These fits are somewhat better than those for infrahuman VI data. For the 9 sets of data from pigeons and rats reported by de Villiers and Herrnstein (1976), Equation 1 accounted for between 77% and 100% of the individual subjects' data variance, with a mean of 90% of the variance accounted for by the equation. In addition, the range of reinforcement rates for the humans (5-445 reinforcements/hour) was 50% larger than that for the infrahumans (8-300 reinforcements/hour). Bradshaw et al. further showed that the hyperbolic form of Equation 1 for

² Figure 2 actually shows fits of McDowell and Kessel's (1979) multivariate rate equation. McDowell (1980) has shown, however, that the two equations are indistinguishable when fitted to data. The parameters in Table 1 are for fits of Herrnstein's hyperbola.

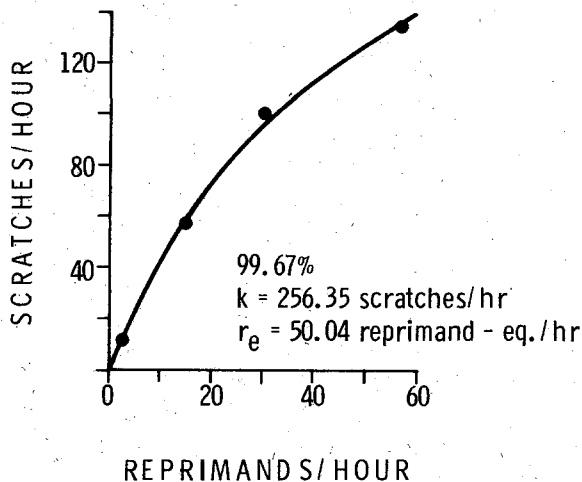


Figure 3. Fit of Equation 1 to data from self-injurious client. The top number is the percentage of variance accounted for by the equation. The estimated value of r_e is expressed in reprimand-equivalents/hour. From "On the Validity and Utility of Herrnstein's Hyperbola in Applied Behavior Analysis" by J. J. McDowell, in C. M. Bradshaw, E. Szabadi, and C. F. Lowe (Eds.), *Quantification of Steady-State Operant Behaviour*. Copyright 1981 by Elsevier/North-Holland Biomedical Press. Reprinted by permission.

human subjects was retained even when reinforcement was arranged for a second response (concurrent schedule), and also when variable-ratio (VR) 34 or VI 170-second schedules of punishment (response cost) were superimposed on the VI schedules of monetary reinforcement.

In an interesting clinical experiment, Szabadi, Bradshaw, and Ruddle (1981) studied the behavior of a 33-year-old woman and a 24-year-old man, both with histories of manic and depressive episodes that required periodic hospitalization. Szabadi et al. arranged monetary reinforcement for button pressing on various VI schedules for these subjects during manic, depressive, and normal phases of their illness. They showed that the response rate versus reinforcement rate function was hyperbolic regardless of mood state. Equation 1 was fitted to eight sets of data from these subjects (one obtained during a manic phase, three during depressed phases, and four during normal phases) and was found to account for between 85% and 99% of the individual-subject data variance, with a mean of 92% of the variance accounted for by the equation. Although the hyperbolic form of the equation for these subjects was independent of mood state, the magnitudes of its parameters were

not. Compared to the magnitudes during normal phases, k was larger and r_e smaller during the manic phase, and k was smaller and r_e larger during the depressed phases. These results suggest that a given reinforcer's "value" (Baum & Rachlin, 1969) or "power" (McDowell, 1980) may be greater than normal during manic periods and less than normal during depressive ones.

The studies by Bradshaw, Szabadi, and their colleagues provide strong support for the hyperbolic form of Equation 1 for human button pressing on VI schedules of monetary reinforcement. McDowell (1981b) extended these findings to naturally occurring human behavior of clinical interest. He reported a case of self-injurious scratching in an otherwise normal 11-year-old boy that was shown to be supported (i.e., reinforced) by contingent verbal reprimands from family members (Carr & McDowell, 1980). Figure 3 shows the boy's rate of scratching plotted against the obtained rate of contingent reprimanding during four 20-30-minute periods of continuous observation conducted in the boy's living room while the family was watching television. The data represent naturally occurring scratch and reprimand rates inasmuch as they were obtained prior to treatment and in the absence of instructions to the family. The smooth curve drawn through the points is Equation 1, fitted by the method described by McDowell (1981c). As noted in the figure, the equation accounts for virtually all of the data variance. Notice that the relatively large value of r_e (cf. Table 1) is consistent with the "richness" of the environment in which the scratching occurred.

EVIDENCE SUPPORTING r_e AS EXTRANEOUS REINFORCEMENT

De Villiers (1977) and Rachlin and Baum (1972) have discussed infrahuman data supporting the interpretation of r_e as all reinforcement that the environment provides exclusive of r . At least one study has confirmed this interpretation of r_e for button pressing in humans (Bradshaw, Szabadi, & Bevan, 1976), and no contradictory evidence has been published. All of these experiments demonstrate that response rate on a given response alternative varies inversely with the reinforcement rate obtained for responding on a second alternative (concurrent schedule), or with the reinforcement rate delivered independent of behavior (free reinforcement). This effect is illustrated in Figure 4, which shows data from two of Bradshaw et al.'s

(1976) human subjects who pressed buttons for money on concurrent VI VI schedules. Subjects on these schedules could press either of two buttons at any time. Reinforcement for pressing the first button was held constant at 70 reinforcements/hour, while reinforcement for pressing the second was varied from 5 to 211 reinforcements/hour. Figure 4 shows plots of the response rate on the first button as a function of the reinforcement rate obtained for responding on the second. The smooth curves drawn through the points represent Equation 1, with r_e as the independent variable, and were fitted by the method described by McDowell (1981c). The figure shows that response rate on the first button varied inversely with reinforcement rate obtained on the second, in the manner required by the equation. It may be worth emphasizing that these data show that responding on the first button depended not only on the reinforcement contingent on that response but also on the reinforcement obtained from another concurrent source.

McDowell (1981b) has reviewed the clinical evidence supporting the interpretation of r_e as extraneous reinforcement. As one example, Ayllon and Roberts (1974) arranged token reinforcement for the reading test performance of five fifth-grade boys. Disruptive behavior during the reading periods was also recorded. Figure 5 shows Ayllon and Roberts' results. The filled circles represent the mean percentage of observation intervals during which disruptive behavior was observed. The open circles represent the mean percentage correct on the reading test. During sessions 9-25 and 41-46, points which could be exchanged for backup reinforcers were given for correct responses on the reading test. This arrangement is a concurrent schedule, with disruptive behavior being the behavior of interest and reading test performance being the second alternative. As can be seen from the figure, when reinforcement for the second alternative (r_e) was increased, responding on the first alternative declined. Any reinforcement accruing to the disruptive behavior is assumed to remain constant across sessions, since no instructions were given to the teacher about responding to the disruptive behavior.

McDowell (1981b) presented additional clinical evidence showing that responding increased when r_e declined and decreased when free reinforcement was made available to human subjects in applied situations. Both effects are predicted by Equation 1 when r_e is interpreted as extraneous reinforcement.³

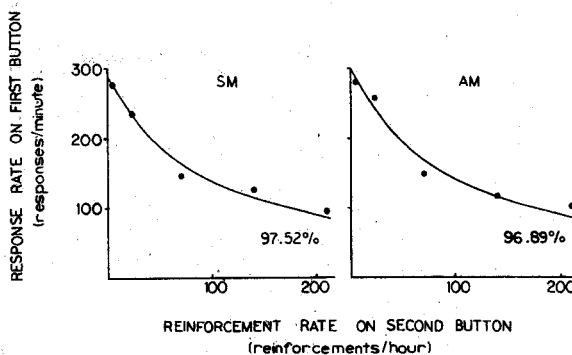


Figure 4. Fits of Equation 1, with r_e as the independent variable, to data from two of Bradshaw et al.'s (1976) human subjects. The percentage of variance accounted for by the equation is given for each fit.

The Clinical Relevance of Herrnstein's Hyperbola

As noted earlier, Herrnstein's hyperbola represents a more advanced understanding of the effects of reinforcement on the behavior of organisms. Skinner's view is of course not contradicted by the equation, which still asserts that response rate is governed by the rate of contingent reinforcement. But the equation improves on Skinner's view by asserting that the response rate versus reinforcement rate function is hyperbolic and by asserting that response rate is also governed by the rate of reinforcement supplied to the organism from all other concurrent sources. The data reviewed above strongly support this new view of reinforcement for human behavior, whether it is the simple button pressing that is studied in the laboratory or the more complicated and clinically interesting behavior that occurs in the natural environment.

CONCEPTUAL ISSUES

Herrnstein's hyperbola evidently requires a new approach to conceptualizing cases behaviorally

³ Several recent mathematical accounts (e.g., McDowell, 1980; Staddon, 1977) suggest that r_e may play a somewhat more complicated role than indicated in this section. One important additional consideration is reinforcer "substitutability" (Rachlin, Kagel, & Battalio, 1980), which is the extent to which reinforcers are alike or can be used interchangeably. In extreme cases of highly nonsubstitutable reinforcers (as, for example, food and water where an increase in free water delivery may result in an increase rather than a decrease in food consumption and hence in responding for food), Equation 1 may require certain modifications. When reinforcers are alike or nearly alike, however, there is little doubt that an inverse relationship between response rate and extraneous reinforcement holds in the laboratory and in the natural environment.

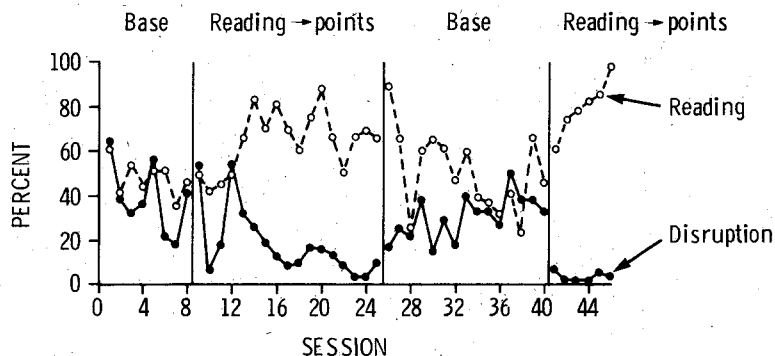


Figure 5. Mean percentage of intervals during which disruptive behavior occurred, and mean percentage correct on reading test for five fifth-grade boys. Reading performance was reinforced during sessions 9–25 and 41–46. From "On the Validity and Utility of Herrnstein's Hyperbola in Applied Behavior Analysis" by J. J. McDowell, in C. M. Bradshaw, E. Szabadi, and C. F. Lowe (Eds.), *Quantification of Steady-State Operant Behaviour*. Copyright 1981 by Elsevier/North-Holland Biomedical Press. Reprinted by permission.

and to applying reinforcement principles in therapy. In particular, the equation requires broader environmental conceptualizations of problem behavior that take into account sources of reinforcement other than the one in direct contact with (i.e., contingent upon) the behavior. Since some of these sources may provide reinforcement for behaviors other than the target behavior, the equation also requires that concurrently available response alternatives be taken into account. This broader view of reinforcement and behavior may make clinical problems that are difficult to understand in strictly Skinnerian terms more comprehensible.

The issue of so-called side effects of reinforcement regimens is an important example of the type of problem that may be clarified by Herrnstein's equation. Both desirable and undesirable side effects of reinforcement regimens have been reported in the clinical literature (McDowell, 1981b). Sajwaj, Twardosz, and Burke (1972), for example, observed an increase in conversation with peers when a boy's conversation with his teacher was put on extinction. An example of an undesirable side effect of treatment is a decline in appropriate collateral behavior when a regimen of positive reinforcement is instituted for a specific target behavior. Neither effect is easily understood in strict Skinnerian terms, since the consequences for social behavior with peers in the one case and for collateral behavior in the other presumably remained the same. But both effects are easily understood in terms of Equation 1. In the first case, the extinction procedure decreased the total amount of reinforcement in the environment. Since reinforcement for social behavior with peers remained the

same, the *relative* rate of reinforcement for this behavior increased, and hence the behavior increased in frequency. In the second case, the reinforcement procedure increased the total amount of reinforcement in the environment and thus decreased the *relative* rates of reinforcement for the collateral behaviors. It is evident that these effects, and others like them, are not necessarily "side effects" of treatment, but may instead be direct consequences of changing the environmental context of reinforcement.

The broader environmental conceptualization of reinforcement and behavior provided by Herrnstein's hyperbola can also lead to more efficient treatment designs. For example, lavish reinforcement procedures may be unnecessary in lean environments, since minimal amounts of reinforcement can support large amounts of behavior when r_e is small. Conversely, reinforcement procedures may be relatively unsuccessful in rich environments, because responding approaches its asymptote only at extremely high reinforcement rates when r_e is large.

INTERVENTION STRATEGIES

The clinical relevance of Equation 1 is not limited to improvements in conceptualization. The equation suggests four new intervention strategies, all of which follow from the equation's assertion that responding is governed by relative rather than by absolute reinforcement rates.

Consider first the problem of reducing the frequency of a problem behavior. The standard treatment procedures are extinction and punishment. Herrnstein's hyperbola suggests two additional in-

terventions: (a) increasing the rate of reinforcement for a concurrently available response alternative and (b) increasing the rate of free or noncontingent reinforcement.

McDowell (1981b) provided an example of the first intervention. He treated a 22-year-old mildly retarded male for oppositional behavior that frequently escalated in intensity and sometimes culminated in assault. The oppositional behavior appeared to be supported by social reinforcement from the client's parents. Extinction was contraindicated in this case because of the likely possibility of assault. Punishment was also ruled out because of the understandable reluctance of the parents to attempt to enforce a time-out contingency. The treatment selected involved introduction of a token economy for behaviors unrelated to the client's oppositional behavior. In other words, the treatment strategy was to increase r_e , which according to Equation 1, should cause the rate of oppositional behavior to decline. The client earned points, which could be exchanged for money, for various personal hygiene, job, and educational tasks. Figure 6 shows the effect of this intervention on the client's oppositional behavior. At baseline, he showed almost daily episodes of escalating oppositional behavior. During the first eight weeks of treatment the behavior declined by about 80% to approximately 1 episode per week. During the next six weeks of treatment, a response cost contingency (point deduction for oppositional behavior) was added to the treatment regimen, but this failed to reduce the oppositional behavior much further.

There are, of course, many variations of this type of intervention. The case described here is especially interesting because the standard behavioral interventions could not be used. Some applied workers may be inclined to refer to this type of intervention as "reinforcing an incompatible response," but the incompatible response rationale is faulty. The argument usually advanced is that the client cannot possibly show the undesirable behavior while executing the incompatible response. The hyperactive child, for example, cannot possibly be out of his or her seat while working on arithmetic problems at a desk. But this sort of incompatibility is trivial, since the child may at any moment stop working on the problems and leave the desk. Similarly, a pigeon on a concurrent schedule cannot peck both keys at the same time but can at any moment stop pecking one key and start pecking the other. Herrnstein's hyperbola suggests that one must arrange the environment

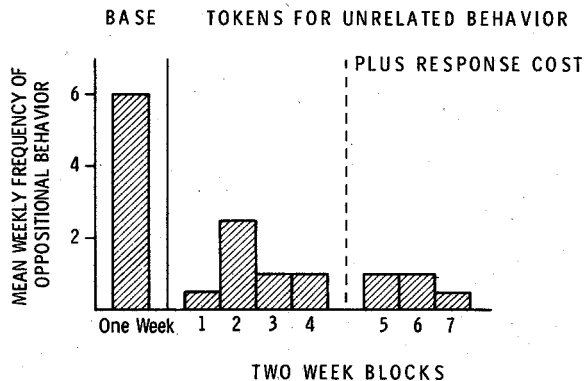


Figure 6. Mean weekly frequency of oppositional behavior when token reinforcement was provided for unrelated behaviors. From "On the Validity and Utility of Herrnstein's Hyperbola in Applied Behavior Analysis" by J. J. McDowell, in C. M. Bradshaw, E. Szabadi, and C. F. Lowe (Eds.), *Quantification of Steady-State Operant Behaviour*. Copyright 1981 by Elsevier/North-Holland Biomedical Press. Reprinted by permission.

such that the relative rate of reinforcement for working on arithmetic problems is higher than the relative rate of reinforcement for being away from the desk. In other words, the equation argues that the important variable is not the incompatibility of the two behaviors but the disparity between the relative rates of reinforcement accruing to them. If the disparity is large and in favor of the desired alternative, then the child will "choose" that alternative more frequently. If the disparity is not large, then the treatment may have no effect even though the two behaviors are incompatible.

The second intervention suggested by Herrnstein's hyperbola—increasing the rate of free or noncontingent reinforcement—may also be implemented in various ways. "Unconditional positive regard" (provided that it is in fact independent of the problem behavior) is an interesting, although behaviorally unconventional, manner of increasing r_e . According to Equation 1 this intervention should reduce the frequency of a problem behavior, although it may also reduce the frequencies of all behaviors concurrently exhibited in that environment.

In addition to suggesting new methods of reducing the frequency of undesirable behavior, Herrnstein's hyperbola can be applied to the problem of increasing the frequency of a desired behavior. The standard treatment is, of course, to increase the rate of contingent reinforcement for the behavior. Herrnstein's equation suggests two additional interventions: (a) decreasing the rate of reinforcement for a concurrently available re-

sponse alternative and (b) decreasing the rate of free or noncontingent reinforcement. Neither intervention need involve the possibly objectionable removal of reinforcement from the environment. Instead, r_e can be decreased by taking reinforcement removed from other sources and making it contingent on the desired behavior. In this way, the reinforcement already available in the environment is simply redistributed. This type of intervention may be especially powerful in rich environments where the simple addition of contingent reinforcement may have little effect. If the added reinforcement comes from elsewhere in the environment, r is increased and r_e is decreased in a single stroke, an intervention that seems likely to be more effective than changing either r or r_e alone.

Discussion

Herrnstein's equation is considerably more descriptive of natural human environments than Skinner's earlier view of reinforcement. It is not always easy to isolate Skinnerian response-reinforcement units in the natural environment. Herrnstein's equation makes efforts to do so unnecessary and, moreover, obsolete. The equation can help clinicians conceptualize cases more effectively and design treatment regimens more efficiently. It also suggests new intervention strategies that may be especially useful in difficult cases (e.g., when standard procedures are contraindicated or in environments that already supply large amounts of reinforcement).

Although Herrnstein's equation improves on earlier views of reinforcement, there are several important features of reinforcement and responding that it does not take into account. For example, the equation does not permit differentiation among types of responses (e.g., self-injurious scratching vs. conversation with peers) or among types of reinforcers (e.g., tokens vs. praise), since it relates only the time rates of occurrence of these events. The problem of response and reinforcer "value," however, is being studied in operant laboratories (Baum, 1973; Baum & Rachlin, 1969; Killeen, 1972; Rachlin, 1971). As one example, McDowell and Kessel (1979) have applied the mathematical theory of linear systems to operant behavior. The theory yields a "transfer function" that relates certain properties of responding to certain properties of reinforcement. Among these properties are quantities that reflect response and reinforcer values. Although this theory has been shown to be

empirically correct in many respects (McDowell, 1979, 1980, 1981a; McDowell & Kessel, 1979; McDowell & Sulzen, 1981), its formal solution of the problem of value has yet to be confirmed. Nevertheless, this and other approaches to the problem of value (e.g., Rachlin & Burkhard, 1978) show promise of being directly applicable to clinical problems (Winkler, 1981), perhaps in the near future.

It is clear that basic behavioral science is moving in the direction of increased formality. It must be encouraging to applied workers to find that the operant formalisms are providing descriptions, not of simpler situations, but of more complex and naturalistic ones. Perhaps this formal work will provide a basis for the continued improvement of therapeutic regimens in behavior therapy.

REFERENCES

- Ayllon, T., & Roberts, M. D. Eliminating discipline problems by strengthening academic performance. *Journal of Applied Behavior Analysis*, 1974, 7, 71-76.
- Baum, W. M. The correlation-based law of effect. *Journal of the Experimental Analysis of Behavior*, 1973, 20, 137-153.
- Baum, W. M., & Rachlin, H. Choice as time allocation. *Journal of the Experimental Analysis of Behavior*, 1969, 12, 861-874.
- Bradshaw, C. M., Szabadi, E., & Bevan, P. Behavior of humans in variable-interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1976, 26, 135-141.
- Bradshaw, C. M., Szabadi, E., & Bevan, P. Effect of punishment on human variable-interval performance. *Journal of the Experimental Analysis of Behavior*, 1977, 27, 275-279.
- Bradshaw, C. M., Szabadi, E., & Bevan, P. Effect of variable-interval punishment on the behavior of humans in variable-interval schedules of monetary reinforcement. *Journal of the Experimental Analysis of Behavior*, 1978, 29, 161-166.
- Carr, E. G., & McDowell, J. J. Social control of self-injurious behavior of organic etiology. *Behavior Therapy*, 1980, 11, 402-409.
- de Villiers, P. A. Choice in concurrent schedules and a quantitative formulation of the law of effect. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior*. Englewood Cliffs, N.J.: Prentice-Hall, 1977.
- de Villiers, P. A., & Herrnstein, R. J. Toward a law of response strength. *Psychological Bulletin*, 1976, 83, 1131-1153.
- Herrnstein, R. J. On the law of effect. *Journal of the Experimental Analysis of Behavior*, 1970, 13, 243-266.
- Herrnstein, R. J. Derivatives of matching. *Psychological Review*, 1979, 86, 486-495.
- Killeen, P. R. The matching law. *Journal of the Experimental Analysis of Behavior*, 1972, 17, 489-495.
- Killeen, P. R. Arousal: Its genesis, modulation, and extinction. In M. D. Zeiler & P. Harzem (Eds.), *Reinforcement and the organization of behavior*. New York: Wiley, 1979.
- McDowell, J. J. Some contributions to the mathematical description of behavior (Doctoral dissertation, State University of New York at Stony Brook, 1979). *Dissertation Abstracts International*, 1979, 40, 1931B-1932B. (University Microfilms No. 79-22,058)
- McDowell, J. J. An analytic comparison of Herrnstein's equations and a multivariate rate equation. *Journal of the Experimental Analysis of Behavior*, 1980, 33, 397-408.
- McDowell, J. J. A gross rate-transfer function for fixed-interval performance. In C. M. Bradshaw, E. Szabadi, & C. F. Lowe

- (Eds.), *Quantification of steady-state operant behaviour*. Amsterdam: Elsevier/North-Holland, 1981. (a)
- McDowell, J. J. On the validity and utility of Herrnstein's hyperbola in applied behavior analysis. In C. M. Bradshaw, E. Szabadi, & C. F. Lowe (Eds.), *Quantification of steady-state operant behaviour*. Amsterdam: Elsevier/North-Holland, 1981. (b)
- McDowell, J. J. Wilkinson's method of estimating the parameters of Herrnstein's hyperbola. *Journal of the Experimental Analysis of Behavior*, 1981, 35, 413-414. (c)
- McDowell, J. J., & Kessel, R. A multivariate rate equation for variable-interval performance. *Journal of the Experimental Analysis of Behavior*, 1979, 31, 267-283.
- McDowell, J. J., & Sulzen, H. M. Dynamic equilibrium on a cyclic-interval schedule with a ramp. *Journal of the Experimental Analysis of Behavior*, 1981, 36, 9-19.
- Myerson, J., & Miezin, F. M. The kinetics of choice: An operant systems analysis. *Psychological Review*, 1980, 87, 160-174.
- Pedersen, O., & Pihl, M. *Early physics and astronomy*. New York: American Elsevier, 1974.
- Rachlin, H. C. On the tautology of the matching law. *Journal of the Experimental Analysis of Behavior*, 1971, 15, 249-251.
- Rachlin, H. C., & Baum, W. M. Effects of alternative reinforcement: Does the source matter? *Journal of the Experimental Analysis of Behavior*, 1972, 18, 231-241.
- Rachlin, H. C., & Burkhard, B. The temporal triangle: Response substitution in instrumental conditioning. *Psychological Review*, 1978, 85, 22-47.
- Rachlin, H. C., Kagel, J. H., & Battalio, R. C. Substitutability in time allocation. *Psychological Review*, 1980, 87, 355-374.
- Sajwaj, T., Twardosz, S., & Burke, M. Side effects of extinction procedures in a remedial preschool. *Journal of Applied Behavior Analysis*, 1972, 5, 163-175.
- Skinner, B. F. *The behavior of organisms*. Englewood Cliffs, N.J.: Prentice-Hall, 1938.
- Staddon, J. E. R. On Herrnstein's equation and related forms. *Journal of the Experimental Analysis of Behavior*, 1977, 28, 163-170.
- Szabadi, E., Bradshaw, C. M., & Ruddle, H. V. Reinforcement processes in affective illness: Towards a quantitative analysis. In C. M. Bradshaw, E. Szabadi, & C. F. Lowe (Eds.), *Quantification of steady-state operant behaviour*. Amsterdam: Elsevier/North-Holland, 1981.
- Winkler, R. C. Target behaviour changes in behavioural economics. In C. M. Bradshaw, E. Szabadi, & C. F. Lowe (Eds.), *Quantification of steady-state operant behaviour*. Amsterdam: Elsevier/North-Holland, 1981.